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POLYNOMIAL CONVERGENCE OF PREDICTOR-CORRECTOR ALGORITHMS FOR SDLCP BASED ON THE M-Z FAMILY OF DIRECTIONS†

FEIXIANG CHEN* AND RUIYIN XIANG

Abstract. We establishes the polynomial convergence of a new class of path-following methods for semidefinite linear complementarity problems (SDLCP) whose search directions belong to the class of directions introduced by Monteiro [9]. Namely, we show that the polynomial iteration-complexity bound of the well known algorithms for linear programming, namely the predictor-corrector algorithm of Mizuno and Ye, carry over to the context of SDLCP.

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1. Introduction

Several authors have discussed generalizations of interior-point algorithms for linear programming (LP) to the context of semidefinite programming (SDP). The landmark work in this direction is due to Nesterov and Nemirovskii [1, 2], where a general approach for using interior-point algorithms for solving convex programs is proposed, based on the notion of self-concordant functions. They show that the problem of minimizing a linear function over a convex set can be solved inpolynomial timeas long as a self-concordant barrier function for the convex set is known. On the other hand, Alizadeh [3] extends Yeprojective potential reduction algorithm for LP to SDP and argues that many known interior point algorithms for LP can also be transformed into algorithms for SDP in a mechanical way. Since then many authors have proposed interior-point algorithms for solving the SDP problems and SDLCP, including Alizadeh, Haeberly

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*Corresponding author.
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and Overton [4], Freund [5], Helmberg, Rendl, Vanderbei and Wolkowicz [6], Kojima, Shida and Shindoh [7], Kojima, Shindoh and Hara [8], Monteiro [9,10], Monteiro and Zhang [11,12], and Zhang [13].

2. Notation and Terminology

The set of all symmetric $n \times n$ matrices is denoted by S^n. For $Q \in S^n$, $Q \succeq 0$ means Q is positive semidefinite and $Q \succ 0$ means Q is positive definite. The inner product between them in the vector space $R^{m\times n}$ is defined as $P \bullet Q \equiv \mathrm{Tr}\ P^T Q$. The Euclidean norm and its associated operator norm are both denoted by $\|\circ\|$; The Frobenius norm of $Q \in R^{n\times n}$ is $\|Q\|_F \equiv (Q \bullet Q)^{1/2}$. For $Q, R \in R^{n\times n}$. S^n_+ and S^n_{++} denote the set of all matrices in S^n which are positive semidefinite and positive definite, respectively.

3. The SDLCP problem and preliminary discussion

This section describes the SDLCP problem and the corresponding assumptions. It also contains some notations and terminology that are used throughout our presentation. Semidefinite linear complementarity problems (SDLCP) determines a matrix pair $(X, S) \in S^n \times S^n$ satisfying

$$(X, S) \in \mathcal{F}, \quad X \succeq 0, \quad Y \succeq 0, \quad X \bullet Y = 0. \tag{1}$$

Here $\mathcal{F}$ is an $n(n+1)/2$-dimensional affine subspace of $S^n \times S^n$. We call $(X, S) \in \mathcal{F}$ with $X \succeq 0$ and $Y \succeq 0$ a feasible solution of the SDLCP (1) and $(X, S) \in \mathcal{F}$ with $X \succ 0$ and $Y \succ 0$ an interior feasible solution of the SDLCP (1) denoted by $\mathcal{F}_+$ and $\mathcal{F}_{++}$, respectively.

Throughout our presentation, we assume that

[**A1**] $\mathcal{F}$ is monotone, that is $(X_1 - X_2) \bullet (S_1 - S_2) \geq 0$ for any $(X_1, S_1) \in \mathcal{F}$ and $(X_2, S_2) \in \mathcal{F}$.

[**A2**] $\mathcal{F}_{++}$ is nonempty.

