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THE DYNAMIC OF TWO-SPECIES IMPULSIVE DELAY GILPIN-AYALA COMPETITION SYSTEM WITH PERIODIC COEFFICIENTS†

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Abstract. In this paper, we consider two-species periodic Gilpin-Ayala competition system with delay and impulsive effect. By using some analysis methods, sufficient conditions for the permanence of the system are derived. Further, we give the conditions of the existence and global asymptotic stable of positive periodic solution.

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1. Introduction

Impulsive differential equations are suitable for the mathematical simulation of evolutionary process whose states are to sudden change at certain moments. Equations of this kind are found in almost every domain of applied sciences[1-9]. Recently, theory and applications of impulsive delay differential equations have developed[10-13]. Mathematical models of various biological process and phenomena in the study of population dynamics, biology, ecology,etc. can be expressed by impulsive delay differential equations.

In this paper, we consider the following two-species Gilpin-Ayala competition system with delay and impulsive effects.

$$\begin{cases} \left.\begin{array}{l} y_1'(t) = y_1(t)(b_1(t) - b_{11}(t)y_1^{\theta_{11}}(t-\tau_{11}) - b_{12}(t)y_2^{\theta_{12}}(t-\tau_{12})) \\ y_2'(t) = y_2(t)(b_2(t) - b_{21}(t)y_1^{\theta_{21}}(t-\tau_{21}) - b_{22}(t)y_2^{\theta_{22}}(t-\tau_{22})) \end{array}\right\} & t \neq \tau_k, \\ \left.\begin{array}{l} \Delta y_1(\tau_k^+) = (1+h_1^k)y_1(\tau_k) \\ \Delta y_2(\tau_k^+) = (1+h_2^k)y_2(\tau_k) \end{array}\right. & t = \tau_k \end{cases} \tag{1.1}$$

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with

$$(y_1(t), y_2(t)) = (\varphi_1(t), \varphi_2(t)) = \varphi(t), -\tau \le t \le 0, \varphi \in L([-\tau, 0], [0, +\infty)), \varphi(0) > 0 \qquad (1.2)$$

where $L([-\tau, 0], [0, +\infty))$denotes the set of Lebesgue measurable functions on $[-\tau, 0], \tau = max\{\tau_{11}, \tau_{12}, \tau_{21}, \tau_{22}\}$. $y_1(t), y_2(t)$ denotes the concentration of two competition species at time t, $b_i(t)(i = 1, 2)$ is ith growth rate at this time, $b_{ij}(i, j = 1, 2)$ denotes the competitive coefficient between the ith species and the jth species, $k \in Z_+ = \{1, 2, \cdots\}$.

If system (1.1) without impulsive effect, delays and $\theta_{ij} = 1, i \ne j$, then system (1.1) becomes the following competition model:

$$\begin{cases} y_1'(t) &= y_1(t)(b_1(t) - b_{11}(t)y_1^{\theta_{11}}(t) - b_{12}(t)y_2(t)) \\ y_2'(t) &= y_2(t)(b_2(t) - b_{21}(t)y_1(t) - b_{22}(t)y_2^{\theta_{22}}(t)) \end{cases} \qquad (1.3)$$

If the coefficients of system (1.2) are positive constants, then system (1.2) was proposed and studied by Gilpin and Ayala [14]. To consider the periodic environmental factors in real population, it is reasonable to study Gilpin-Ayala competition system with periodic coefficients. Fan and Wang [15] have investigated a generalized periodic n-species competition system with delays, they have obtained the sufficient conditions for the existence of positive periodic solution.

As a mathematical model, system (1.1) is more general and includes many ecology models as special. If $\theta_{ij} = 1, i, j = 1, 2$, the system (1.1) becomes the following competition system:

$$\begin{cases} \left.\begin{array}{l} y_1'(t) = y_1(t)(b_1(t) - b_{11}(t)y_1(t - \tau_{11}) - b_{12}(t)y_2(t - \tau_{12})) \\ y_2'(t) = y_2(t)(b_2(t) - b_{21}(t)y_1(t - \tau_{21}) - b_{22}(t)y_2(t - \tau_{22})) \end{array}\right\} & t \ne \tau_k, \\ \left.\begin{array}{l} \Delta y_1(\tau_k^+) = (1 + h_1^k)y_1(\tau_k) \\ \Delta y_2(\tau_k^+) = (1 + h_2^k)y_2(\tau_k) \end{array}\right. & t = \tau_k \end{cases} \qquad (1.4)$$

The organization of the paper is as follows. In Section 1, a two-species periodic Gilpin-Ayala competition system with delay and impulsive effect is proposed. In Section 2, we will give some notations and lemmas. In Section 3, we establish sufficient conditions for existence and global asympotic stability of positive periodic solutions of system (1.1). Lastly, we give a brief discussion.

