Performance Analysis of Coordinated Random Beamforming Technique in Multi-cell Environments

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***Abstract*— For multi-cell environments, coordinated random beamforming technique in multiuser MIMO (multiple-input multiple-output) broadcast channel is considered. In order to mitigate severe interference at receivers, the multi-cell environments might require complex transmitter and receiver design because the scheduler decision based on full channel state information (CSI) in one cell must be intertwined with decision made by other cells' CSI. With limited CSI, however, this paper considers a scheme of randomizing transmitters' beamforming but being coordinated with other cell transmitters. The transmitters in each cell share random beamforming patterns and schedule data transmission within coherent scheduling period. The co-randomized beams allow the users to be selected with the highest SINRs even in multi-cell environments. We analyze the performance of the proposed scheme. And numerical results show that the scheme achieves better performance than the conventional random beamforming when applying to multi-cell environments.**

***Index Terms*— multiple-input multiple-output (MIMO), opportunistic beamforming, coordinated random beamforming, multi-cell environment, coordinated multiple point transmission and reception (CoMP).**

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) wireless communications can increase throughput in multiuser cellular systems by using multiple antennas to provide simultaneous transmission to multiple users [1]-[3]. Recent theoretical results have found that dirty paper coding (DPC) achieves the capacity region for the MIMO broadcast channel. In practical, however, the DPC do not yet exist and thus it is difficult to implement due to high computational complexity and perfect channel state information (CSI).

In order to reduce the feedback overhead, several approaches for informing the base station (BS) about the transmit channels for each user have been proposed. One of the approaches is to use limited feedback for beamforming or space-division multiple access (SDMA).

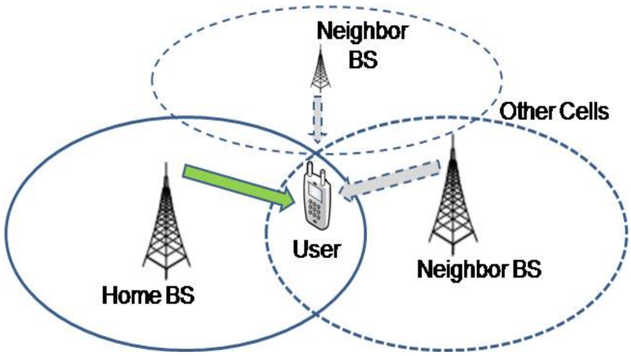

Fig. 1. Cell-edge users in interference limited multi-cell environments.

The impact of limited feedback on the performance of multiuser MIMO channels has been presented in [4]-[7]. Although the limited feedback methods may reduce the overhead effectively, they require higher resolution codebook to quantize precisely in the multiuser case than the single user case. In multi-cell environments, furthermore, the feedback requirements for higher resolution will be more complicated.

Another approach to reduce the feedback requirements is to employ the opportunism inherent in multiuser communication systems [8][9]. With opportunistic beamforming, the BS randomly selects beams to transmit data streams. The users feed back their signal-to-noise ratio (SNR) corresponding to the beam and thus the BS schedules the users with the highest SNR. Since the opportunistic beamforming [8] (or random beamforming [9]) sends only single data stream, it does not fully exploit the advantages of the MIMO broadcast channel capacity gains. An alternative is the opportunistic SDMA (OSDMA) where the BS transmits orthogonal beams [10][11]. In this case, each user reports the best beam and their signal-to-interference-plus-noise ratio (SINR) to the BS. Then, the BS schedules transmissions to the multiple users based on the received SINR.

When it comes to multi-cell environments as shown in Fig. 1, due to the fact that interference is affected by other cells as well as inner-cell, the feedback requirement for each cell will increase and thus the multiple cellular systems must be very complex. In LTE-Advanced system, for instance, coordinated multiple point transmission and reception (CoMP) was

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introduced in order to mitigate the severe interference of cell-edge users in multi-cell environments. Joint transmission or coordinated beamforming techniques allow the cell-edge users to reduce interference. However, it should be noted that the scheduler decision in one cell or a group of cells intertwines with decision made by another cell.

