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The Mathematical Knowledge of Elementary School Teachers: A Comparative Perspective[1]

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This study examines the differences and similarities of mathematics teachers' subject matter knowledge among England, the Chinese mainland and Hong Kong. Data were collected from a ten-item test in the SKIMA subject matter audit instrument [Rowland, T.; Martyn, S.; Barber, P. & Heal, C. (2000). Primary teacher trainees' mathematics subject knowledge and classroom performance. In: T. Rowland & C. Morgan (eds.), *Research in Mathematics Education, Volume 2* (pp. 3–18). ME **2000e.**03066] from over 500 participants. Results showed that participants from England performed consistently better, with those from Hong Kong being next and then followed by those from the Chinese mainland. The qualitative data revealed that participants from Hong Kong and

[1] An earlier shorter version was presented at the Discussion Group 14 of the 11th International Congress on Mathematical Education (ICME-11) at the Universidad Autonoma de Nuevo Leon (UANL), Monterrey, Mexico; July 6–13, 2008.

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the Chinese mainland were fluent in applying routines to solve problems, but had some difficulties in offering explanations or justifications.

Keywords: subject matter knowledge, pedagogical content knowledge, teacher education, cross-cultural comparison, the Chinese learner
MESC Classification: B50
MSC2010 Classification: 97B50

INTRODUCTION

Background

Teacher knowledge is receiving increasing attention in studies of curriculum and pedagogy. It is indispensable to teacher professionalism and crucial to curriculum implementation and educational reforms. In his oft-cited article, Shulman (1986) set out the multi-dimensional nature of teacher knowledge. The term "missing paradigm" was coined, and seven aspects of teacher knowledge were identified, among which the most frequently researched have been "subject matter knowledge" (SMK) and "pedagogical content knowledge" (PCK).

SMK is structured into substantive and syntactic areas (Grossman, 1990; Schwab, 1978), where substantive content knowledge refers to the concepts, principles, laws, and models in particular content areas of science, and syntactic content knowledge refers to the agreements, norms, paradigms, and ways of establishing new knowledge that are held as acceptable (Smith, 1999). In the United Kingdom (UK), specific concerns about elementary teachers' SMK and PCK have been a recurrent theme in reports by the government inspection agency, the Office for Standards in Education (1994; 2000; 2005). More recently, Williams (2008) recommended a national program to nurture specialist teachers of primary mathematics in recognition of the current deficit in teacher knowledge in the subject. This recommendation is already being implemented.

Ma (1999) presented compelling evidence that the adequacy of elementary teachers' mathematical knowledge, for their own professional purposes, cannot by any means be taken for granted.

Recent government initiatives to enhance the mathematical content knowledge (especially SMK) of prospective and serving elementary teachers have been taken in a number of countries. The rather direct approach to tackle the "problem" in England is captured by an edict in the first set of government "standards" for Initial Teacher Training (ITT) issued in 1997:

> All providers of ITT must audit trainees' knowledge and understanding of the mathematics contained in the National Curriculum programmes of study for

> mathematics at KS1 and KS2, and that specified in paragraph 13 of this document. Where gaps in trainees' subject knowledge are identified, providers of ITT must make arrangements to ensure that trainees gain that knowledge during the course. (Department for Education and Employment, 1997, p. 27)

The process of audit and remediation of SMK within primary ITT became a high-profile issue following the introduction of these requirements. Within the UK teacher education community, few could be found to support the imposition of the "audit and remediation" culture, yet the introduction of this "testing" regime provoked a body of UK research on prospective elementary teachers' mathematics SMK (*e.g.*, Goulding, Rowland & Barber, 2002; Jones, Mooney & Harris, 2002; Morris, 2001; Rowland, Martyn, Barber & Heal, 2000).

A team based at the London Institute of Education in London developed a written audit instrument for the diagnostic assessment of prospective elementary teachers' SMK (Rowland *et al*, 2000). They also investigated the relationship between student teachers' SMK (as assessed by the audit instrument) and their mathematics teaching competence in classroom (as assessed on a three-point scale by a small, cohesive team of experienced teacher educators). A chi-square test of the grouped data showed that the association between audit score and teaching performance is significant ($p < 0.05$). This finding turned out to be robust when the study was replicated with a different cohort of student teachers (Rowland, Martyn, Barber & Heal, 2001). Students obtaining high (or even middle) audit scores were more likely to be assessed as strong numeracy teachers than those with low scores; students with low audit scores were more likely than other students to be assessed as weak numeracy teachers.

In 2000, the London-based team joined with researchers in Cambridge, York, and Durham to form an ongoing consortium named SKIMA (subject knowledge in mathematics).

Educational and curriculum reforms in different parts of the world have similar concerns and face similar challenges (Wong, Han & Lee, 2004). Given that students in Eastern educational regions such as the Chinese mainland and Hong Kong[2] do better than their counterparts in international studies like TIMSS (Trends in International Mathematics and Science Study) and PISA (Programme for International Student Assessment), it would be interesting to investigate the differences and similarities in teachers' SMK in different regions (Wong, 2004). This is the purpose of the present study. The three regions have particular interest since they situates at different points in the Eastern-Western spectrum. Obviously with the Chinese mainland and the UK at different

[2] The terms "the Chinese mainland" and "Hong Kong" were usually used to distinguish between them. Since Hong Kong is a Special Administrative Region of China and is not a country, we use "region" all through.

ends, Hong Kong, being an Eastern regions strongly influenced by Western culture, situates somewhere in the middle.

