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The Comparison of the Unconditional and Conditional Exact Power of Fisher's Exact Test

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Abstract

Since Fisher's exact test is conducted conditional on the observed value of the margin, there are two kinds of the exact power, the conditional and the unconditional exact power. The conditional exact power is computed at a given value of the margin whereas the unconditional exact power is calculated by incorporating the uncertainty of the margin. Although the sample size is determined based on the unconditional exact power, the actual power which Fisher's exact test has is the conditional power after the experiment is finished. This paper investigates differences between the conditional and unconditional exact power of Fisher's exact test. We conclude that such discrepancy is a disadvantage of Fisher's exact test.

Keywords: Conditional test, sample size determination, homogeneity, binomial.

1. Introduction

In this paper we focus on testing the homogeneity of two independent binomial proportions when the sample size is small. When the sample size is large enough, the normal approximation to the binomial distribution may be employed. However, when the sample size is small, such approximation may not be valid and Fisher's exact test is often employed as an alternative. The main advantage of exact tests (including Fisher's exact test) is that it is guaranteed to control type I error rates under the nominal level.

A key feature of Fisher's exact test is that the test is conducted conditional on the observed value of the margin. Therefore, there are two kinds of the exact power, the conditional and unconditional exact power. The conditional exact power is computed at a given value of the margin whereas the unconditional exact power is calculated by incorporating the uncertainty of the margin. The sample size should be determined based on the unconditional exact power, because a value of the margin is not observed yet before an experiment is conducted. However, after the experiment is finished, the actual power which Fisher's exact test has is the conditional power, because the test is a conditional test. Since in general the conditional exact power is not as same as the unconditional exact power, the actual power (the conditional exact power) which Fisher's exact test has after the experiment

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is completed may not be equal to the power such as 80% or 90% which is targeted through the sample size determination with the unconditional exact power. Several articles have been devoted to the study of the sample size determination based on the unconditional exact power over the past few decades (Gail and Gart, 1973; Haseman, 1978; Sahai and Khurshid, 1996; Crans and Shuster, 2008). However, no studies have ever tried to examine discrepancies between the two exact powers of Fisher's exact test. Therefore, in this paper, we investigate differences between the conditional and unconditional exact power of Fisher's exact test.

2. Notations and Review

Let X_1 and X_2 be two independent binomial random variables, $X_k \sim B(n_k, p_k)$, $k = 1, 2$. Then data can be summarized in the 2×2 contingency table as in Table 1 where $M = X_1 + X_2$, $n = n_1 + n_2$. We would like to test $H_0 : p_1 = p_2 = p$ $(0 < p < 1)$ versus $H_1 : p_1 \neq p_2$ where p is an unknown nuisance parameter. Under the null hypothesis, the likelihood of (X_1, X_2) depends on p, because it is given by

$$P(X_1 = x_1, X_2 = x_2 | H_0) = \prod_{i=1}^{2} \binom{n_i}{x_i} p^{x_i} (1-p)^{n_i - x_i}.$$

In order to remove the dependency of the unknown nuisance parameter p, Fisher's exact test employs the conditional null distribution of a test statistic, given a sufficient statistic $(M = X_1 + X_2)$ of an unknown nuisance parameter p. Hence, the following hypergeometric distribution is used to construct Fisher's exact test

$$P(X_1 = x_1, X_2 = x_2 | M = m, H_0) = \binom{n_1}{x_1}\binom{n_2}{x_2} \Big/ \binom{n_1 + n_2}{m}.$$

For Fisher's exact test the test statistic is a reciprocal of the conditional null probability which is given by

$$T(x_1, x_2) = \binom{n_1 + n_2}{m} \Big/ \left[\binom{n_1}{x_1}\binom{n_2}{x_2}\right].$$

Let (x_1^0, x_2^0) be the observed vector of (X_1, X_2) and $m = x_1^0 + x_2^0$. When m, an observed value of M, is given, Fisher's exact test considers only the following set of 2×2 tables as a sample space.

