Economic Reliability Group Acceptance Sampling Based on Truncated Life Tests Using Pareto Distribution of the Second Kind

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Abstract

Economic Reliability test plans(ERTP) are proposed considering that the life time of the submitted items follow the Pareto distribution of the second kind. For various specified acceptance number, sample size and producer's risk, a minimum test termination time is obtained. A comparison of proposed plan has been made with the existing plan developed by Aslam *et al.* (2010). The results are explained by tables and example.

Keywords: Grouped data, reliability test plan, minimum sample size, truncated life test, Pareto distribution of the second kind.

1. Introduction

The acceptance sampling plans are essential tools for the inspection of the final product. The different types of acceptance sampling schemes are available in practice for the testing purpose. The main advantage of these schemes is to accept or reject the submitted product. The ordinary acceptance sampling scheme is used when the experimenter has the facility to test one item at the time. For example, if an experimenter wants to test 100 items then they need 100 testers to equip this product. So, this approach needs more time and testers to install this product. On the other hand, group acceptance sampling plans are used when the experimenter has the facility to install more than one item at the same time in a single tester. This will require less time and effort to install the items in the testers. To test the 100 items in group case, for example, the experimenter has the facility to install 10 items in single testers. Then he needs 10 testers to test the 100 items. The other advantage of the group acceptance sampling plan is that it provides the strict inspection before it can be sent for consumer's use. For more details, about the group acceptance sampling plan based on truncated life tests, the reader can refer to Aslam and Jun (2009). It is important to note that whatever the type of sampling plan, the producer's risk and the consumer's risk are always attached with the sampling plan. The probability of rejection of a good product is called the producer's risk and the probability of acceptance of a bad product is termed as the consumer's risk. The purpose of well a design plan is to minimize these risks.

Reliability Sampling test are considered a necessary and pre-requisite to determine the trust worthiness of an item with regard to its life time. One of the objectives of such an inspection is to ascertain a confident and perfect limit with respect to the average (mean) life of an item. When the

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confidence limit on the mean life is established, it becomes easier to arrive at a definite decision as to whether the submitted lot may be accepted or rejected. The submitted lot is accepted in case the mean life of the product is found above the desired standard and otherwise the same is rejected. Such tests are deemed to be economical. The ordinary acceptance sampling plan have been developed by Epstein (1954) for exponential distribution, Goode and Kao (1961) for Weibull distribution, Kantam and Rosaiah (1998) for half logistic distribution, Kantam *et al.* (2001) for log-logistic distribution, Baklizi (2003) for the Pareto distribution of the second kind, Rosaiah and Kantam (2005) for inverse Rayleigh distribution, Rosaiah *et al.* (2006) for exponentiated log-logistic distribution, Rosaiah *et al.* (2007) developed the reliability plans for exponentiated log-logistic distribution, Aslam (2007) for the Rayleigh distribution, Balakrishnan *et al.* (2007) for the generalized Birnbaum-Saunders distribution, Aslam and Shahbaz (2007) for the generalized exponential distribution, Rosaiah *et al.* (2008) for the inverse Rayleigh, and Aslam and Kantam (2008) for the Birnbaum-Saunders distribution. Aslam *et al.* (2010) also developed a group acceptance sampling plan for a truncated life test based on the Pareto distribution of the second kind. As mentioned earlier, in the ordinary acceptance sampling plan a single item is inspected, where as in the group acceptance sampling plan, a multiple number of items are inspected on the basis of availability of the testers (denoted by r). It is a well known factor that a group sampling plan performs better than an ordinary sampling plan in terms of reduction of time, cost, energy and labor.