Teacher Education Systems in the Three Regions

In the UK, after their undergraduate years, the majority of prospective elementary teacher trainees follow a one-year, full-time course leading to a Postgraduate Certificate in Education (PGCE) in a university education department, with about half of the year being working in a school under the guidance of a school-based mentor. All elementary teacher trainees are trained to be generalist teachers of the whole elementary school curriculum. Later in their careers, most take on responsibility for leadership in one curriculum area (such as mathematics) in their school, but, almost without exception, they remain generalists, teaching the whole curriculum to one class.

In the Chinese mainland, teachers were prepared by teacher education departments in universities and in teacher training colleges. According to an ordinance issued in 1995, all teachers should be licensed. The basic qualifications for elementary, junior secondary, and senior secondary teachers are passes of the relevant subjects at secondary, tertiary, and university levels respectively. A more comprehensive teacher education system was established in 2000. Expert teachers are also rewarded with the grades of advanced rank and special rank.

Hong Kong is a Special Administrative Region of China. While over 99% of Hong Kong citizens are Chinese, they were much influenced by Western culture during the British colonial governance for over a century. Previously, most elementary school teachers were not university graduates. On the other hand, those who obtained a first degree (in mathematics or related topics) were eligible to teach (mainly secondary) school mathematics. The situation has improved greatly starting from the mid-1990s. At present, around 90% of school teachers are university graduates, and most of them went through teacher education programs (*i.e.*, having Teacher Cert., PGDE or PGCE3).[3] The idea of "having subject specialists teaching the subject" was also proposed. It is envisaged that all teachers teaching mathematics would have a substantial qualification in mathematics in the future.

Research Questions

The aims of the present study are to:

a. Investigate how well mathematics teachers who were brought up in the Eastern educational systems of the Chinese mainland and Hong Kong are equipped with SMK

[3] Teacher Cert. stands for Teacher Certificate; PGDE stands for Postgraduate Diploma in Education; and PGCE stands for Postgraduate Certificate in Education.

when compared with their Western counterparts in England; and

b. Find out some features of responses to SMK items among mathematics teachers in the Chinese mainland and Hong Kong.

METHODOLOGY

Instrument

Items relevant to the mathematics curricula in the Chinese mainland and Hong Kong were chosen from the SKIMA subject matter audit instrument (Rowland *et al*, 2000), which consists of 10 items in the three categories of basic arithmetic competence (**Category I**: Items 1–4), mathematical exploration and justification (**Category II**: Items 5–8), and geometric knowledge (**Category III**: Items 9–10). The items are listed below.

Item 1. Arrange the following numbers in order from the largest to the smallest.

$$0.203, \text{ two hundredths, } 2.19\times10^{-1}, \ 0.026, \ \frac{2}{9}$$

Item 2. Use any written method to multiply 63 and 37. Does your method use the distributive law? If so, explain how.

Item 3. Work out 2915 ÷ 14 without using a calculator. Show your method and give your answer in remainder form.

Item 4. In a supermarket, there are two brands of washing powder on offer:

Economy: 2.1 kg per box for \$35.00; Standard: 840 g per bag for \$13.40

Which brand of washing powder is cheaper? Explain how you reached your decision.

Item 5. Check that: $3 + 4 + 5 = 3 \times 4$, $8 + 9 + 10 = 3 \times 9$, $29 + 30 + 31 = 3 \times 30$.

Write down a statement (in words) which describes the generalization behind these three examples. Express your generalization using symbolic (algebraic) notation.

Item 6. Three numbers are written in the bottom row of a pyramid as shown in Figure 1.

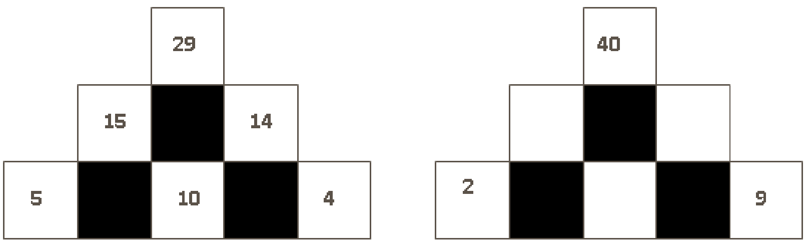

Figure 1. Item 6

Each number in the rows above is the sum of the two numbers directly below it. All the numbers in the pyramid on the left of Figure 1 have been filled in for reference. Find the missing number in the bottom row of the pyramid on the right of Figure 1. Show how you did it.

Item 7. 120 square tiles can be made into a rectangular mosaic. The sides of each tile are 1 cm. The shape of the rectangle can vary. For example, it might be 10 tiles by 12 tiles. State whether each of the following three statements is true or false. Justify your claims in an appropriate way.

(a) The perimeter (in cm) of every such rectangle is an even number.
(b) The perimeter (in cm) of every such rectangle is a multiple of 4.
(c) No such rectangle is a square.

Item 8. In Figure 2, the number in each rectangle is the sum of the two number s in the circles at either end of the line segment through the rectangle.

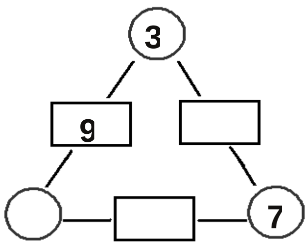

Figure 2. Item 8

(a) Calculate the sum of the numbers in the 3 rectangles.
(b) Calculate the sum of the numbers in the 3 circles.
(c) State the relationship between the sums in parts (a) and (b). Will this relationship hold if there were different numbers in the 3 circles? Justify your answer.