$$\Gamma_m = \{(x_1, x_2) | x_1 + x_2 = m, 0 \leq x_1 \leq n_1, 0 \leq x_2 \leq n_2\}.$$

Since the large value of T is significant, for the given m, the rejection region of Fisher's exact test is given by

$$\Gamma_m(t) = \{(x_1, x_2) \in \Gamma_m | T(x_1, x_2) \geq t\}.$$

Then, the exact null distribution of T is evaluated in order to calculate the exact power

$$P(T \geq t | m, H_0) = \sum_{(x_1, x_2) \in \Gamma_m(t)} \binom{n_1}{x_1}\binom{n_2}{x_2} \Big/ \binom{n_1 + n_2}{m}$$

for each possible value of t. Let α be the given nominal level and $t_{\alpha(m)}$ be the smallest possible value such that

$$P(T \geq t_\alpha(m)|m, H_0) \leq \alpha.$$

For a given m, Fisher's exact test rejects H_0 if $T(x_1^0, x_2^0) \geq t_\alpha(m)$. The conditional exact power is given by (Cytel, 2006)

$$\begin{aligned}\beta^c(m, p_1, p_2) &= P(T \geq t_\alpha(m)|m, p_1, p_2) \\ &= \sum_{(x_1,x_2)\in\Gamma_m(t_\alpha(m))} \left[\frac{\prod_{i=1}^{2} \frac{n_i!}{x_i!(n_i - x_i)!} p_i^{x_i}(1-p_i)^{n_i-x_i}}{\sum_{(x_1,x_2)\in\Gamma_m} \prod_{i=1}^{2} \frac{n_i!}{x_i!(n_i - x_i)!} p_i^{x_i}(1-p_i)^{n_i-x_i}} \right].\end{aligned}$$

The unconditional exact power is a weighted average of the conditional exact power.

$$\beta^u(p_1, p_2) = \sum_{m=0}^{n_1+n_2} \beta^c(m, p_1, p_2) P(X_1 + X_2 = m|H_1).$$

The unconditional exact power has an alternative expression (Kang and Ahn, 2008)

$$\begin{aligned}\beta^u(p_1, p_2) &= \sum_{m=0}^{n} P(T \geq t_\alpha(m)|m, p_1, p_2) P(M = m|p_1, p_2) \\ &= \sum_{(x_1,x_2)\in\Gamma_\alpha} \prod_{i=1}^{2} \binom{n_i}{x_i} p_i^{x_i}(1-p_i)^{n_i-x_i},\end{aligned}$$

where

$$\Gamma_\alpha = \cup_{m=0}^{n} \Gamma_m(t_\alpha(m)).$$

3. Numerical Results

Numerical results in this section were obtained in the following order. We selected the combinations of (n_1, n_2) and (p_1, p_2) whose unconditional exact powers are between 80% and 95%, because we are usually interested in such cases with high power. We used commercial software StatXact 6.0 (Cytel, 2006) to calculate the unconditional exact powers for given values of (n_1, n_2) and (p_1, p_2). Since Fisher's exact test is employed in small sample problems, the sample size in each group was chosen to be smaller than 100 in our investigation. For the given values of (n_1, n_2) and (p_1, p_2), we generated all possible combinations ($N = (n_1 + 1)(n_2 + 1)$) of (x_1, x_2) with $0 \leq x_1 \leq n_1$ and $0 \leq x_2 \leq n_2$. Since this study was done based on complete enumeration, it is important to recognize that this is not a simulation study. For the given value of (x_1, x_2), the conditional exact power ($\beta_i^c(m, p_1, p_2)$) was computed with $m = x_1 + x_2$, and the probability of observing (x_1, x_2) was also calculated by

$$q_i \equiv P(X_1 = x_1, X_2 = x_2|H_1) = \prod_{i=1}^{2} \binom{n_i}{x_i} p_i^{x_i}(1-p_i)^{n_i-x_i}.$$