No attention from the researchers has been paid to propose an economic reliability test plan for Pareto distribution of the second kind assuming that the life time of a product follows the this distribution with known shape parameter. Hence, there is a need to propose economic reliability test plans (ERTP) for a life test using groups, which will be called an economic reliability group acceptance sampling plan. This study is to reduce the experimental time in view of given design parameters and satisfy both the consumer's as well as the producer's risks. The rest of the paper is arranged as: the design of ERTP is given in Section 2. Termination ratio is given in Section 3. Illustrated example is given in Section 4. Some tables of comparison are given in Section 5. In section 6, we compare the results of the proposed design with the existing one. In the last section, some concluding remarks are given.

2. Design of ERTP

The group acceptance sampling plan based on truncated life tests proposed by Aslam and Jun (2009) is

1) Select the number of groups g and allocate predefined r items to each group so that the sample size for a lot will be $n = gr$.

2) Select the acceptance number c for a group and the experiment time t_0.

3) Perform the experiment for the g groups simultaneously and record the number of failures for each group.

4) Accept the lot if at most c failures occur in each of all groups.

5) Truncate the experiment if more than c failures occur in any group and reject the lot.

The above group plan can be explained as: let required sample size is $n = r \times g$ (where r and g denotes the pre-defined testers and the number of groups, respectively). If 'n' number of products put on test from a lot and c denote the acceptance number for the experiment will be terminated if more

than c or $(c + 1)$ failure occurs during the experimental time. It is assumed that the life time of the products followed the Pareto distribution of the second kind. As mentioned in the introduction section, two types of risks, a good lot is rejected (producer's risk, α) and a bad lot is accepted (consumer's risk, β) are also considered to reach a valid inference. The quality of a product is good or acceptable if $\mu \geq \mu_0$ (μ is the true average life and μ_0 is specified life), otherwise rejectable. Pareto (1897) introduced the Pareto distribution as a model for incomes. Applications of the Pareto distribution have been discussed in the field of survival and biomedical sciences. The distribution function and the density function of the Pareto distribution of the second kind are

$$F(t; \sigma, \lambda) = 1 - \left(1 + \frac{t}{\sigma}\right)^{-\lambda}, \quad t > 0, \ \sigma > 0, \ \lambda > 0, \tag{2.1}$$

$$f(t; \sigma, \lambda) = \frac{\lambda}{\sigma}\left(1 + \frac{t}{\sigma}\right)^{-(\lambda+1)}, \quad t > 0, \ \sigma > 0, \ \lambda > 0, \tag{2.2}$$

where 'σ' and 'λ' are the scale and shape parameters respectively. The mean of this distribution is

$$\mu = \frac{\sigma}{\lambda - 1}, \quad \lambda > 1. \tag{2.3}$$

The lot size is large and the decision about the submitted lot is either accepted or rejected so that binomial law could be applied, for more justification one may refer to Stephens (2001). The lot acceptance probability for the proposed plan is given by

$$L(p) = \left[\sum_{i=0}^{c}\binom{r}{i}p^{i}(1 - p)^{r-i}\right]^{g}, \tag{2.4}$$

where p is the probability that a product fails before the termination time if the termination time is a multiple of specified life μ_0, then $t_0 = a\mu_0$, where a is pre-fix constant.

The probability p for the Pareto distribution of the second kind is

$$p = F(t; \sigma, \lambda) = 1 - \left[1 + \frac{a}{(\lambda - 1)(\mu/\mu_0)}\right]^{-\lambda}. \tag{2.5}$$

The minimum values of sample size $n = r \times g$ is found by satisfying the following inequality (2.6)

$$\left[\sum_{i=0}^{c}\binom{r}{i}p^{i}(1 - p)^{r-i}\right]^{g} \leq \beta. \tag{2.6}$$

Now, we find the minimum termination time for a given producer's risk, sample size ($n = r \times g$) and the acceptance number c, when this inequality (2.7) is satisfied,

$$\left[\sum_{i=0}^{c}\binom{r}{i}p^{i}(1 - p)^{r-i}\right]^{g} \geq 1 - \alpha. \tag{2.7}$$