Item 9. Find the perimeter and area of the parallelogram drawn in the square grid below (each square presents a square of length 1 cm) (Figure 3). Explain your methods.

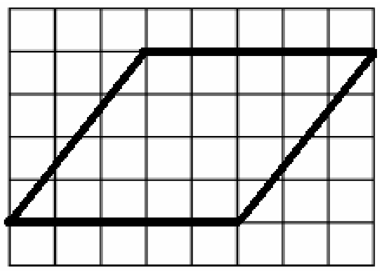

Figure 3. Item 9

Item 10. The shape labeled X is drawn on the coordinate grid shown below (Figure 4).

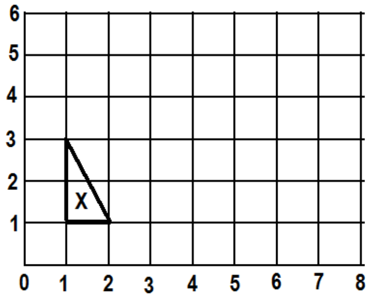

Figure 4. Item 10

On the same coordinate grid, draw the positions of the shape X after the following transformations.

(a) Translation through $x \to x + 5$, $y \to y + 5$; label this shape P.
(b) A reflection in the line $y = 3$; label this shape Q.
(c) A clockwise rotation through 90º about (4, 3); label this shape R.
(d) An enlargement with center at the origin and scale factor 2; label this shape S.

Besides these quantitative (scores) and qualitative (respondents' workings) data, demographic data were also collected.

Participants

Both pre-service and in-service teachers from the Chinese mainland and Hong Kong were invited to participate in the study. In the Chinese mainland, data were collected from two large industrial cities, Changchun and Guangzhou. In all these regions, the whole cohorts of teacher education programs were invited to participate in the study. Thus, in the UK, all students in a typical PGCE program in an institution in Southern England participated. In Hong Kong, the participants comprised seven complete classes in the two teacher education institutions that prepare elementary mathematics teachers. For the Chinese mainland, the participants comprised whole classes from teacher education programs in one institution each from Changchun and Guangzhou. Although they are typical elementary teacher education programs in each region and student participation in the study was wholesale, it cannot be claimed that each sample "represents" a national population of pre-service elementary students. Nevertheless, because the sampling methods were open and comprehensive, we suggest that these three groups offer a basis for a meaningful comparison of the mathematics SMK of such students in the three regions. The characteristics of the participants are shown in Tables 1 and 2.

Table 1. Participants of the Study across the Three Regions

	Hong Kong	Chinese mainland	England
Pre-service	88	79	149
In-service	70	119	0
Total	158	198	149

Table 2. Gender of the Participants

	Hong Kong	Chinese mainland	England
Male	41	68	25
Female	106	126	116
Unidentifiable	11	4	8

Scoring

The general scoring principles developed by Rowland *et al* (2000; 2001) were employed (Table 3). A specific marking scheme was also developed for each item. Sample scripts were marked and markers came to a consensus on the details of the markings. Marks for each item ranged from 0 to 4. Participants who obtained a correct answer with a partial explanation were awarded 3 marks. Participants who did not get the correct answer or gave no explanation were awarded 0 to 2 marks depending on their workings. If participants got 3 or 4, their mastery of SMK was deemed "secure."

Although the question items were marked by different persons, inter-rater consistency was secured through double-checking by members of the research team. In general, the discrepancies were less than 1 mark. Teachers' workings were also analyzed qualitatively.

Table 3. Scoring Rubrics of Items

Score	Mastery of SMK	General scoring principles
0	Insecure	Not attempted, no progress toward a solution
1	Insecure	Partial and incorrect solution
2	Insecure	Correct in part, incorrect in part
3	Secure	Correct solution with small errors, explanations acceptable but not completely convincing
4	Secure	Full solution with convincing and rigorous explanations (not necessarily using algebra)

STATISTICAL ANALYSES OF THE RESULTS

Mastery of SMK in Various Regions

The percentages of participants in the three regions who secured mastery of each of the 10 audit items were calculated and are shown in Table 4.

Table 4. Percentages of Student Teachers Showing Secured Mastery of Various SMK Items

Item	England *N* = 149	Hong Kong *N* = 158	Chinese mainland *N* = 198
1	**83.2**	69.6	68.2
2	**68.4**	24.7	27.4
3	79.8	**81.0**	57.0
4	77.2	**88.0**	71.7
5	**72.5**	41.8	34.8
6	**69.8**	67.8	65.6
7	47.0	**56.3**	39.4
8	68.5	**72.2**	63.1
9	44.3	**69.6**	54.5
10	**38.3**	20.9	37.9

Note: Figures in boldface are the highest percentages among the three regions.

Results revealed that, in general, the participants in the three regions performed well in Items 1, 3, 4, 6, and 8. Though Items 1–4 all concerns basic arithmetic, Item 2 requires explanations of the procedures involved. Items 6 and 8 concern number patterns in which one can make use of the assistance of a diagram. Over 50% secured mastery in these items in each of the three regions. In some of them, the percentages were higher than 80. This indicates that these types of items were not a challenge for the participants.