In this paper, we propose a coordinated random beamforming technique for the multi-cell environments. Two scenarios of cell-edge and inner-cell users are considered to analyze the performance of the coordinated random beamforming scheme. The coordinated random beamforming has better performance than uncoordinated random beamforming in multi-cell environments. The performance of the coordinated random beamforming technique has greater throughput with limited feedback than uncoordinated but can be closed to the channel capacity in large number of users.

The remainder of this paper is organized as follows. Section II describes the system model to analyze the performance of the coordinated random beamforming technique in multi-cell environments. In Section III, we provide the capacity analysis of the proposed scheme with information theoretic perspective. Section IV evaluates the performance of the scheme. Finally, Section V concludes the paper.

II. SYSTEM MODEL

Throughout the paper, we consider the MIMO broadcast channel capacity with M transmit antennas per a single cell and U users with N receive antennas in multi-cell environment. We assume that each BS of the multi-cell is identical. And the BSs transmit multiple symbols $X(t)$ multiplying a set of random beamforming vectors $\phi = (\phi_1 \ldots \phi_M)$ which are generated by a shared random beamforming generator, while each user feeds back the highest SINR and the best beamforming weight into their home BS. Then, the home BS selects the set of users and beamforming weight that maximize the sum capacity based on a certain scheduling scheme. Random beamforming technique is well known to effectively not only exploit multiuser diversity via user selection but also improve multiplexing gain by transmitting parallel data streams over orthonormal beams [9]. In this paper, based on the concept of random beamforming, we consider a scheme of coordinating the random orthonormal beams for multiple cellular environments.

Let $X(t)$ denotes the $M \times 1$ vector of the transmit symbols at time$t$,and let $Y(t)$ denotes the $N \times 1$ vector of received signal at ith user. Then, the received signal at time t is

$$Y_i(t) = \sqrt{\rho_i} H_i X(t) + W_i, \quad i = 1, \ldots, U \tag{1}$$

where H_i and W_i are an $N \times M$ complex channel matrix and an $N \times 1$ additive noise respectively, and the entries of H_i and W_i are independent and identically distributed (i.i.d.) complex Gaussian with zero mean and unit variance $CN(0,1)$.Moreover, we assume $E\{X^*X\} = P/M$, such that the total transmit power is P. The transmitted signal $X(t)$ at time slot t is represented as below

$$X(t) = \sum_{m=1}^{M} \phi_m(t) x_m(t), \quad t = 1, \ldots, T \tag{2}$$

where the mth orthonormal beamforming vector ϕ_m is multiplied by the mth transmit symbol $x_m(t)$. The orthonormal vectors ϕ_m is generated according to an isotropic distribution [12], and it represents random beamforming weight vectors that construct M random beams and transmit to the users with the highest SINR. We assume that channels for all the users are static during a time slot. And the symbols $\{x_m\}$ are selected from a Gaussian codebook. For simplicity, we can drop the time index t, thus, the received signal at the ith receiver is

$$Y_i = \sum_{m=1}^{M} H_i \phi_m x_m + W_i, \quad i = 1, \ldots, U. \tag{3}$$

When it comes to a multi-cell environment that might cause significant interference to users at each cell edge, we consider a random beamforming technique. But, that is still coordinated with the multiple cells. Fig. 2 describes the block diagram of the coordinated random beamforming technique in a multi-cell environment. Both the home cell BS and other cell neighbor BSs share the random bemforming generator.

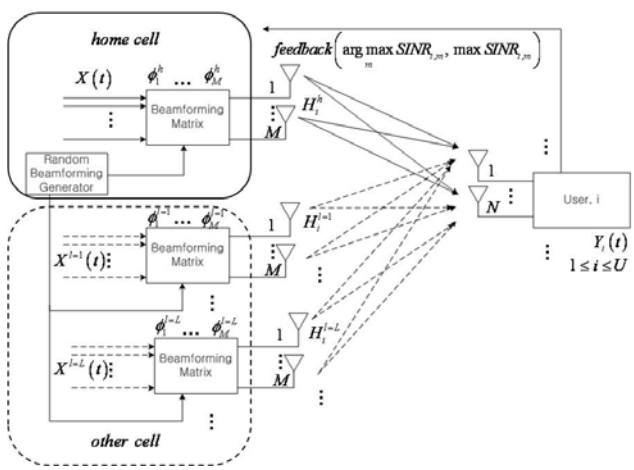

Fig. 2. Block diagram of the coordinated random beamforming technique: the coordinator generates random beams in pseudo-random manner, which are shared in multiple BSs. And maximum SINR users are scheduled.