Table 1 shows the termination time for given information (defined above) at the various values of $\alpha = 0.05, 0.01$, for more detail see Aslam *et al.* (2010). It is interesting to note that, as the sample size increases for a fixed value of c the termination time decreases

Table 1: Test termination time for $\alpha = 0.05$

	$\lambda = 2$							
c	r / g	$2g$	$3g$	$4g$	$5g$	$6g$	$7g$	$8g$
0	1	0.0129	0.0085	0.0064	0.0551	0.0042	0.0036	0.0032
1	2	0.0905	0.0511	0.0358	0.0276	0.0225	0.0190	0.0164
2	3		0.1599	0.0972	0.0705	0.0555	0.0458	0.0390
3	4			0.2270	0.1447	0.1076	0.0860	0.0718
4	5				0.2907	0.1915	0.1451	0.1174
5	6					0.3506	0.2370	0.1822
6	7						0.4074	0.2810
7	8							0.4613

	$\lambda = 3$							
c	r / g	$2g$	$3g$	$4g$	$5g$	$6g$	$7g$	$8g$
0	1	0.0171	0.0114	0.0080	0.0068	0.0057	0.0048	0.0042
1	2	0.1190	0.0676	0.0475	0.0367	0.0299	0.0252	0.0218
2	3		0.2080	0.1276	0.0930	0.0734	0.0606	0.0517
3	4			0.2924	0.1885	0.1410	0.1131	0.0946
4	5				0.3710	0.2478	0.1891	0.1536
5	6					0.4437	0.3046	0.2361
6	7						0.5117	0.3590
7	8							0.5755

Table 2: Test termination time for $\alpha = 0.01$

	$\lambda = 2$							
c	r / g	$2g$	$3g$	$4g$	$5g$	$6g$	$7g$	$8g$
0	1	0.0025	0.0016	0.0012	0.0010	0.0008	0.0007	0.0006
1	2	0.0373	0.0214	0.0151	0.0116	0.0095	0.0080	0.0069
2	3		0.0843	0.0521	0.0381	0.0301	0.0249	0.0212
3	4			0.1350	0.0876	0.0657	0.0528	0.0442
4	5				0.1856	0.1250	0.0954	0.0776
5	6					0.2350	0.1622	0.1260
6	7						0.2830	0.1992
7	8							0.3290

	$\lambda = 3$							
c	r / g	$2g$	$3g$	$4g$	$5g$	$6g$	$7g$	$8g$
0	1	0.0033	0.0022	0.0016	0.0013	0.0011	0.0009	0.0008
1	2	0.0495	0.0284	0.0200	0.0155	0.0126	0.0107	0.0092
2	3		0.1110	0.0690	0.0504	0.0399	0.0331	0.0282
3	4			0.1760	0.1151	0.0867	0.0697	0.0585
4	5				0.2403	0.1631	0.1253	0.1022
5	6					0.3020	0.2110	0.1646
6	7						0.3611	0.2575
7	8							0.4180

3. Termination Ratio

It is assumed that the shape parameter of the Pareto distribution is known, for given values of μ_0 and t/μ_0, we can find the minimum values of sample size ($n = r \times g$). Aslam *et al.* (2010) found the minimum values of sample size by solving the inequality (2.6), given design parameters and known values of shape parameter $\lambda = 2, 3$. The termination times are determined and placed in Tables 1–

Table 3: Comparisons of test termination time for Pareto distribution of the second kind using binomial law for $\alpha = 0.05$

	$\lambda = 2$							
c	r / g	$2g$	$3g$	$4g$	$5g$	$6g$	$7g$	$8g$
0	1	0.0129 0.8000						
1	2		0.0511 1.0000					
2	3			0.0972 0.8000				
3	4							
4	5							
5	6						0.2370 1.0000	
6	7							0.2810 1.0000

	$\lambda = 3$							
c	r / g	$2g$	$3g$	$4g$	$5g$	$6g$	$7g$	$8g$
0	1	0.0171 1.5000						
1	2		0.0676 1.0000					
2	3			0.1276 1.0000				
3	4				0.1885 1.0000			
4	5							
5	6							
6	7							0.3590 1.2000