As for other items, their performances were least satisfactory in Item 10, which concerns geometric transformation. The performances in Items 7 and 9 were not as poor, but the percentages of secured mastery were only around 39% to 69%. However, the actual discrepancies of the percentages were not too large. They were 16.9%, 25.6%, and 17.4% for Items 7, 9 and 10 respectively.

As a whole, in some items, participants from England had a higher percentage of secured mastery, and those from Hong Kong had higher ones in the others. Those from the Chinese mainland had the lowest percentage in general, though we are fully aware that they cannot be viewed as a representative sample from the Chinese mainland. Virtually it is not realistic to expect a representative sample from such a big country. Yet the discrepancies between those from England and their counterparts were notably big for Items 2 and 5. Both required respondents to explain or justify their working or arguments.

We performed further analyses by stratifying the pre-service and in-service participants. The percentages of secured mastery in these groups are given in Table 5.

Table 5. Percentages of Student Teachers Showing Secured Mastery of Various SMK Items, With Pre-service and In-service Separated

Item	Pre-Service			In-Service	
	England N = 149	Hong Kong N = 88	Chinese mainland N = 79	Hong Kong N = 70	Chinese mainland N = 119
1	**83.2**	75.0	79.8	62.9	60.5
2	**68.4**	23.9	50.7	25.7	11.8
3	79.8	**86.3**	64.6	74.3	52.1
4	77.2	**88.6**	87.3	87.2	61.3
5	**72.5**	46.6	40.5	35.7	31.1
6	69.8	81.8	**82.3**	50.0	54.6
7	47.0	**57.9**	32.9	54.3	43.7
8	68.5	**78.4**	67.1	64.3	60.5
9	44.3	**76.1**	56.9	61.5	53.0
10	38.3	31.9	33.0	7.2	**41.2**

Note: Figures in boldface are the highest percentages among the three regions.

It is clear from Table 5 that the results were quite consistent with the general picture as depicted in Table 4, except for Items 2, 6, and 10. Item 2 drew our attention, in which our in-service participants from the Chinese mainland scored particularly low. The low percentage of secured mastery of participants from the Chinese mainland as a whole can then be attributed to these practicing teachers. The percentage among the pre-service teachers from the Chinese mainland was not that low. So, if we confine ourselves to pre-service participants, the results shown in Table 5 are more or less the same as those shown in Table 4.

To take a closer look, we make further analysis by considering the performances in the three categories. Results revealed the overall performance across all regions were highest for Category I and least well for Category III. Participants from England performed consistently better, with those from Hong Kong being next and then followed by those from the Chinese mainland. Yet the discrepancy was smallest for Category III. In fact, participants from the Chinese mainland performed better in that category than those from Hong Kong and the performance of these Chinese mainlanders was very close to that from England (Figure 5).

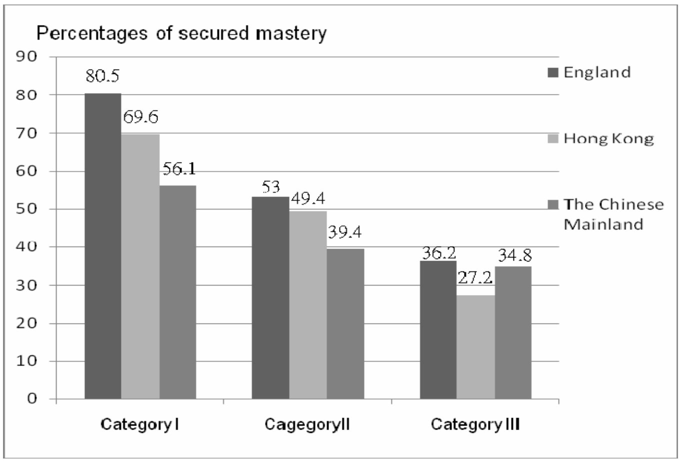

Figure 5. Percentages of Student Teachers Showing Secured Mastery of Each Category in Different Regions

Again we separate the in-service from the pre-service teachers. It is interesting to note that if we put the in-service teachers aside, the performances of the partic ipants from the three regions were more or less the same. The weak performance of the in-service Hong Kong participants can be attributed to their low scores (per centage of secured mastery) in geometry and the weak performance of the in-servi ce Chinese mainland participants can be attributed to their comparative low scores in arithmetic computation (Figure 6).

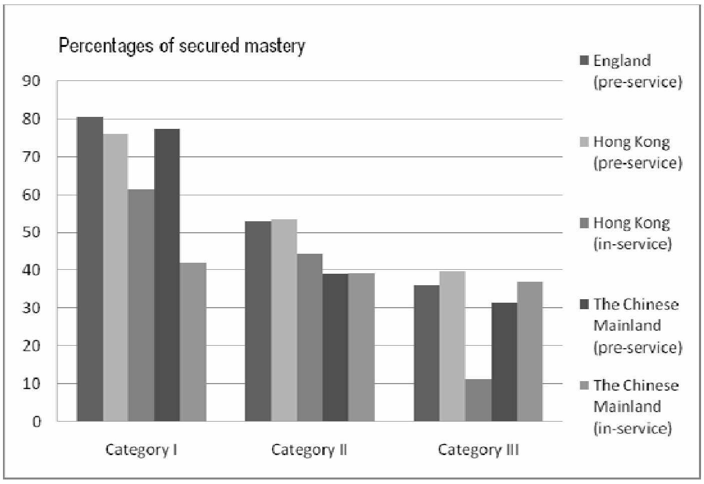

Figure 6. Percentages of Secured Mastery of Each Category among Pre-service and In-service Teachers in Different Regions

ANALYSES OF PERFORMANCE AND THE PROBLEM-SOLVING STRATEGIES

While the above painted a picture in broad strokes the strengths and weaknesses of elementary mathematics teachers in these regions who participated in the study, we also made some observations on the participants' working. Some common features were identified.