Since it is assumed that the received signal at the home cell is interfered by the neighbor BSs in the other cells, the received signal at user i, Y_i, can be expressed as

$$Y_i = \sum_{m=1}^{M} H_i^h \phi_m^h x_m^h + \alpha \sum_{l=1}^{L} \sum_{m=1}^{M} H_i^l \phi_m^l x_m^l + W_i, \quad i = 1, \ldots, U \quad (4)$$

where L denotes the number of interfering neighbor BSs, and $\alpha (0 \le \alpha \le 1)$ represents the attenuation of the neighbor BSs' signals received at the home cell user. Without even having complete CSI in BSs, the ith receiver knows the coordination of $H_i^h \phi_m^h, H_i^l \phi_m^l$ for $m = 1, \ldots, M$ at every scheduling period by the pseudo-randomness. M SINRs, therefore, are computed at the ith receiver. We assume that, since the multiple cell BSs are identical and the most significant interferer is adjacent one or two cells, the system model is simplified into two cells--home cell and the other cell. When x_m is the desired signal and the other signals are interference, thus, the signal-to-interference-plus-noise ratio (SINR) of mth stream at ith receiver, $\gamma_{i,m}$, can be

$$\gamma_{i,m} = \frac{\left| H_i^h \phi_m^h \right|^2}{\frac{1}{\rho} + \sum_{k \ne m}^{M-1} \left| H_i^h \phi_k^h \right|^2 + \alpha \sum_{m=1}^{M} H_i^l \phi_m^l} \quad (5)$$

where h and l indicate respectively home cell and the other cell, and ρ denotes the input SNR.

From the information theoretic perspective, the average throughput R of the system that randomly assigns beams to users can be described as

$$R = \sum_{m=1}^{M} \log\left(1 + \gamma_{i,m}\right) \quad (6)$$

As explained in opportunistic beamforming and random beamforming [8]-[10], we consider each user feeds back its maximum SINR along with index m in which the SINR maximized. Instead of assigning each beam to one of the users randomly, thus, the transmitter assigns the symbol x_m to the user with the highest corresponding SINR. In the sense of maximum ergodic capacity, which is a function with variables of a number of multiple antennas, users, and the SINR level, the sum rate capacity based on the above scheduling can be expressed as

$$E[R] = E\left[\sum_{m=1}^{M} \log\left(1 + \max_{1 \le i \le U} \gamma_{i,m}\right)\right] \quad (7)$$

$$= M\, E\left[\log\left(1 + \max_{1 \le i \le U} \gamma_{i,m}\right)\right]. \quad (8)$$

We assume that there is a small probability that user i may be the strongest user for more than one signal x_m. Thus we neglect this small probability since this is very unlikely as the number of user U increases.

III. CAPACITY ANALYSIS OF THE COORDINATED RANDOM BEAMFORMING TECHNIQUE

The opportunism proposed in [8]-[10] achieves effectively multi-user diversity with a number of users. And multiplexing gain is improved via orthonormal beamforming vectors [10][11]. In the context of opportunism, it is believed that a random beamforming technique even for multi-cell environments will achieve the same gain. However, we need to consider more complicated beamforming schemes due to the fact that neighbor BSs as well as home BS should jointly schedule users' selection. The optimal joint beamforming scheme, moreover, might not be found at once. Thus, we rather present a coordinated random beamforming technique for the multi-cell environments and analyze its performance.

A. Coordinated Random Beamforming

The coordinated random beamforming scheme does not necessarily require transmitters and receivers to find the optimal joint beamforming vectors of the home BS and neighbor BSs in order to mitigate the interference of users in multi-cell environments. Instead, based on co-randomized orthonormal beamforming vectors of the BSs, we can exploit the multiuser diversity on simple user scheduling and multiplexing gain even in the multi-cell environments. That is because the coordinated random beams of the multiple BSs allow a set of users on the multi-cells to be selected with the maximum SINRs of (5).