Under assumptions [A1] and [A2], it is known that problem (1) has at least one solution. Since for $(X, S) \in S^n_+ \times S^n_+$, we have $X \bullet Y = 0$ if and only if $XY = 0$, problem (1) is equivalent to find a pair (X, S) such that

$$(X, S) \in \mathcal{F}_+, \quad XS = 0.$$

It has been shown by Kojima, Shindoh and Hara [8] that the perturbed system

$$(X, S) \in \mathcal{F}_+, \quad XS = \mu I. \tag{2}$$

has a unique solution in $\mathcal{F}_+$, denoted by (X_μ, S_μ), for every $\mu > 0$, and $\lim_{\mu\to 0}(X_\mu, S_\mu)$ exists and is a solution of (1). The set $\{(X_\mu, S_\mu) : \mu > 0\}$ is called the central path associated with (1) and plays a fundamental role in the development of interior point algorithms for solving SDP and SDCLP. Using the square root $X^{1/2}$, (2) can also be alternatively expressed in the following symmetric form:

$$(X, S) \in \mathcal{F}_+, \quad X^{1/2}SX^{1/2} = \mu I \quad (or, S^{1/2}XS^{1/2} = \mu I).$$

The path-following algorithms studied in this paper are all based on the following centrality measures of a point for $(X, S) \in \mathcal{F}_+$:

$$\|X^{1/2}SX^{1/2} - \mu I\|_F \leq \gamma\mu.$$

Path following algorithms for solving (1) are based on the idea of approximately tracing the central path. Application of Newton method for computing the solution of (2) with $\mu = \hat{\mu}$ leads to the Newton search direction $(\widehat{\Delta X}, \widehat{\Delta S})$ which solves the linear system

$$X\widehat{\Delta S} + \widehat{\Delta X}S = \hat{\mu}I - XS, \qquad (X + \widehat{\Delta X}, S + \widehat{\Delta S}) \in \mathcal{F}. \tag{3}$$

This system does not always have a solution. To overcome this bottleneck, if we adapt the M-Z search directions to the monotone SDLCP, we can describe it as a solution of the system of equations:

$$X^{-1/2}(X\Delta S + \Delta XS)X^{1/2} + X^{1/2}(\Delta SX + S\Delta X)X^{-1/2} = 2(\hat{\mu}I - X^{1/2}SX^{1/2}). \tag{4}$$

Here $(X, S) \in \mathcal{F}_{++}$ denotes an iterate and $\mu = X \bullet S/n$. It was shown in paper [14] that the system (4) of equations above has the unique solution $(\Delta X, \Delta S) \in S^n \times S^n$.

We let throughout this section that $(X, S) \in \mathcal{F}_{++}$ and that $(\Delta X, \Delta S)$ is a solution of system (4) with $\hat{\mu} = \sigma\mu$ for some $\mu > 0$ and $\sigma \in [0, 1]$. Moreover, we define for every $\alpha \in R$,

$$X(\alpha) \equiv X + \alpha\Delta X, S(\alpha) \equiv S + \alpha\Delta S, \tag{5}$$

$$\mu(\alpha) \equiv (1 - \alpha + \sigma\alpha)\mu. \tag{6}$$

Lemma 2.2. *For every $\alpha \in R$, we have*

$$X(\alpha)S(\alpha) - \mu(\alpha)I = (1-\alpha)(XS - \mu I) + \alpha(XS - \sigma\mu I) + \alpha(X\Delta S + \Delta XS) + \alpha^2\Delta X\Delta S. \tag{7}$$

Proof. Follows immediately from (5), (6) and (4) with $\hat{\mu} = \sigma\mu$.

For a nonsingular matrix $P \in R^{n\times n}$, consider the following operator $H_P : R^{n\times n} \longrightarrow S^n$ defined as

$$H_P(M) \equiv \frac{1}{2}[PMP^{-1} + (PMP^{-1})^T, \quad \forall M \in R^{n\times n}.$$

The operator H_P has been used by Zhang [13] to characterize the central path of SDP problems. □

Lemma 2.3. *For every $\alpha \in [0, 1]$, we have*

$$\|H_{X^{-1/2}}[X(\alpha)S(\alpha) - \mu(\alpha)I]\|_F \leq (1-\alpha)\|X^{1/2}SX^{1/2} - \mu I\|_F + \alpha^2\delta_x\delta_s/2\mu, \tag{8}$$

where

$$\delta_x = \mu\|X^{-1/2}\Delta XX^{-1/2}\|_F, \qquad \delta_s = \|X^{1/2}\Delta SX^{1/2}\|_F. \tag{9}$$