For Item 1, over 90% of the Hong Kong participants attempted to convert some of the given numbers into their equivalent decimal values before ordering them, while over 57% of the participants from the Chinese mainland did not show any working and arrived at the answer simply by inspection of the given numbers. However, a small portion of the participants (7%) made a mistake during the conversion and took the value of

$$\frac{2}{9} = 0.\dot{2}$$

as 0.2, thus resulting at an incorrect ordering (Figure 7).

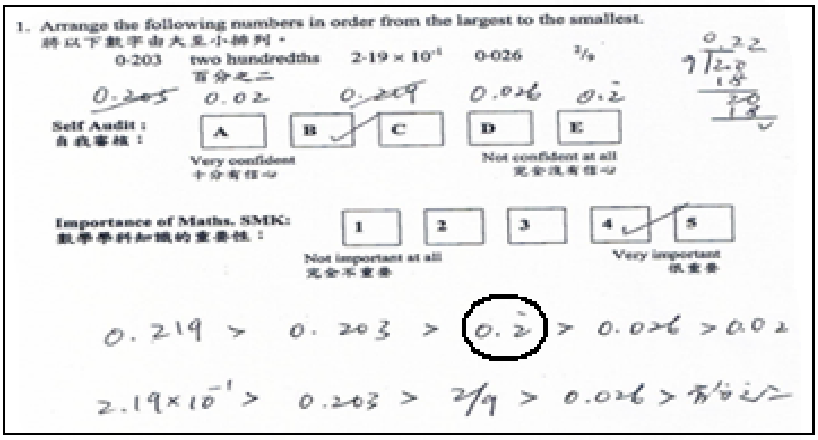

Figure 7. Mistake in Item 1: Error Generated During the Conversion

For Item 2, over 69% of the participants (participants from Hong Kong and the Chinese mainland pooled together) did not respond to whether the distributive law was used. There are a number of possibilities. Not knowing the term "distributive law" is one. Another more serious possibility is that the participants only knew "how" but were unable to state "why." Even worse, over 7% of the participants claimed that no distributive law was used.

For Item 3, almost all participants who got the right answer employed the long division method, with only a few participants from the Chinese mainland used “chunking” or “partitioning of the dividend” to carry out the division (Figure 8). Many participants used the “standardized” way of tackling this item. It is worth noticing that a few (3%) participants carried on the division after the decimal point and thus ending up with a decimal number as the answer (Figure 9).

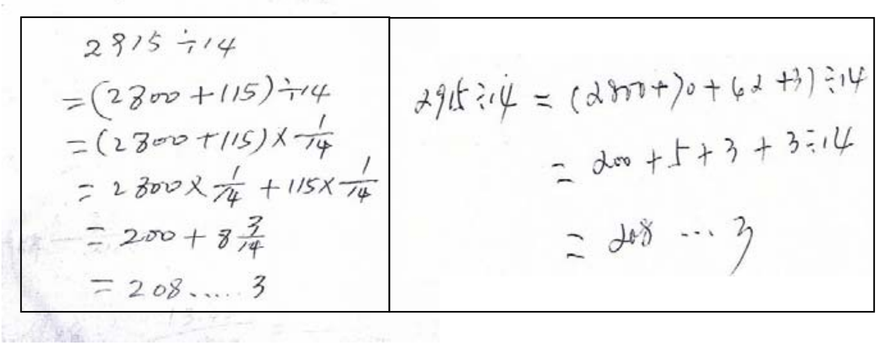

Figure 8. Tackling Item 3 in Non-standardized Ways

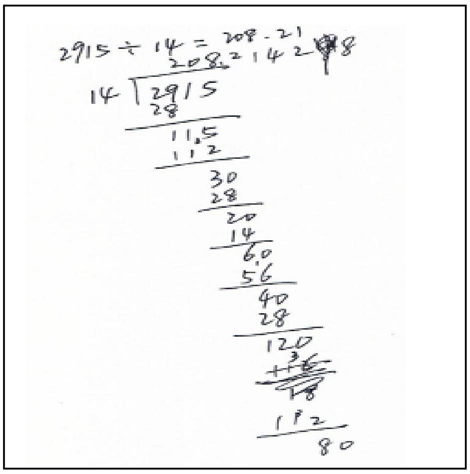

Figure 9. Continued With the Division after the Decimal Place

For Item 4, over 76% of the participants noticed the inconsistency of the units and converted the prices into a common unit (*e.g.*, dollars per kilogram or grams per dollar) first before making the comparison. Only 12% of the participants were flexible or original in their approach, for example, by comparing \$13.4 with

$$\$\frac{35}{10} \times 4 = \$14 .$$

This suggests that most participants looked for a standard method of some kind rather than seeking a creative, non-standard solution.

For Item 5, more than 71% of the participants could come up with the relation "$n+(n+1)=(n+2)=3(n+1)$" (or in similar form), but not as many could express it in words. Only around 46% did it. Some 13% of the participants provided justifications (for instance "$a-1+a+a+1=3a$"), and 7% offered what Rowland *et al* (2000) said of "partial algebraic attempts." This again shows that though the participants could be familiar with routines, many have trouble with providing justifications. They are not used to expressing mathematics by common language too. This may be due to the common conception of mathematics as a subject of "calculables" found in earlier studies (Wong, Marton, Wong & Lam, 2002).