As shown in Fig. 2, the random beamforming coordinator generates random beams and shares with a certain set of BSs. From users' feedback information, i.e., effective SINRs, the coordinator determines scheduling period during which each BS transmits their own data streams to users. Through the coordination of random beams in the multi-cell environment, data streams can be transmitted to users with the lowest interference in a certain period of scheduling time. And the optimal period of scheduling time is an implementation issue depending on feedback delay and channel variation.

B. Capacity of Cell-edge Users ($\alpha \approx 1$)

In order to evaluate the capacity of the coordinated random beamforming technique, we have to obtain the distribution of SINR $\gamma_{i,m}$. Because the orthonormal ϕ_m vectors are unitary and H_i is i.i.d. $CN(0,1)$, we can write the SINR as $\gamma_{i,m} = \frac{z}{1/\rho + y}$ where z has $\chi^2(2N)$ and y has $\chi^2\left(2N(M-1)\right)$ distributions that are independent, similar to [10]. Thus, the probability density function (pdf) of $\gamma_{i,m}$ is

$$f_{\gamma_{i,m}}(x)=\frac{x^{2N-2}e^{-x/\rho}}{(2N-1)!(NM-N-1)!}\sum_{i=0}^{2N-1}\binom{2N-1}{i}$$
$$\times\frac{1}{\rho^{2N-i-1}}\times\frac{(N(M-1)+i-1)!}{(1+x)^{N(M-1)+i}}. \tag{9}$$

Considering the capacity of cell-edge users *($\alpha \approx 1$)*, we rewrite (5) into $\gamma_{i,m}=\frac{z}{1/\rho+y}$ where z has $\chi^2(2)$ and y has $\chi^2(2N(2M'-1))$ distributions in which $N=1$ and $M=2M'$. According to [10], the pdf of mth SINR at the ith cell-edge user is as follows:

$$f_{\gamma_{i,m}}(x)\approx\frac{e^{-\frac{x}{\rho}}}{(1+x)^{2M'}}\left(\tfrac{1}{\rho}(1+x)+2M'-1\right). \tag{10}$$

An upper bound of the sum rate capacity of (8) with the (10) is defined as follows

$$E[R]\le M'\log\left(1+E\left[\max_{1\le i\le u}\gamma_{i,m}\right]\right) \tag{11}$$
$$\le M'\log\left(1+\mu+\frac{U-1}{\sqrt{2U-1}}\sigma\right) \tag{12}$$

where (11) is used by Jensen's inequality and (12) by order static theory [15]. μ and σ respectively the mean and standard deviation of $\gamma_{i,m}$, which are given by

$$\mu=\int_0^\infty\frac{x\,e^{-\frac{x}{\rho}}}{(1+x)^{2M'}}\left(\tfrac{1}{\rho}(1+x)+2M'-1\right)dx \tag{13}$$
$$\sigma^2=\int_0^\infty\frac{x^2\,e^{-\frac{x}{\rho}}}{(1+x)^{2M'}}\left(\tfrac{1}{\rho}(1+x)+2M'-1\right)dx-\mu^2. \tag{14}$$

The closed-form equations of μ and σ can be obtained by the integral formula as shown in [11][14]:

$$\int_0^\infty\frac{e^{-\frac{x}{\rho}}}{(1+x)^n}dx=\begin{cases}-e^{1/\rho}\mathrm{Ei}\left(-\frac{1}{\rho}\right), & n=1\\ \rho e^{1/\rho}\mathrm{Ei}\left(-\frac{1}{\rho}\right)+1, & n=2\\ \frac{1}{(n-1)!}\sum_{k=1}^{n-1}(k-1)!\left(-\frac{1}{\rho}\right)^{n-k-1} & \\ \quad-\frac{(-1/\rho)^{n-1}}{(n-1)!}e^{1/\rho}\mathrm{Ei}\left(-\frac{1}{\rho}\right), & n>2\end{cases} \tag{15}$$

where $\mathrm{Ei}(x)$ is the exponential integral function defined by $\mathrm{Ei}(x)=-\int_{-x}^{\infty}e^{-t}/t\ dt$.