Proof. Using (7), we can obtain

$$\begin{aligned}
&2H_{X^{-1/2}}[X(\alpha)S(\alpha)-\mu(\alpha)I]\\
&\quad= 2(1-\alpha)(X^{1/2}SX^{1/2}-\mu I)+2\alpha(X^{1/2}SX^{1/2}-\sigma\mu I)\\
&\qquad+\alpha[X^{-1/2}(X\Delta S+\Delta XS)X^{1/2}+X^{1/2}(\Delta SX+S\Delta X)X^{-1/2}]\\
&\qquad+\alpha^2(X^{-1/2}\Delta X\Delta SX^{1/2}+X^{1/2}\Delta S\Delta XX^{-1/2})\\
&\quad= 2(1-\alpha)(X^{1/2}SX^{1/2}-\mu I)\\
&\qquad+\alpha^2(X^{-1/2}\Delta X\Delta SX^{1/2}+X^{1/2}\Delta S\Delta XX^{-1/2}).
\end{aligned}$$

Then take Frobenius norm on both sides, we can prove the (8) holds. □

Lemma 2.4. *Let $(X,S)\in\mathcal{F}_{++}$ be such that $\|X^{1/2}SX^{1/2}-\mu I\|\le\mu\gamma$ for some $\gamma\in[0,1)$ and $\mu>0$. Suppose that $(\Delta X,\Delta S)\in S^{n\times n}\times S^{n\times n}$ is a solution of (4) for $\mathcal{W}\in R^{n\times n}$, where $\mathcal{W}=\sigma\mu I-X^{1/2}SX^{1/2}$. Let $\delta_x=\mu\|X^{-1/2}\Delta XX^{-1/2}\|_F$ and $\delta_s=\|X^{1/2}\Delta SX^{1/2}\|_F$. Then,*

$$\delta_x\delta_s\le\frac{1}{2}(\delta_x^2+\delta_s^2)\le\frac{\|\mathcal{W}\|_F^2}{2(1-\gamma)^2}.$$

Proof. We let $\mathcal{W}=H_{X^{-1/2}}[X\Delta S+\Delta XS]$. Using (4) and simple algebraic manipulation, we can obtain

$$\begin{aligned}
\mathcal{W}&=X^{1/2}\Delta SX^{1/2}+\mu X^{-1/2}\Delta XX^{-1/2}+\frac{1}{2}X^{-1/2}\Delta XX^{-1/2}(X^{1/2}SX^{1/2}-\mu I)\\
&\quad+\frac{1}{2}(X^{1/2}SX^{1/2}-\mu I)X^{-1/2}\Delta XX^{-1/2},
\end{aligned}$$

from which it follows that

$$\begin{aligned}
\|\mathcal{W}\|_F&\ge\|X^{1/2}\Delta SX^{1/2}+\mu X^{-1/2}\Delta XX^{-1/2}\|_F\\
&\quad-\|X^{1/2}SX^{1/2}-\mu I\|\|X^{-1/2}\Delta XX^{-1/2}\|_F\\
&\ge(\|X^{1/2}\Delta SX^{1/2}\|_F^2+\mu^2\|X^{-1/2}\Delta eXX^{-1/2}\|_F^2)^{1/2}-\gamma\mu\delta_x/\mu\\
&\ge\sqrt{\delta_x^2+\delta_s^2}-\gamma\delta_x\ge(1-\gamma)\sqrt{\delta_x^2+\delta_s^2},
\end{aligned}$$

where the second inequality follows from the assumption that $\|X^{1/2}SX^{1/2}-\mu I\|\le\mu\gamma$ and the fact that $(X^{-1/2}\Delta XX^{-1/2})\bullet(X^{1/2}\Delta SX^{1/2})=\Delta X\bullet\Delta S\ge 0$, due to the monotonicity of $\mathcal{F}$. The result now follows trivially from the last inequality. □

Now we are ready to state the main result of this section.