Let us turn to Item 6. More than 58% of the participants tackled the problem by setting algebraic equation(s) (Figure 10). It is worth noticing that more participants from Hong Kong (16%) than the Chinese mainland (1%) adopted the try-and-error method (Figure 11). Only 7% of the participants tried to search for a pattern (Figure 12).

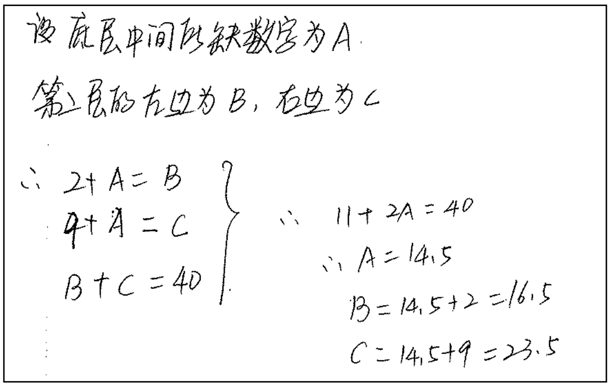

Figure 10. Setting up of Equation Is a Common Technique

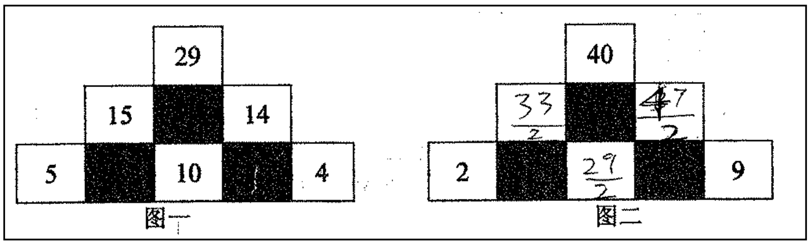

Figure 11. Try-and-error

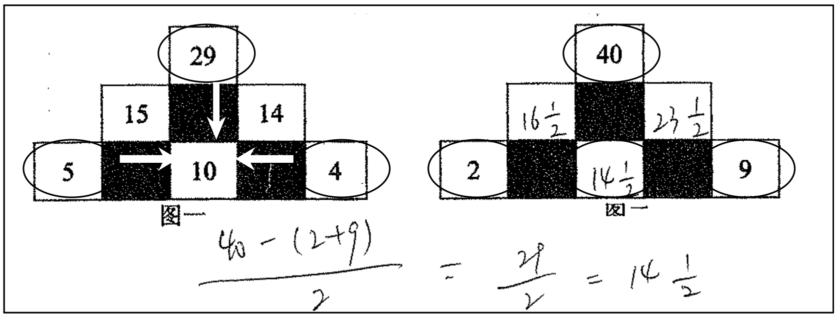

Figure 12. Search for a Pattern

For Item 7, more participants from Hong Kong (40%) than the Chinese mainland (7%) solved the problem by exhausting all possible cases. On the contrary, over 70% of the participants from the Chinese mainland adopted the logical reasoning strategy to deal with the problem. However, some got the wrong answer due to assuming unwarranted assumptions. Figure 13 shows such a case from a Hong Kong participant.

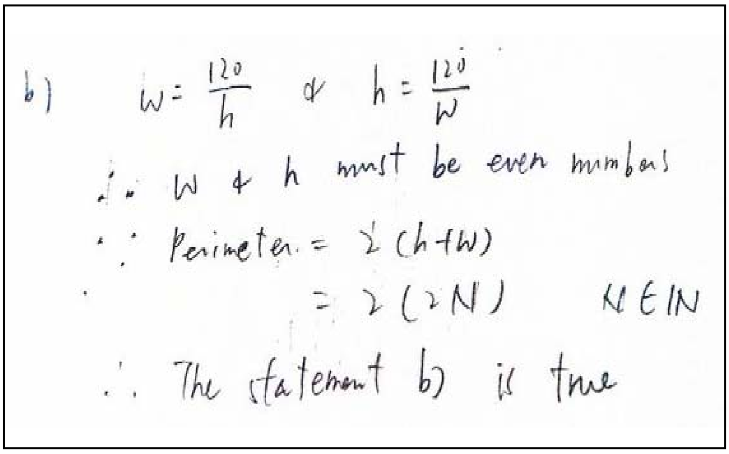

Figure 13. Wrong Answer Obtained From Unwarranted Assumptions

As for Item 8, over 60% of the participants were able to offer a correct justification. Their preferences in using symbols or words vary. Around 33% of them used both words and symbols, while around 29% used only words and 14% used only symbols.

For Item 9, more participants from Hong Kong (76%) than the Chinese mainland (48%) quoted Pythagorean Theorem in the calculation of the perimeter. However a considerable amount (10%) of the Hong Kong participants were confused with the conservation of area through cut and paste and took for granted that it applies to perimeters as well (Figure 14), thus resulting in the wrong answer (Figure 15).