The SAS code was developed to compute $\beta_i^c(m, p_1, p_2)$ and q_i based on complete enumeration. The standard deviation of the distributions of the conditional exact powers is computed by

$$\sqrt{\sum_{i=1}^{N}[\beta_i^c(m, p_1, p_2)]^2 q_i - \left[\sum_{i=1}^{N}\beta_i^c(m, p_1, p_2) q_i\right]^2}.$$

Table A.2$\sim$A.6 display the distributions of the conditional exact powers for various combinations of (n_1, n_2) and (p_1, p_2). For example, Table A.2 presents the distribution of the conditional exact power in each column at $(p_1, p_2) = (0.5, 0.1), (0.6, 0.2), (0.7, 0.3), (0.8, 0.4)$ and $(0.9, 0.5)$ when $(n_1, n_2) = (30, 30)$. Although the conditional exact powers are real numbers between 0 and 1, they are categorized into intervals with length 0.02 for convenience of presentation. The numbers displayed in Table A.2 are the probabilities that the conditional exact powers belong to a specific interval. For instance, in the second column of Table A.2,

$$P\left(0.92 \leq \beta^c < 0.94 | (p_1, p_2) = (0.5, 0.1)\right) = 0.278.$$

Note that the unconditional exact power is a weighted average of the conditional exact powers. Therefore, from Table A.2$\sim$A.6, we see that the distributions of the conditional exact powers are distributed around the unconditional exact power. But, the distributions are skewed to the left. The standard deviations of the distributions are computed for each case. When $(n_1, n_2) = (30, 30)$, the standard deviations range from 0.03 to 0.06 approximately. The standard deviations decrease as the sample size increases.

4. Discussion

When an experiment is planned to compare two independent binomial proportions, Fisher's exact test might be employed, if the sample size is expected to be small. Since the value of the margin is not observed yet prior to the experiment, the uncertainty of the margin should be incorporated. Therefore, the sample size determination should be based on the unconditional exact power. Commercial software StatXact 6.0 (Cytel, 2006) is available to compute the sample size of Fisher's exact test based on the unconditional exact power. However, after the experiment is finished, a specific value of the margin is observed. The actual power which Fisher's exact test has is the conditional exact power for a given observed value of the margin, because Fisher's exact test is a conditional test. In this paper, we investigate differences between the unconditional and conditional exact powers of Fisher's exact test when the sample size is small.

The numerical results in Section 3 show discrepancies between two exact powers of Fisher's exact test. It is very likely that Fisher's exact test does not have the targeted power. For example, when $(n_1, n_2) = (30, 30)$ and $(p_1, p_2) = (0.5, 0.1)$, the targeted power (the unconditional exact power) is 0.914. But, the probability that the actual power (the conditional exact test) is greater than or equal to 0.940 is 0.283, and the probability that the actual power is less than 0.900 is 0.215. The cause of this problem is that Fisher's exact test is a conditional test. Therefore, similar problems are expected to occur in other conditional tests.

It is very difficult to give a general guideline which fits all situations, because discrepancies between two exact powers vary depending on the values of (n_1, n_2) and (p_1, p_2). It is almost infeasible to tabulate all possible cases. The authors are willing to provide the SAS code upon request so that any statistician can investigate the discrepancies between two exact powers for the values (n_1, n_2) and (p_1, p_2).