2. Preliminaries

In what follows, for a continuous $\omega-$ periodic function $g(t)$, we shall introduce the notations

$$\overline{g} = \frac{1}{\omega}\int_0^\omega g(t)dt, g^l = \min_{t\in[0,\omega]} g(t), g^m = \max_{t\in[0,\omega]} g(t).$$

We shall make following hypotheses:

(H_1) $0 < \tau_1 < \tau_2 < \cdots < \tau_k < \tau_{k+1} < \cdots$ and $\lim_{k\to+\infty} \tau_k = +\infty$.

(H_2) $\{h_i^k\}$is a real sequence and $h_i^k > -1, i = 1, 2, k = 1, 2, \cdots$.

(H_3) The function $b_i(t), b_{ij}(t), \imath, j = 1, 2$ are positive continuous $\omega-$ periodic functions. $\theta_{ij}, i, j = 1, 2$ are positive constants, time delays $\tau_{ij}, i, j = 1, 2$ are nonnegative constants.

Definition 2.1. Functions $y_1(t), y_2(t) \in ([-\tau, 0], [0, +\infty))$ are said to be solution of system (1.1) on $[-\tau, +\infty)$ if:

(1) $y_1(t), y_2(t)$ are absolutely continuous on each interval $[0, \tau_1]$ and $[\tau_k, \tau_{k+1}]$, $k = 1, 2, \cdots$.

(2) $y_1(\tau_k^+), y_2(\tau_k^+)$ and $y_1(\tau_k^-), y_2(\tau_k^-)$ exist and $y_1(\tau_k^+) = y_1(\tau_k^-), y_2(\tau_k^+) = y_2(\tau_k^-)$ for any $\tau_k, k = 1, 2, \cdots$.

(3) $y_1(t)$ and $y_2(t)$ satisfy system (1.1) for almost everywhere in $[0, +\infty)$ and at impulsive points τ_k may have discontinuity of the first kind.

Lemma 2.1 ([17]). *Consider a single-species periodic logistic system with impulses*

$$\begin{cases} x'(t) = x(t)(a(t) - b(t)x^{\theta}(t)), t \neq \tau_k \\ x(\tau_k^+) = (1 + h_k), \qquad t = \tau_k, k \in N \end{cases} \tag{2.1}$$

where θ is a positive constant, $a(t), b(t)$ are continuous ω-periodic functions with $b(t) > 0, \omega\overline{a} > 0$ and if there exists a positive integer q such that $t_{k+q} = t_k + \omega, hk + q = h_k$. Let $x(t)$ be any solution of system (2.1) with the initial value $x(0^+) > 0$.

(1) If $\sum_{k=1}^{q} \ln(1 + h_k) + \omega\overline{a} > 0$, then system (2.1) has a unique ω-periodic solution $x^*(t)$, which is globally asymptotically stable in the sense that $\lim_{t\to+\infty} \mid x(t) - x^*(t) \mid= 0$;

(2) If $\sum_{k=1}^{q} \ln(1 + h_k) + \omega\overline{a} < 0$, then $\lim_{t\to+\infty} x(t) = 0$.

Obviously, if there exist a positive integer q such that $\tau_{k+q} = \tau_k + \omega, h_i^{k+q} = h_i^k, i = 1, 2.$ and

$$\sum_{k=1}^{q} \ln(1 + h_i^k) + \omega\overline{b_i} > 0, i = 1, 2.$$

then it follows forms lemma 2.1 that

$$\begin{cases} x_1'(t) = x_1(t)(b_1(t) - b_{11}(t)x_1^{\theta_{11}}(t)), t \neq \tau_k \\ x_1(\tau_k^+) = (1 + h_1^k), \qquad t = \tau_k, k \in N \end{cases} \tag{2.2}$$

and

$$\begin{cases} x_2'(t) = x_2(t)(b_2(t) - b_{22}(t)x_2^{\theta_{22}}(t)), t \neq \tau_k \\ x_2(\tau_k^+) = (1 + h_2^k), \qquad t = \tau_k, k \in N \end{cases} \tag{2.3}$$

have the unique positive globally asymptotically stable ω-periodic solutions $x_1^*(t)$ and $x_2^*(t)$ respectively.