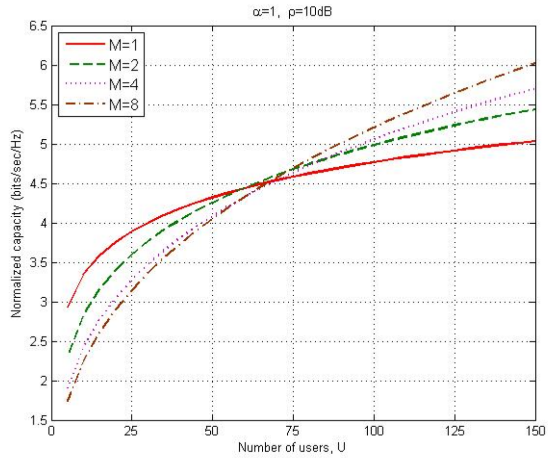

Fig. 3. Upper bounds of capacity on the proposed scheme, at cell-edge users and high SNR ($\alpha=1,\rho=10$dB)

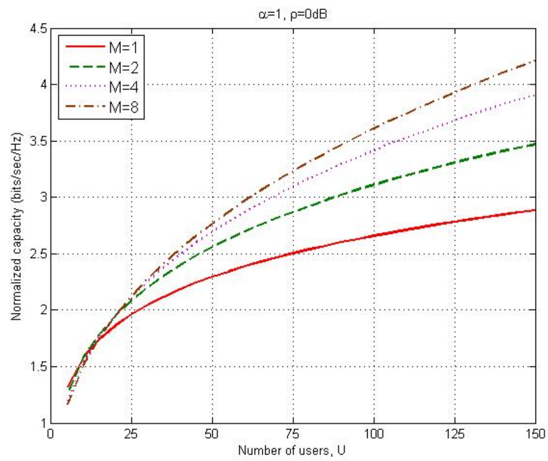

Fig. 4. Upper bounds of capacity on the proposed scheme, at cell-edge users and low SNR ($\alpha=1,\rho=0$dB)

C. Capacity of Inner-cell Users ($\alpha \ll 1$)

From the SINR of (5) with $\alpha \ll 1$ for inner-cell users, we can analyze the capacity of the coordinated random beamforming technique at inner-cell. Note that α is the attenuation factor of the neighbor BSs' signal strength. In practical, since the value of α is far less than one ($\alpha \ll 1$), the users are less interference-limited. We can rewrite the SINR of (5) as the form of

$$\gamma_{i,m}=\frac{z}{\frac{1}{\rho}+y+\alpha\upsilon} \tag{16}$$

where the z has $\chi^2(2)$ distribution, and $y+\alpha\nu$ is a weighted sum of $\chi^2(2(M-1))$ and $\chi^2(2M)$ random variable. Similar to (9)-(14), the capacity analysis of the inner-cell users can be derived, and represented in Fig. 7.

IV. NUMERICAL RESULTS

In this section, we evaluate the performance of the coordinated random beamforming scheme in multiple cellular networks. The average values of SNR are set to 0dB and 10dB, which indicate the received signal-to-noise ratio (SNR) at users. The attenuation factor α is set to 0.2 for the inner-cell users and 1 for the cell-edges. The number of antennas of a BS M and users U are considered respectively upto 10 antennas and 150 users for multi-user MIMO systems. In this paper, we investigate a two-cell model that the number of antennas is upto $2M$; multiple cells, more than two, can be easily calculated with higher degree of freedom (DoF) in (5). We effectively have NU single antenna users, considering each receive antenna as an independent user.

Fig. 3 shows the upper bounds of capacity for the coordinated random beamforming technique in an interference limited multi-cell environments. The larger the users are, the higher the capacity achieves; but, with the very small number of users, a multiplexing gain of MIMO system would not be improved significantly. One of the rationale is interference with high SNR (e.g., $\rho = 10$dB in Fig. 3). In general, the signal composed of a few but strong interference terms have a larger variance than that of many weak interference. The small variance of the other cell interference as well as home cell inter-stream interference in SINR reduces the ergodic capacity [13]. The other one of the rationale is the low probability of user selection that the randomly generated beam vectors match the receive antennas. Fig. 4 describes the capacity of cell-edge users with low SNR (i.e., $\rho = 0$dB). Both of Fig. 3 and Fig. 4 are considered as the upper bound of cell-edge user capacity.