For testing the homogeneity of two independent binomial proportions, there are several procedures available such as the conditional exact tests (for example, Fisher's exact test), the unconditional exact test and the asymptotic tests (Lydersen *et al.*, 2009). A considerable number of studies have been made on the comparison of these procedures over the past few decades (Lyderson and Laake, 2003; Suissa and Shuster, 1985; Berger and Boos, 1994; Martin Andres *et al.*, 1998, 2004; Kang and Ahn, 2008). All exact tests including Fisher's exact test and the unconditional exact tests guarantee to control type I error rates under the nominal level. Therefore, a next natural question is which exact test is more powerful. Lydersen *et al.* (2009) recommend the use of the unconditional exact tests, because they preserve the significance level and generally are more powerful than Fisher's exact test for moderate to small samples. Lydersen *et al.* (2009) argued that Fisher's exact test should practically never be used, because the test is conservative. This paper adds another disadvantage of Fisher's exact test.

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Appendix

Table A.1. A 2×2 contingency table

	group 1	group 2	Totals
response	X_1	X_2	M
no response	$n_1 - X_1$	$n_2 - X_2$	$n - M$
Totals	n_1	n_2	n

Table A.2. The distributions of the conditional exact power, $(n_1, n_2) = (30, 30)$

conditional	(p_1, p_2)				
exact power(β^c)	$(0.5, 0.1)$	$(0.6, 0.2)$	$(0.7, 0.3)$	$(0.8, 0.4)$	$(0.9, 0.5)$
$0.00 \leq \beta^c < 0.70$	0.006	0.004	0.004	0.004	0.006
$0.70 \leq \beta^c < 0.72$	0.000	0.000	0.000	0.000	0.000
$0.72 \leq \beta^c < 0.74$	0.000	0.000	0.000	0.000	0.000
$0.74 \leq \beta^c < 0.76$	0.037	0.076	0.018	0.076	0.037
$0.76 \leq \beta^c < 0.78$	0.000	0.000	0.002	0.000	0.000
$0.78 \leq \beta^c < 0.80$	0.021	0.098	0.054	0.098	0.021
$0.80 \leq \beta^c < 0.82$	0.000	0.049	0.432	0.049	0.000
$0.82 \leq \beta^c < 0.84$	0.000	0.115	0.001	0.115	0.000
$0.84 \leq \beta^c < 0.86$	0.093	0.245	0.032	0.245	0.093
$0.86 \leq \beta^c < 0.88$	0.000	0.052	0.241	0.052	0.000
$0.88 \leq \beta^c < 0.90$	0.058	0.221	0.216	0.221	0.058
$0.90 \leq \beta^c < 0.92$	0.221	0.138	0.000	0.138	0.221
$0.92 \leq \beta^c < 0.94$	0.278	0.000	0.000	0.000	0.278
$0.94 \leq \beta^c < 0.96$	0.106	0.000	0.000	0.000	0.106
$0.96 \leq \beta^c < 0.98$	0.174	0.000	0.000	0.000	0.174
$0.98 \leq \beta^c < 1.00$	0.003	0.000	0.000	0.000	0.003
unconditional exact power	0.914	0.851	0.838	0.851	0.914
standard deviation	0.059	0.046	0.037	0.046	0.059

* The unconditional exact power is a weighted average of the conditional exact powers.

* The standard deviation denotes the standard deviation of the distribution of the conditional exact powers.

Table A.3. The distributions of the conditional exact power, $(n_1, n_2) = (50, 50)$