We will discuss the existence of positive solution of system (1.1). We give the following assumption:

(H_4) $\prod_{0<\tau_k<t} (1 + h_i^k)$ is an ω-periodic function and there exist four positive constants $m_i, M_i, i = 1, 2$ such that $m_1 \leq \prod_{0<\tau_k<t} (1 + h_1^k) \leq M_1$ and $m_2 \leq$

$\prod_{0<\tau_k<t}(1+h_2^k) \leq M_2$ *for* $t>0$.

Under the above assumptions $(H_1)-(H_4)$,*we consider non-impulsive differential equation:*

$$\begin{cases} x_1'(t) = x_1(t)(b_1(t) - b_{11}(t)p_{11}(t)x_1^{\theta_{11}}(t-\tau_{11})) - b_{12}(t)p_{12}(t)x_2^{\theta_{12}}(t-\tau_{12})), \\ x_2'(t) = x_2(t)(b_2(t) - b_{21}(t)p_{21}(t)x_1^{\theta_{21}}(t-\tau_{21})) - b_{22}(t)p_{22}(t)x_2^{\theta_{22}}(t-\tau_{22})) \end{cases} \tag{2.4}$$

where

$$p_{ij}(t) = \prod_{0<\tau_k<t-\tau_{ij}} (1+h_j^k)^{\theta_{ij}}, i,j=1,2 \tag{2.5}$$

with initial condition $(x_1(t),x_2(t)) = (\varphi_1(t),\varphi_2(t))$ *for* $-\tau \leq t \leq 0, \varphi_1(0) > 0, \varphi_2(0)) > 0, (\varphi_1(t),\varphi_2(t) \in L([-\tau,0],[0,+\infty))$.

By a solution $(x_1(t),x_2(t))$ *of (2.2) and (2.4), we mean an absolutely continuous functions* $x_1(t),x_2(t)$ *defined on* $[-\tau,0]$ *which satisfies (2.2) a.e. for* $t>0$ *and* $x_1(t)=\varphi(t), x_2(t)=\psi(t)$ *on* $[-\tau,0]$.

The following lemma will be used in the proofs of our results.

Lemma 2.2. *Assume that* $(H_1)-(H_4)$ *hold. Then*

(1) $x_i(t),(i=1,2)$ *is a solution of system (2.2) on* $[-\tau,+\infty)$, *then* $y_i(t) = \prod_{0<\tau_k<t}(1+h_i^k)x_i(t),(i=1,2)$ *is a solution of (1.1) on* $[-\tau,+\infty)$.

(2) $y_i(t),(i=1,2)$ *is a solution of system (1.1) on* $[-\tau,+\infty)$, *then* $x_i(t) = \prod_{0<\tau_k<t}(1+h_i^k)^{-1}y_i(t),(i=1,2)$ *is a solution of (2.2) on* $[-\tau,+\infty)$.

Proof. First, we prove (1). Let $x_i(t), i=1,2$ be a solution of system (2.2). It is easy to see $y_i(t) = \prod_{0<\tau_k<t}(1+h_i^k)x_i(t), i=1,2$ is absolutely continuous on the interval $(\tau_k,\tau_{k+1}]$ and for any $t\neq\tau_k, k=1,2,\cdots$,

$$\begin{aligned} \prod_{0<\tau_k<t}(1+h_1^k)x_1'(t) &= \prod_{0<\tau_k<t}(1+h_1^k)x_1(t)(b_1(t) - b_{11}(t)p_{11}(t)x_1^{\theta_{11}}(t-\tau_{11})) \\ &\quad - b_{12}(t)p_{12}(t)x_2^{\theta_{12}}(t-\tau_{12})), \\ \prod_{0<\tau_k<t}(1+h_2^k)x_2'(t) &= \prod_{0<\tau_k<t}(1+h_2^k)x_2(t)(b_2(t) - b_{21}(t)p_{21}(t)x_1^{\theta_{21}}(t-\tau_{21})) \\ &\quad - b_{22}(t)p_{22}(t)x_2^{\theta_{22}}(t-\tau_{22})), \end{aligned}$$