Lemma 2.5. *Suppose that $(X,S)\in N_F(\mu,\gamma)$ for some $\gamma\in(0,1)$ and let $(\Delta X,\Delta S)\in S^{n\times n}\times S^{n\times n}$ be the solution of (4). Then,*

$$\|H_{X^{-1/2}}[X(\alpha)S(\alpha)-\mu(\alpha)I]\|_F\quad\le\quad\{(1-\alpha)\gamma+\alpha^2\frac{n(1-\sigma)^2+\gamma^2}{4(1-\gamma)^2}\}\mu$$

Proof. Follows immediately from (8), the assumption that $(X,S) \in N_F(\mu,\gamma)$ and Lemma 2.3, we can obtain

$$\begin{aligned}
&\|H_{X^{-1/2}}[X(\alpha)S(\alpha) - \mu(\alpha)I]\|_F \\
&\quad \le \{(1-\alpha)\gamma + \alpha^2 \frac{\|\sigma\mu I - X^{1/2}SX^{1/2}\|_F^2}{4(1-\gamma)^2\mu^2}\}\mu \\
&\quad = \{(1-\alpha)\gamma + \alpha^2 \frac{\|(\sigma-1)\mu I\|_F^2 + \|\mu I - X^{1/2}SX^{1/2}\|_F^2}{4(1-\gamma)^2\mu^2}\}\mu \\
&\quad \le \{(1-\alpha)\gamma + \alpha^2 \frac{n(\sigma-1)^2+\gamma^2}{4(1-\gamma)^2}\}\mu.
\end{aligned}$$

The equality holds from the fact $(X^{1/2}SX^{1/2} - \mu I) \bullet I = 0$, then we complete the proof. □

We start by stating two technical results. The first one is due to Monteiro (see Lemma 2.1 of [10])and plays a crucial role in our analysis.

Lemma 2.6. *Suppose that $(X,S) \in S_{++}^n \times S_{++}^n$ and $\mathcal{M} \in R^{n\times n}$ is a nonsingular matrix. Then, for every $\mu \in R$, we have*

$$\|X^{1/2}SX^{1/2} - \mu I\|_F \le \|H_{\mathcal{M}}(XS - \mu I)\|_F,$$

with equality holding if $\mathcal{M}XS\mathcal{M}^{-1} \in S^n$.

Lemma 2.7. *Suppose $V, Q \in R^{n\times n}$ be given, and $\mathcal{M}$ is nonsingular which satisfying*

$$\|H_{\mathcal{M}}(V) - I\|_\infty < 1, \tag{10}$$

then, the matrix V is nonsingular.

Proof. Define $M \equiv \mathcal{M}V\mathcal{M}^{-1}/2$. Condition (10) implies that $M + M^T \succ 0$, and this clearly implies that M is nonsingular . Hence, V is also nonsingular.

When the constant Γ defined in (11) is such that $\Gamma \le \gamma$, the Theorem below implies that the sequence $\{(X^k, S^k)\}$ generated by Algorithm-I is contained in the neighborhood $N_F(\mu^k, \gamma)$. This Theorem is also used in the analysis of the corrector steps of the predictor-corrector algorithm presented in the next section. □

Theorem 2.1. *Suppose $\gamma \in (0,1)$ and $\delta \in [0, \sqrt{n})$ be constants satisfying*

$$\Gamma \equiv \frac{\gamma^2+\delta^2}{4(1-\gamma)^2}(1 - \frac{\delta}{\sqrt{n}})^{-1} \le 1. \tag{11}$$

Suppose that $(X,S) \in N_F(\mu,\gamma)$ for some $\mu > 0$, and that $(\Delta X, , \Delta S)$ denote the solution of system (4) with $\hat{\mu} = \sigma\mu$ and $\sigma = 1 - \delta/\sqrt{n}$. Then,

(1) $(\widetilde{X}, \widetilde{S}) = (X + \Delta X, S + \Delta S) \in N_F(\sigma\mu, \Gamma)$; *(2)* $\widetilde{X} \bullet \widetilde{S} = (1 - \delta/\sqrt{n}) X \bullet S$.