conditional exact power(β^c)	(p_1, p_2)				
	$(0.4, 0.1)$	$(0.5, 0.2)$	$(0.6, 0.3)$	$(0.7, 0.4)$	$(0.8, 0.5)$
$0.00 \leq \beta^c < 0.70$	0.001	0.002	0.002	0.002	0.002
$0.70 \leq \beta^c < 0.72$	0.008	0.000	0.000	0.000	0.000
$0.72 \leq \beta^c < 0.74$	0.000	0.009	0.006	0.006	0.009
$0.74 \leq \beta^c < 0.76$	0.004	0.000	0.015	0.015	0.000
$0.76 \leq \beta^c < 0.78$	0.000	0.023	0.029	0.029	0.023
$0.78 \leq \beta^c < 0.80$	0.000	0.072	0.124	0.124	0.072
$0.80 \leq \beta^c < 0.82$	0.024	0.136	0.338	0.338	0.136
$0.82 \leq \beta^c < 0.84$	0.000	0.146	0.059	0.059	0.146
$0.84 \leq \beta^c < 0.86$	0.014	0.177	0.141	0.141	0.177
$0.86 \leq \beta^c < 0.88$	0.048	0.210	0.286	0.286	0.210
$0.88 \leq \beta^c < 0.90$	0.089	0.098	0.000	0.000	0.098
$0.90 \leq \beta^c < 0.92$	0.208	0.114	0.000	0.000	0.114
$0.92 \leq \beta^c < 0.94$	0.158	0.012	0.000	0.000	0.012
$0.94 \leq \beta^c < 0.96$	0.260	0.000	0.000	0.000	0.000
$0.96 \leq \beta^c < 0.98$	0.178	0.000	0.000	0.000	0.000
$0.98 \leq \beta^c < 1.00$	0.007	0.000	0.000	0.000	0.000
unconditional exact power	0.927	0.853	0.829	0.829	0.853
standard deviation	0.045	0.040	0.032	0.032	0.040

* The unconditional exact power is a weighted average of the conditional exact powers.

* The standard deviation denotes the standard deviation of the distribution of the conditional exact powers.

Table A.4. The distributions of the conditional exact power, $(n_1, n_2) = (70, 70)$

conditional exact power(β^c)	(p_1, p_2)				
	$(0.4, 0.1)$	$(0.5, 0.2)$	$(0.6, 0.3)$	$(0.7, 0.4)$	$(0.8, 0.5)$
$0.00 \leq \beta^c < 0.70$	0.000	0.000	0.000	0.000	0.000
$0.70 \leq \beta^c < 0.72$	0.000	0.000	0.000	0.000	0.000
$0.72 \leq \beta^c < 0.74$	0.000	0.000	0.000	0.000	0.000
$0.74 \leq \beta^c < 0.76$	0.000	0.000	0.000	0.000	0.000
$0.76 \leq \beta^c < 0.78$	0.000	0.001	0.000	0.000	0.001
$0.78 \leq \beta^c < 0.80$	0.000	0.000	0.001	0.001	0.000
$0.80 \leq \beta^c < 0.82$	0.000	0.005	0.003	0.003	0.005
$0.82 \leq \beta^c < 0.84$	0.002	0.000	0.009	0.009	0.000
$0.84 \leq \beta^c < 0.86$	0.011	0.013	0.051	0.051	0.013
$0.86 \leq \beta^c < 0.88$	0.001	0.097	0.200	0.200	0.097
$0.88 \leq \beta^c < 0.90$	0.007	0.134	0.280	0.280	0.134
$0.90 \leq \beta^c < 0.92$	0.030	0.308	0.456	0.456	0.308
$0.92 \leq \beta^c < 0.94$	0.067	0.306	0.000	0.000	0.306
$0.94 \leq \beta^c < 0.96$	0.282	0.136	0.000	0.000	0.136
$0.96 \leq \beta^c < 0.98$	0.374	0.001	0.000	0.000	0.001
$0.98 \leq \beta^c < 1.00$	0.226	0.000	0.000	0.000	0.000
unconditional exact power	0.984	0.954	0.934	0.934	0.954
standard deviation	0.012	0.015	0.013	0.013	0.015

* The unconditional exact power is a weighted average of the conditional exact powers.

* The standard deviation denotes the standard deviation of the distribution of the conditional exact powers.