that is

$$\begin{aligned} y_1'(t) &= y_1(t)(b_1(t) - b_{11}(t)y_1^{\theta_{11}}(t-\tau_{11}) - b_{12}(t)y_2^{\theta_{12}}(t-\tau_{12})), \\ y_2'(t) &= y_2(t)(b_2(t) - b_{21}(t)y_1^{\theta_{21}}(t-\tau_{21}) - b_{22}(t)y_2^{\theta_{22}}(t-\tau_{22})). \end{aligned}$$

On the other hand, for every $\tau_k, k=1,2,\cdots$,

$$y_1(\tau_k^+) = \lim_{t\to\tau_k^+}\prod_{0<\tau_j<t}(1+h_1^j)x_1(t) = \prod_{0<\tau_j\leq\tau_k}(1+h_1^k)x_1(\tau_k) = (1+h_1^k)\prod_{0<\tau_j<\tau_k}x_1(\tau_k),$$

$$y_2(\tau_k^+) = \lim_{t\to\tau_k^+}\prod_{0<\tau_j<t}(1+h_2^j)x_2(t) = \prod_{0<\tau_j\leq\tau_k}(1+h_2^k)x_2(\tau_k) = (1+h_2^k)\prod_{0<\tau_j<\tau_k}x_2(\tau_k),$$

and

$$y_1(\tau_k) = \prod_{0<\tau_j<\tau_k}(1+h_1^j)x_1(\tau_k), y_2(\tau_k) = \prod_{0<\tau_j<\tau_k}(1+h_2^j)x_2(\tau_k).$$

Thus, for every $k = 1, 2, \cdots$,

$$y_1(\tau_k^+) = (1+h_1^k)y_1(\tau_k), y_2(\tau_k^+) = (1+h_2^k)y_2(\tau_k). \tag{2.6}$$

Next, we prove (2). Since $y_1(t), y_2(t)$ is absolutely continuous on $(\tau_k, \tau_{k+1}]$ and, in view of (2.6), it follows that for any $k = 1, 2, \cdots$,

$$x_1(\tau_k^+) = \prod_{\tau^*\le\tau_j\le\tau_k}(1+h_1^j)^{-1}y_1(\tau_k^+) = \prod_{\tau^*\le\tau_j<\tau_k}(1+h_1^j)^{-1}y_1(\tau_k),$$

$$x_2(\tau_k^+) = \prod_{\tau^*\le\tau_j\le\tau_k}(1+h_2^j)^{-1}y_2(\tau_k^+) = \prod_{\tau^*\le\tau_j<\tau_k}(1+h_2^j)^{-1}y_2(\tau_k),$$

and

$$x_1(\tau_k^-) = \prod_{\tau^*\le\tau_j\le\tau_{k-1}}(1+h_1^j)^{-1}y_1(\tau_k^-) = \prod_{\tau^*\le\tau_j<\tau_k}(1+h_1^j)^{-1}y_1(\tau_k) = x_1(\tau_k),$$

$$x_2(\tau_k^-) = \prod_{\tau^*\le\tau_j\le\tau_{k-1}}(1+h_2^j)^{-1}y_2(\tau_k^-) = \prod_{\tau^*\le\tau_j<\tau_k}(1+h_2^j)^{-1}y_2(\tau_k) = x_2(\tau_k),$$

where $k = 1, 2, \cdots$. Which implies that $x_1(t), x_2(t)$ is continuous on $[\tau, +\infty)$. It is easy to prove that $x_1(t), x_2(t)$ is absolutely continuous on $[\tau, +\infty)$. Now, one can easily obtain $x_1(t) = \prod_{0\le\tau_k<t}(1+h_1^k)^{-1}y_1(t)$,
$x_2(t) = \prod_{0\le\tau_k<t}(1+h_2^k)^{-1}y_2(t)$ is a solution of (2.2). This completes the proof. □

Let X, Z be normed vector space, be a linear mapping, and $N : X \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $dimKerL$ =codim $ImL < +\infty$ and ImL is closed in Z. If L is a Fredholm mapping of index zero there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $ImP = KerL, KerQ = ImL = Im(I-Q)$. It follows that $L|domL \cap KerP : (I-P)X \to ImL$ is invertible. We denote the inverse of that map by K_p. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_p(I-Q)N : \bar{\Omega} \to X$ is compact. Since ImQ is isomorphic to $KerL$, there exist isomorphisms $J : ImQ \to KerL$.