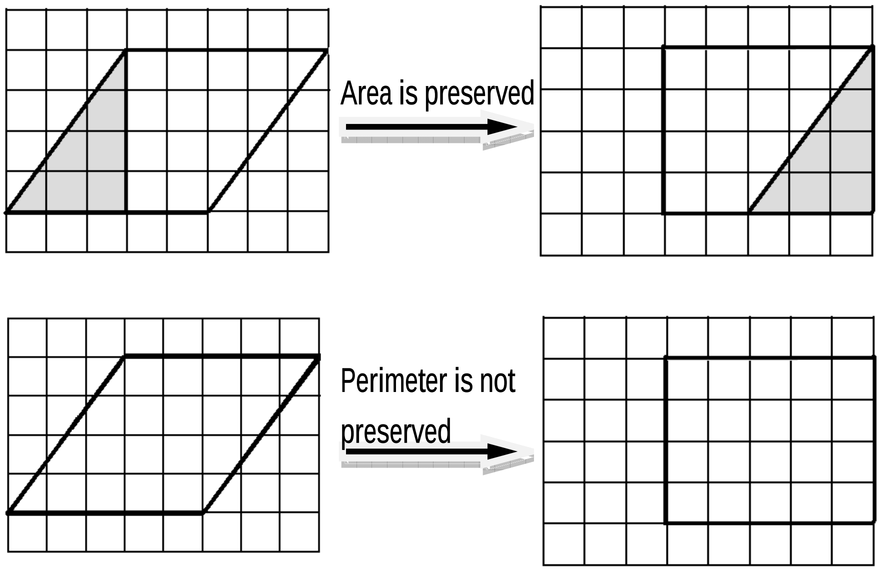

Figure 14. Wrong Conservation Assumptions

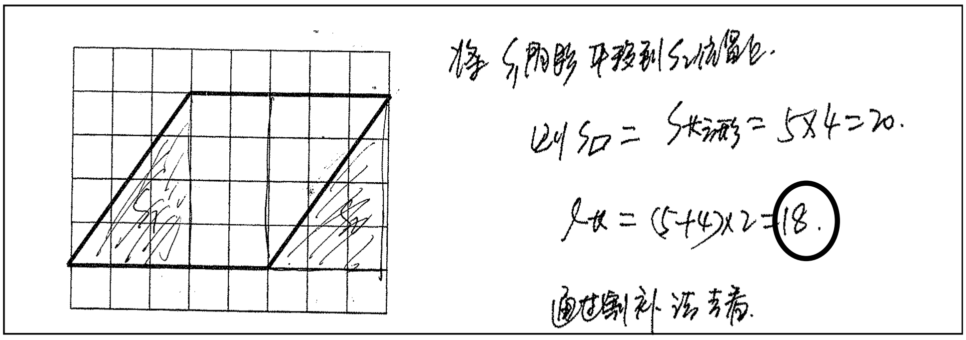

Figure 15. Erroneous Methods Resulting in Wrong Answers

Over 33% of the participants from the Chinese mainland missed the units in the answers and 7% of the Hong Kong participants mixed up the units. They took “cm” for area and “cm2” for perimeter. Even worse, a few of the participants (from Hong Kong and the Chinese mainland inclusive) mixed up the different measuration formulas, namely the area formulas for triangles, parallelograms, and trapezia (Figure 16).

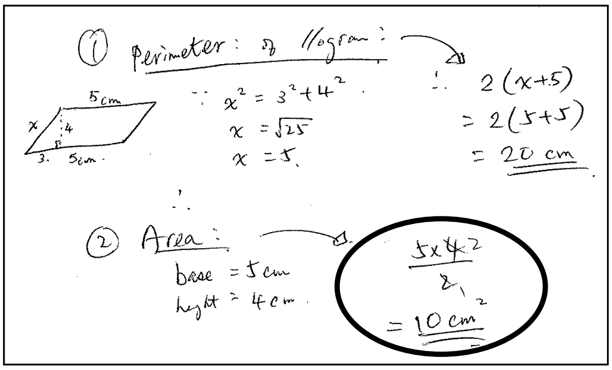

Figure 16. Mixing up of Formulas

Finally, for Item 10, first of all, the attempt rate was low (about 16%). One possibility is that the participants became tired toward the end of the test. Another possibility is that such contents on geometric transformations are not covered in the current mathematics curriculum both in Hong Kong and the Chinese mainland. For the various parts, the correct attempt rate for reflection, translation, rotation, and enlargement were 50%, 49%, 35%, and 20% respectively.

SUMMARY

It is a common understanding that one does not compare for comparison's sake (Wong, 2009). By contrasting a region with another, we understand that region more. That could provide us with food for thought and ideas for improvement. Naturally there are limitations of the study. First of all, the performances of the Eastern regions were checked across with an instrument developed in the West. However, the audit items designed by the English team were quite universal and these items can precisely help us to see whether the participating mathematics teachers in the Eastern regions reached a reasonable standard as compared with their Western counterparts. Second, as mentioned earlier, though the Hong Kong participants do provide a cross-sectional snapshot of the region, it is impractical to look for a representative sample of the Chinese mainland in view of its large population. Yet we can still make sense of the data as it can let us know the strengths and weaknesses of those participants from the Chinese mainland. Teacher educators and policy makers can reflect on how one can strengthen teacher education programs. For instance, in the UK, those who scored not enough marks (less than 3) on an item were directed to undertake further study on that topic. This is one way how assessment can enhance learning.

Results in the present study revealed that, by and large, the Eastern participants scored lower than their Western counterparts in England. The scores of those from the Chinese mainland were particularly unsatisfactory, especially for those practicing teachers. As said, we should not conclude here that this reflect the general situation, but at least we can say that, despite the strong teacher education system in the Chinese mainland, it is still possible to find a group of teachers that need improvement.