Table A.5. The distributions of the conditional exact power, $(n_1, n_2) = (30, 60)$

conditional exact power (β^c)	(p_1, p_2) $(0.45, 0.1)$	$(0.55, 0.2)$	$(0.65, 0.3)$	$(0.75, 0.4)$	$(0.85, 0.5)$
$0.00 \le \beta^c < 0.70$	0.001	0.000	0.000	0.000	0.000
$0.70 \le \beta^c < 0.72$	0.000	0.000	0.000	0.000	0.000
$0.72 \le \beta^c < 0.74$	0.000	0.000	0.000	0.002	0.000
$0.74 \le \beta^c < 0.76$	0.000	0.000	0.000	0.000	0.000
$0.76 \le \beta^c < 0.78$	0.003	0.000	0.000	0.001	0.003
$0.78 \le \beta^c < 0.80$	0.000	0.026	0.001	0.009	0.030
$0.80 \le \beta^c < 0.82$	0.000	0.000	0.014	0.000	0.001
$0.82 \le \beta^c < 0.84$	0.012	0.000	0.049	0.037	0.021
$0.84 \le \beta^c < 0.86$	0.000	0.098	0.230	0.091	0.081
$0.86 \le \beta^c < 0.88$	0.035	0.095	0.135	0.279	0.063
$0.88 \le \beta^c < 0.90$	0.006	0.227	0.279	0.100	0.096
$0.90 \le \beta^c < 0.92$	0.021	0.262	0.292	0.284	0.206
$0.92 \le \beta^c < 0.94$	0.143	0.237	0.000	0.196	0.205
$0.94 \le \beta^c < 0.96$	0.210	0.054	0.000	0.000	0.182
$0.96 \le \beta^c < 0.98$	0.436	0.000	0.000	0.000	0.112
$0.98 \le \beta^c < 1.00$	0.133	0.000	0.000	0.000	0.000
unconditional exact power	0.951	0.898	0.881	0.890	0.914
standard deviation	0.032	0.032	0.027	0.030	0.041

* The unconditional exact power is a weighted average of the conditional exact powers.

* The standard deviation denotes the standard deviation of the distribution of the conditional exact powers.

Table A.6. The distributions of the conditional exact power, $(n_1, n_2) = (60, 30)$

conditional exact power (β^c)	(p_1, p_2) $(0.45, 0.1)$	$(0.55, 0.2)$	$(0.65, 0.3)$	$(0.75, 0.4)$	$(0.85, 0.5)$
$0.00 \le \beta^c < 0.70$	0.000	0.000	0.000	0.000	0.000
$0.70 \le \beta^c < 0.72$	0.000	0.004	0.000	0.000	0.001
$0.72 \le \beta^c < 0.74$	0.000	0.000	0.000	0.000	0.000
$0.74 \le \beta^c < 0.76$	0.000	0.002	0.000	0.002	0.003
$0.76 \le \beta^c < 0.78$	0.005	0.000	0.001	0.000	0.000
$0.78 \le \beta^c < 0.80$	0.000	0.019	0.000	0.000	0.000
$0.80 \le \beta^c < 0.82$	0.003	0.011	0.006	0.014	0.000
$0.82 \le \beta^c < 0.84$	0.001	0.049	0.028	0.061	0.001
$0.84 \le \beta^c < 0.86$	0.001	0.007	0.217	0.143	0.027
$0.86 \le \beta^c < 0.88$	0.024	0.118	0.149	0.229	0.125
$0.88 \le \beta^c < 0.90$	0.101	0.245	0.280	0.141	0.000
$0.90 \le \beta^c < 0.92$	0.084	0.185	0.320	0.228	0.300
$0.92 \le \beta^c < 0.94$	0.153	0.290	0.000	0.181	0.229
$0.94 \le \beta^c < 0.96$	0.330	0.070	0.000	0.000	0.200
$0.96 \le \beta^c < 0.98$	0.220	0.000	0.000	0.000	0.114
$0.98 \le \beta^c < 1.00$	0.078	0.000	0.000	0.000	0.000
unconditional exact power	0.943	0.899	0.884	0.885	0.921
standard deviation	0.033	0.038	0.026	0.030	0.034

* The unconditional exact power is a weighted average of the conditional exact powers.

* The standard deviation denotes the standard deviation of the distribution of the conditional exact powers.

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