In the proof of our existence theorem below, we will use the continuation theorem of Gaines and mawhin [16].

Lemma 2.3. *Let L be a Fredholm mapping of index zero and N be L-compact on $\bar{\Omega}$. Suppose*

(a) For each $\lambda \in (0,1)$, every solution x of $Lx = \lambda Nx$ is such that $x \in \partial\Omega$.

(b) $QNx \neq 0$ for each $x \in \partial\Omega \cap KerL$ and $deg\{JQN, \Omega \cap KerL, 0\} \neq 0$,

Then the equation $Lx = Nx$ has at least one solution lying in $DomL \cap \bar{\Omega}$.

3. Main results

In this section, we will prove the permanence of the system (1.1) and give the conditions for the existence and global asymptotic stability of positive periodic solution of system (1.1).

Theorem 3.1. *Assume that ($H_1 - H_3$) and $\tau_{ii} = 0, i = 1, 2$, hold. If there exist a positive integer q such that $\tau_{k+q} = \tau_k + \omega, h_i^{k+q} = h_i^k, i = 1, 2$ and*

$$\sum_{k=1}^{q} \ln(1 + h_1^k) + \omega\overline{b_1 - b_{12}x_2^*(t)} > 0, \sum_{k=1}^{q} \ln(1 + h_2^k) + \omega\overline{b_2 - b_{21}x_1^*(t)} > 0.$$

Then system (1.1) is permanent. where $x_i^*(t), i = 1, 2$ are described above.

Theorem 3.2. *Assume that ($H_1 - H_3$) and $\tau_{ii} = 0, i = 1, 2$, hold. If there exist a positive integer q such that $\tau_{k+q} = \tau_k + \omega, h_i^{k+q} = h_i^k, i = 1, 2$ and $\sum\limits_{k=1}^{q} \ln(1 + h_i^k) + \omega\overline{b_i}/ < 0$. Then system (1.1) is extinct.*

Theorem 3.3. *For system (1.1), we assume that $(H_1) - (H_4)$ hold and $P_1 > 0, Q_1 > 0$. If t is large enough, then $K_1 \le y_1(t) \le K_2, N_1 \le y_2(t) \le N_2$, Where*

$$K_1 = m_1(\frac{P_1}{b_{11}^m M_1^{\theta_{11}}})^{\frac{1}{\theta_{11}}} \exp((P_1 - K_2)\tau^*), K_2 = M_1(\frac{b_1^m}{b_{11}^l m_1^{\theta_{11}}})^{\frac{1}{\theta_{11}}} \exp(b_1^m \tau^*)$$

$$N_1 = m_2(\frac{Q_1}{b_{22}^m M_2^{\theta_{22}}})^{\frac{1}{\theta_{22}}} \exp((Q_1 - N_2)\tau^*), N_2 = M_2(\frac{b_2^m}{b_{22}^l m_2^{\theta_{22}}})^{\frac{1}{\theta_{22}}} \exp(b_2^m \tau^*)$$

where

$$P_1 = b_1^l - b_{12}^m N_2^{\theta_{12}}, Q_1 = b_2^l - b_{21}^m K_2^{\theta_{21}}$$

Proof. From system (2.2), we have

$$x_1'(t) \le x_1(t)(b_1(t) - b_{11}(t)p_{11}(t)x_1^{\theta_{11}}(t - \tau_{11})) \le x_1(t)(b_1^m - b_{11}^l m_1^{\theta_{11}} x_1^{\theta_{11}}(t - \tau_{11})) \quad (3.1)$$

$$x_2'(t) \le x_2(t)(b_2(t) - b_{22}(t)p_{22}(t)x_2^{\theta_{22}}(t - \tau_{22})) \le x_2(t)(b_2^m - b_{22}^l m_2^{\theta_{22}} x_2^{\theta_{22}}(t - \tau_{22})) \quad (3.2)$$

Take

$$K_2^* = (\frac{b_1^m}{b_{11}^l m_1^{\theta_{11}}})^{\frac{1}{\theta_{11}}}, N_2^* = (\frac{b_2^m}{b_{22}^l m_2^{\theta_{22}}})^{\frac{1}{\theta_{22}}}$$

Firstly, we prove $y_1(t) \le K_2, y_2(t) \le N_2$.