Having said thus, if we put the in-service teachers aside, the performances of the teachers from the three regions were more or less the same. So it seems that the challenge lies in the in-service rather than the pre-service teachers. In particular, those from the Chinese mainland need to strengthen their arithmetic computation fluency and those from Hong Kong their geometric knowledge.

As we investigated the actual workings of the participants, we found that though they did not encounter difficulties in handling problems that concern basic arithmetic, it could be a challenge for them when they are asked to provide explanations. Geometric transformation is another challenge. As for problems concerning number patterns, the participants performed better when there is a diagram provided to assist their thinking. In general, they are more used to tackle routine problems which have a standard way to follow.

Some aspects of the responses of the participants in this study contrast interestingly with those of the English participants to the same items in the earlier studies Rowland *et al*, 2000; 2001). For example, the chunking-type responses to the division in Item 3 are potentially of great interest to UK mathematics educators, who learned this holistic approach to division from their European neighbors in the Netherlands. It appears that this same algorithm, with only superficial differences, is in use in the Chinese mainland. Their work shows sophisticated forms of holistic management of the dividend, and quite explicit use of distributivity (*i.e.*, $(a+b) \div c = a \div c + b \div c$) in the management of the division. Such responses would certainly interest UK-based teacher educators, and provoke enquiries about the place of this chunking-type algorithm in the Chinese elementary curriculum. Given that these chunking methods have only been introduced in the UK since 2000, the pre-service teacher education participants in the English survey would not have learned them at school. However, they did learn chunking in their teacher education program, and a few used it well in their audit responses. By contrast, the standard division algorithm was applied instrumentally, and sometimes erroneously (*e.g.*, with answer “28 the quotient and 3 the remainder”).

The commonalities across the East-West participants’ responses are, however, the most striking. The data suggest that difficulties in understanding and applying some areas of elementary mathematics are commonplace in all three countries. For example, the role of distributivity in multiplication algorithms is not well-understood (Item 3); standard procedures are followed in preference to insightful solutions (Item 4); use of algebraic notation lacks sophistication (Item 5); quasi-logical arguments are presented (Item 7); lengths of diagonal lines are misread and conservation falsely applied (Item 9); understanding of geometrical transformations is rare (Item 10).

From the perspective of anxieties about national "failure" fuelled by comparative studies, this is reassuring. However, from the perspective of improving the learning experiences of pupils worldwide, it is cause for concern.

The above offered a general picture showing to what extent mathematics teachers are equipped with adequate mathematics to teach, and the methods they bring to bear on the items presented to them. The qualitative data further revealed that participants from Hong Kong and the Chinese mainland were fluent in applying routines to solve problems, but had some difficulties in offering explanations or justifications. Their ability in "knowing how" is stronger than that in "knowing why." We notice that these "Eastern" participants were more inclined to employ mechanical means like setting up of equations rather than inspections or other informal strategies. Though the use of formulas plays a central role in such routines, it is quite surprising that some participants mixed formulas up. This led to the failure in arriving at the correct answer.

CONCLUSION

Heddens (1997) summarized students' way of solving problems as a routine of "memorizing facts → applying algorithms → following memorized rules → calculating a result." In a prior study (Wong, Lam & Wong, 1998), it was found that the common strategies employed among Hong Kong students to solve mathematical problems was to: first identify what was given and what are being asked; then by picking up key words or other mathematical or non-mathematical clues, identify to which topic (or chapter in the textbook) the problem situations belong; and finally by narrowing down the search of the formulas to that particular chapter, start solving the problem by imitating what the teacher did in class. This is obviously not a desirable situation.

Yet students' learning outcome, by and large, is a result of the learning experience shaped by the teacher. If the teacher possesses a narrow conception of mathematics and at the same time tends to solve mathematical problems by memorizing routines, it is unlikely that her/his students can have a rich mathematics experience during their learning.

Prior research in the Hong Kong context did reveal that the "lived space" shaped by the teacher is a narrow one, in which every problem in the mathematics classroom has a unique answer, has only one way of tackling, and can be solved within minutes (Wong, Marton, Wong & Lam 2002). This has direct impact on not just the students' problem solving strategies but their conceptions of mathematics too (Wong, Lam & Wong, 1998; Wong, Marton, Wong & Lam 2002).

In the present study, we see again a striking resemblance between the approach of tackling mathematical problems among teachers and that among students (as found in previous studies). As mentioned above, both try to search and apply routines to tackling these problems. If one relies on rote-memorization and blind application of rules, there is not much connections among formulas. More seriously, once they mixed up these formulas, the whole problem-solving procedure collapses.

Several studies show that the major reference for teachers in their teaching is their own experience when they were students (Fosnot, 1996; Goodlad, 1990, Kagan, 1992). If they were brought up in a confined "lived space" in which solving mathematical problems is no more than the search and application of routines, when they become teachers, they will impose the same thought and approach on their students. Inevitably, this is a vicious circle and the reverse of it again lies in education. On the one hand, a new bred of mathematics teachers is needed. These teachers possess professional knowledge, and inspire students with a genuine problem solving environment. On the other hand, teacher education programs should contribute a lot in nurturing a new generation of professional teachers. The present study helps teacher educators and policy makers to address the shortcomings of teachers as reflected by the audit items. We believe that this is only a starting point. More weaknesses about mathematics teachers can be diagnosed with more such items. By addressing to these weaknesses, it is possible to come up with a stronger mathematics teacher profession.

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