Proof. It follows from Lemma 2.5, the definition of σ and [11] that for every $\alpha \in [0,1]$,

$$\begin{aligned}\|H_{X^{-1/2}}[X(\alpha)S(\alpha)-\mu(\alpha)I]\|_F &\leq \{(1-\alpha)\gamma+\alpha^2\frac{n(\sigma-1)^2+\gamma^2}{4(1-\gamma)^2}\}\mu.\\ &\leq \{(1-\alpha)\gamma+\alpha\frac{\delta^2+\gamma^2}{4(1-\gamma)^2}\}\mu.\\ &= \{(1-\alpha)\gamma+\alpha\Gamma(1-\delta/\sqrt{n})\}\mu.\\ &= \{(1-\alpha)\gamma+\sigma\Gamma\alpha\}\mu,\end{aligned}$$

and hence, in view of (6) and (11), we have

$$\|H_{X^{-1/2}}[\frac{X(\alpha)S(\alpha)}{\mu(\alpha)}]-I\|_F \leq \frac{(1-\alpha)\gamma+\sigma\Gamma\alpha}{1-\alpha+\sigma\alpha} \leq max\{\gamma,\Gamma\} < 1.$$

By Lemma 2.7, this implies that $X(\alpha)S(\alpha)$ is nonsingular for every $\alpha \in (0,1]$. Hence, $X(\alpha)$ and $S(\alpha)$ are also nonsingular for every $\alpha \in (0,1]$. Using the fact that $(X,S) \in \mathcal{F}_{++}, (X+\Delta X, S+\Delta S) \in \mathcal{F}$ and a simple continuity argument, we see $(X(\alpha),S(\alpha)) \in \mathcal{F}_{++} \subseteq S^n_{++} \times S^n_{++}$ for every $\alpha \in (0,1]$. Applying Lemma 2.6 with $(X,S) = (X(\alpha),S(\alpha))$ and $\mathcal{M} = X^{-1/2}$, we conclude that for every $\alpha \in (0,1]$,

$$\begin{aligned}\|X(\alpha)^{1/2}S(\alpha)X(\alpha)^{1/2}-\mu(\alpha)I\|_F &\leq \|H_{X^{-1/2}}[X(\alpha)S(\alpha)-\mu(\alpha)I]\|_F.\\ &\leq \|X^{-1/2}X(\alpha)S(\alpha)X^{1/2}-\mu(\alpha)I\|_F\\ &\leq \{(1-\alpha)\gamma+\sigma\Gamma\alpha\}\mu.\end{aligned}$$

Setting $\alpha = 1$ in the last relation and using the fact that $(X(1),S(1)) \in \mathcal{F}_{++}$ together with (5) and (6), we conclude that $(X(1),S(1)) \equiv (X+\Delta X, S+\Delta S) \in N_F(\sigma\mu,\Gamma)$. Statement (2) follows from (6) with $\alpha = 1$ and the definition of σ. □

4. Predictor-corrector algorithm

In this section we give the polynomial convergence of a predictor-corrector algorithm which is a direct extension of the LP predictor-corrector algorithm studied by Minzuno, Todd and Ye. The algorithm considered in this section is as follows.

ALGORITHM-I

Choose a constant $0 < \tau < 1/2$ satisfying the conditions of Theorem 3.1 below. Let $\varepsilon \in (0,1)$ and $(X^0,S^0) \in \mathcal{F}_{++}$ be such that $(X^0,S^0) \in N_F(\mu_0,\tau)$, and set $k := 0$.

Repeat until $\mu^k \leq \varepsilon\mu_0$ **do**

1. **Predictor step:** Let $(X,S) = (X^k,S^k)$ and compute the solution $(\Delta X_P^k, \Delta S_P^k)$ of system (4); compute the largest $\widetilde{\theta}$ so that $(X(\theta),S(\theta)) \in N_F((1-\theta)\mu_k, 2\tau)$ for every $\theta \in [0,\widetilde{\theta}]$, where $(X(\theta),S(\theta)) = (X,S)+\theta(\Delta X_P^k, \Delta S_P^k)$.

2. **Corrector step:** Let $(\widetilde{X}, \widetilde{S}) = (X(\widetilde{\theta}), S(\widetilde{\theta}))$ and compute the solution $(\Delta X_C^k, \Delta S_C^k)$ of system (4) with $(X, S) = (\widetilde{X}, \widetilde{S})$, $\widetilde{\mu} = (1-\widetilde{\theta})\mu^k$; set $(X^{k+1}, S^{k+1}) = (\widetilde{X}, \widetilde{S}) + (\Delta X_C^k, \Delta S_C^k)$.