Case 1. Suppose $x_1(t)$ is oscillatory K_2^*. That is, there exist a time sequence t_k and $\tau^* < t_1 < t_2 < \cdots < t_n < \cdots$ such that $\lim\limits_{k\to\infty} t_k = \infty$ and $x_1(t_k) = K_1^*$. Let $x_1(\xi_k)$ be maximum of $x_1(t)$ on $(t_k, t_{k+1}), k = 1, 2, \cdots$ and $x_1(\xi_k) > K_1^*$. We have

$$0 = \frac{dx_1(t)}{dt}|_{t=\xi_k} \le x_1(\xi_k) b_{11}^l m_1^{\theta_{11}} (k_2^* - x_1^{\theta_{11}}(\xi_k - \tau^*)).$$

This lead to

$$x_1(\xi_k - \tau^*) \le K_2^*.$$

Since $x_1(\xi_k) \geq K_2^*$ and $x_1(\xi_k - \tau^*) \leq K_2^*$, then let η be the first zero of $x_1(t) - K_2^*$ in $[\xi_k - \tau^*, \xi_k)$, that is, $x_1(\eta) = K_2^*$. By integrating (3.1) from η to ξ_k, we obtain

$$\ln \frac{x_1(\xi_k)}{x_1(\eta)} \leq \int_{\eta}^{\xi_k} [b_1^m - b_{11}^l m_1^{\theta_{11}} x_1(t-\tau_{11})]dt \leq \int_{\eta}^{\xi_k} b_1^m dt \leq b_1^m \tau^*.$$

Therefore, we have

$$x_1 \xi_k) \leq K_2^* \exp(b_1^m \tau^*).$$

so

$$x_1(t) \leq K_2^* \exp(b_1^m \tau^*), t > t_1 + 2\tau^*. \tag{3.3}$$

By lemma 2.1 and (3.3), we have

$$y_1(t) = \prod_{0<\tau_k<t} (1 + h_1^k) x_1(t) < M_1 k_2^* \exp(b_1^m \tau^*) = K_2, t > t_1 + 2\tau^*.$$

Case II. Suppose $x_1(t)$ is not oscillatory about k_2^*. Then for any $\varepsilon > 0$, there exists a constant $T_1 > 0$, such that

$$x_1(t) < K_2^* + \varepsilon, t > T_1.$$

Therefore, there exist a constant $T_2 > 0$ such that

$$x_1(t) < k_2^* \exp(b_1^m \tau^*), t > T_2.$$

By lemma 2.1, we have

$$y_1(t) \leq M_1 k_2^* \exp(b_1^m \tau^*) = K_2.$$

Similarly, from (3.2) there exist a constant $T_3 > 0$ such that

$$y_2(t) \leq M_2 N_2^* \exp(b_2^m \tau^*) = N_2.$$

Further, we prove $y_1(t) \geq K_1, y_2(t) \geq N_1$.
From system (2.2),we have

$$x_1'(t) \geq x_1(t)(P_1 - b_{11}^m M_1^{\theta_{11}} x_1^{\theta_{11}}(t - \tau_{11}))) \tag{3.4}$$

$$x_2'(t) \geq x_2(t)(Q_1 - b_{22}^m M_2^{\theta_{22}} x_2^{\theta_{22}}(t - \tau_{22})) \tag{3.5}$$

Similarly, from (3.4) and (3.5) there exist a constant $T_4 > 0, T_5 > 0$ such that

$$x_1(t) \geq (\frac{P_1}{b_{11}^m M_1^{\theta_{11}}})^{\frac{1}{\theta_{11}}} \exp((P_1 - K_2)\tau^*), t > T_4, \tag{3.6}$$

$$x_2(t) \geq (\frac{Q_1}{b_{22}^m M_2^{\theta_{22}}})^{\frac{1}{\theta_{22}}} \exp((Q_1 - N_2)\tau^*), t > T_5. \tag{3.7}$$