2. From these tables, we can observe the different pattern in termination time. For example, when $\alpha = 0.05$, as the shape parameter value increases from 2 to 3, we noted the increases trends in termination time. Other the other hand, for other same values, as the α changes from 0.05 to 0.01, we observed the decreasing trends in the termination ration.
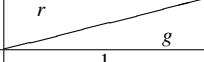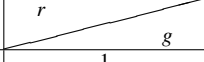
4. Example

A manufacturer would like to run the experiment and knows whether the mean life of their items are longer than the specified life, $\mu_0 = 1000$ hours. Assumed that the lifetime of items follow the Pareto distribution of the second kind, the design parameters of the existing plan are $(g, r, c, t/\mu_0) = (3, 4, 2, 0.8)$ for $\alpha = 0.05$ and $\lambda = 2$. In practice, the manufacture needs to draw a random sample of size 12 items from the lot and allocate 4 items to 3 groups on the life test. The lot is accepted if no more than 2 failed items is found in 800 hours in every group. Otherwise, the lot is rejected. The design parameters of the proposed plan are $(g, r, c, t/\mu_0) = (3, 4, 2, 0.0972)$ for $\alpha = 0.05$ and $\lambda = 2$, which is explained as, one may accept the lot if no more than 2 failed items is found in 97 hours in every group.

5. Tables

From Table 1, the entry against $c = 2$, $r = 5$, $g = 3$, $\alpha = 0.01$, $\lambda = 3$ under the column $5g$ is 0.0504.

Table 4: Comparisons of test termination time for Pareto distribution of the second kind using binomial law for $\alpha = 0.01$

					$\lambda = 2$			
c	r / g	$2g$	$3g$	$4g$	$5g$	$6g$	$7g$	$8g$
0	1							
1	2		0.0214 1.5000					
2	3			0.0521 1.2000				
3	4				0.0876 1.2000			
4	5					0.1250 1.2000		
5	6						0.1622 1.2000	
6	7							

					$\lambda = 3$			
c	r / g	$2g$	$3g$	$4g$	$5g$	$6g$	$7g$	$8g$
0	1							
1	2		0.0284 1.5000					
2	3			0.0690 1.5000				
3	4				0.1151 1.5000			
4	5							
5	6							
6	7							

Since the specified mean life is 1000 hours, the table value say that $t/\mu_0 = 0.0504$ so that $t = 0.0504 \times 1000 = 50.4 = 50$ hours (approximately). It means that, select 15 items from the submitted lot and reject this lot if more than 2 failure occur during the 50 hours, otherwise accept the lot.

6. Comparative Study

In order to compare the proposed plan, our measurements are smaller than the existing plan by Aslam *et al.* (2010) for common (n, c) and various risks. In Tables 3 and 4 upper entries of cells denoting the proposed plan. Suppose that we want to develop an economic reliability test plans(ERTP) with a specified average life of $\mu_0 = 1000$ hours, $\alpha = 0.01$, $c = 3$ and $\lambda = 3$. Select 20 items from the lot and put on test, if a 3rd failure occurs during the termination time 115.10 hours, we reject the lot, otherwise accept it. At the same the measurement termination time using the existing plan is 1500 hours, so the termination time of the proposed plan is smaller than the existing plan of Aslam *et al.* (2010). Now we can say that the proposed plan is more economical in the sense of saving cost, time and energy.

7. Conclusion

In this paper, the methodology to find the minimum termination ratio is given. The tables are constructed for various shape parameters. The results are compared with the existing plan and it is concluded that the proposed method is useful in reducing the experiment time. Hence, the proposed plan

is more economic in cost and time to reach the final decision about the submitted product. There is a need to propose a group economic reliability plan for other distributions such as the Weibull distribution and the gamma distribution in future research.

Acknowledgements

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