3. Increment k by 1.

End

To analyze this method, we start by showing:

Theorem 3.1. *Assume that $\tau \in (0, 1/8]$. Then, algorithm-I satisfies the following statements:*

(1) for every $k \geq 0$, $(X^k, S^k) \in N_F(\mu_k, \tau)$;

(2) for every $k \geq 0$, $X^k \bullet S^k \leq (1-\widetilde{\theta})^k X^0 \bullet S^0$, where $\widetilde{\theta} = 1/\mathcal{O}(\sqrt{n})$;

(3) the algorithm terminates in at most $\mathcal{O}(\sqrt{n} log \varepsilon^{-1})$ iterations.

Proof. Statement (3) and the well-definedness of Algorithm-I follow directly from (1) and (2). In turn, these two statements follow by a simple induction argument, the two lemmas below and relation (6). □

The following lemma analyzes the predictor step of Algorithm-I.

Lemma 3.2. *Suppose that $(X, S) \in N_F(\mu, \tau)$ for some $\tau \in (0, 1/2)$. Let $(\Delta X_P, \Delta S_P)$ denote the solution of (4) with $\widetilde{\mu} = 0$. Let $\widetilde{\theta}$ denote the unique positive root of the second-order polynomial $p(\theta)$ defined as*

$$p(\theta) = \frac{\tau^2 + n}{2(1-\tau)^2}\theta^2 + \tau\theta - \tau. \tag{12}$$

Then for any $\theta \in [0, \widetilde{\theta}]$, we have:

$$(X(\theta), S(\theta)) \equiv (X + \theta\Delta X_P, S + \theta\Delta S_P) \in N_F((1-\theta)\mu, 2\tau). \tag{13}$$

Moveover, $\widetilde{\theta} = 1/\mathcal{O}(\sqrt{n})$.

Proof. Using Lemma 2.5 with $\gamma = \tau$ and $\sigma = 0$, the fact that $p(\theta) \leq 0$ for $\theta \in [0, \widetilde{\theta}]$, $\tau < 1/2$ and Lemma 2.5, we conclude

$$\begin{aligned} \|H_{X^{-1/2}}[X(\theta)S(\theta) - \mu(\theta)]\|_F &\leq \{(1-\theta)\tau + \frac{\tau^2 + n}{2(1-\tau)^2}\theta^2\}\mu \\ &\leq 2\tau\mu(\theta) + p(\theta)\mu \leq 2\tau\mu(\theta). \end{aligned}$$

An argument similar to the one used in Theorem 2.1 together with (6) and the fact that $2\tau < 1$ and $\sigma = 0$ can be used to show that (13) holds. The assertion that $\widetilde{\theta} = 1/\mathcal{O}(\sqrt{n})$ follows by a straightforward verification. □

The following lemma analyzes the corrector step of Algorithm-I.

Lemma 3.3. *Suppose that $(X, S) \in N_F(\mu, 2\tau)$ for some $\tau \in (0, 1/8)$. Let $(\Delta X_C, \Delta S_C)$ denote the solution of (4) with $\widetilde{\mu} = \mu$. Then, we have:*

$$(\widetilde{X} + \Delta X_C, \widetilde{S} + \Delta S_C) \in N_F(\mu, \tau).$$

Proof. Follows immediately from Theorem 2.1 with $\sigma = 1$ (or equivalently, $\delta = 0$) and $\gamma = 2\tau$, and noting that Γ defined by (11) satisfies $\Gamma \leq \tau$ when $\tau \leq 1/8$. □

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Feixiang Chen received M.Sc. from Xi'an Jiaotong University. Since 2009 he has been teaching at Chongqing Three Gorges University. His research interests include numerical optimization and mathematical programming.

College of mathematics and computer science, Chongqing Three-Gorges University, Wanzhou, Chongqing, 404000, P.R.China.
e-mail: cfx2002@126.com

Ruiyin Xiang received M.Sc. from Chongqing University. Since 1995 he has been teaching at Chongqing Three Gorges University. His research interests are finite element and boundary element.

College of mathematics and computer science, Chongqing Three-Gorges University, Wanzhou, Chongqing, 404000, P.R.China.
e-mail: xiangry@163.com