The upper bound of the capacity in (14) is compared with simulation results in Fig. 5. We investigate a number of users and antennas. Because the number of users is less, the capacity decreases as the number of multiple BSs' antennas increases. As shown in Fig. 3 and Fig. 4, we can easily gauge the improvement of capacity as the number of the antennas increases. Fig. 6 and Fig. 7 describe the performance of the coordinated random beamforming and uncoordinated random beamforming techniques. We can find an important consequence: for user selection on maximum SINRs, the coordinated random beamforming in multi-cells adopts appropriate beam vectors that maximize the throughput, due to the fact that the users and BSs know the beam pattern well enough to anticipate the maximum SINR user selection; the uncoordinated random beamforming, on the other hand, cannot find such a maximized user selection because it is difficult to select the highest SINR users without beamforming vector information of other cells in the multi-cell environments where other cell interference varies independently.

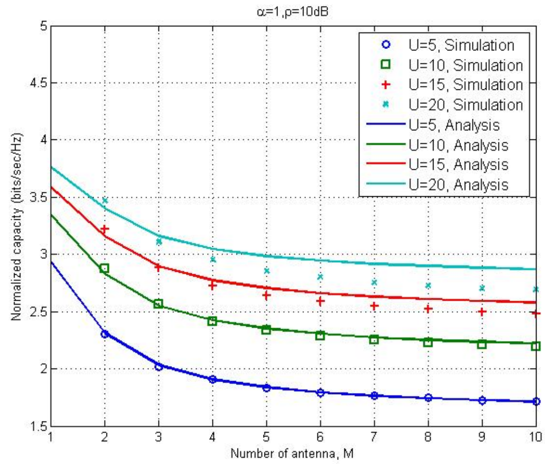

Fig. 5. Simulation vs. analysis

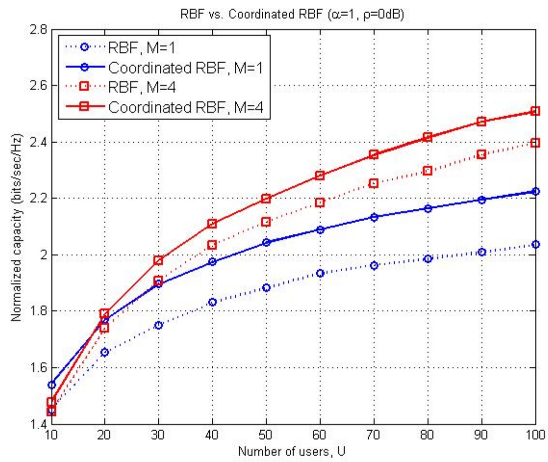

Fig. 6. Random beamforming (RBF) vs. Coordinated RBF (cell-edge)

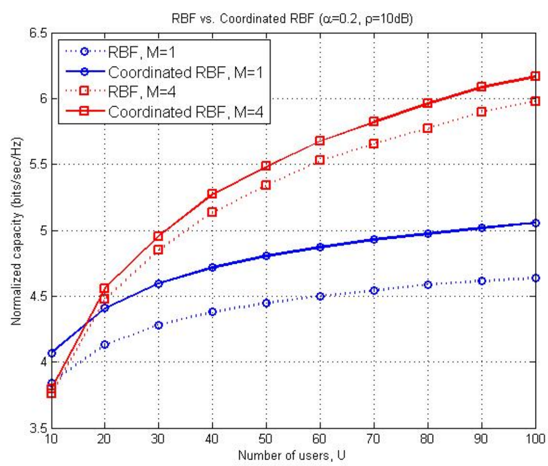

Fig. 7. Random beamforming (RBF) vs. Coordinated RBF (inner-cell)

V. CONCLUSION

We presented a novel transmission technique for the MIMO broadcast channel in multi-cell environments. Even though the multi-cell environments are hardly implemented due to the high complexity for the optimal beamforming with various multi-cell interferences, we proposed a low-complex technique of coordinated random beamforming for the multi-cell environments; the beamforming at each BS is randomized but coordinated with other BSs. The proposed scheme gives throughput gains with a small feedback by co-randomizing even for multi-cell environments. As shown in the numerical results section, the performance of coordinated random beamforming technique has greater throughput with limited feedback than uncoordinated but can be close to the channel capacity in large number of users. The presented scheme could be implemented with low-complexity in multi-cell environments in the LTE-Advanced system.

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