Then by lemma 2.1 and (3.6) (3.7), we have

$$y_1(t) \geq m_1(\frac{P_1}{b_{11}^m M_1^{\theta_{11}}})^{\frac{1}{\theta_{11}}} \exp((P_1 - K_2)\tau^*) = K_1, t > T_4,$$

$$y_2(t) \geq m_2(\frac{Q_1}{b_{22}^m M_2^{\theta_{22}}})^{\frac{1}{\theta_{22}}} \exp((Q_1 - N_2)\tau^*) = N_1, t > T_5.$$

The proof is completed. □

Theorem 3.4. *If* $(H_1)-(H_4)$ *hold and assume that the system of algebraic equations*

$$\begin{array}{l} \overline{b_{11}p_{11}}x_1^{\theta_{11}}+\overline{b_{12}p_{12}}x_2^{\theta_{12}}=\overline{b_1} \\ \overline{b_{21}p_{21}}x_1^{\theta_{21}}+\overline{b_{22}p_{22}}x_2^{\theta_{22}}=\overline{b_2} \end{array} \quad (*)$$

has finite solution $x^*=(x_1^*,x_2^*)$ *with* $x_1^*>0, x_2^*>0$ *and* $\sum_{x^*} signJ(x^*)\neq 0$. *In addition* $\overline{b_i}-\overline{b_{ij}p_{ij}}\exp[\frac{\theta_{ij}}{\theta_{ii}}\ln(\frac{\overline{b_i}}{\overline{b_{ii}p_{ii}}})+2\theta_{ij}\omega\overline{b_i}]>0, i\neq j, i=1,2, j=1,2$. *Then system (1.1) exists at least a periodic solution.*

Proof. We make the change of variables

$$x_1(t)=\exp(u_1(t)), x_2(t)=\exp(u_2(t)). \quad (3.8)$$

Then system (2.2) can be rewritten as

$$\begin{cases} u_1'(t)=b_1(t)-b_{11}(t)p_{11}(t)\exp(\theta_{11}u_1(t-\tau_{11}))-b_{12}(t)p_{12}(t)\exp(\theta_{12}u_2(t-\tau_{12})), \\ u_2'(t)=b_2(t)-b_{21}(t)p_{21}(t)\exp(\theta_{21}u_1(t-\tau_{21}))-b_{22}(t)p_{22}(t)\exp(\theta_{22}u_2(t-\tau_{22})). \end{cases} \quad (3.9)$$

Let

$$X=Z=\{u(t)=(u_1(t),u_2(t))\in C(R,R^2), u(t+\omega)=u(t)\}.$$

Then X and Z are both Banach spaces with the usual norm

$$\|u\|=\max_{t\in[0,\omega]}|u_1(t)|+\max_{t\in[0,\omega]}|u_2(t)|$$

for any $u\in X$ (or Z).
Let

$$Nu(t)=\begin{pmatrix} b_1(t)-b_{11}(t)p_{11}(t)\exp(\theta_{11}u_1(t-\tau_{11}))-b_{12}(t)p_{12}(t)\exp(\theta_{12}u_2(t-\tau_{12})) \\ b_2(t)-b_{21}(t)p_{21}(t)\exp(\theta_{21}u_1(t-\tau_{21}))-b_{22}(t)p_{22}(t)\exp(\theta_{22}u_2(t-\tau_{22})) \end{pmatrix},$$

$$Lu=u', Px=\frac{1}{\omega}\int_0^\omega u(t)dt, u\in X, Qz=\frac{1}{\omega}\int_0^\omega z(t)dt, z\in Z.$$

Obviously,

$$KerL=\{u|u\in X, u=h, h\in R^2\}, ImL=\{z|z\in Z,\int_0^\omega z(t)dt=0\}$$

and dim kerL=codim ImL=2. Since ImL is closed in Z,L is a Fredholm mapping of index zero. It is not difficult to see P and Q are continuous projectors such that $ImP=KerL, KerQ+ImL=Im(I-Q)$. Moreover, the generalized inverse (to L) $K_p: ImL\to KerP\cap DomL$ is given by $K_p(z)=\int_0^t z(s)ds-\frac{1}{\omega}\int\int_0^\omega\int_0^t dsdt$. Thus,

$$QNu=\frac{1}{\omega}\int_0^\omega Nu(t)dt,$$

$$K_p(I-Q)Nu(t)+\int_0^t Nu(s)ds-\frac{1}{\omega}\int_0^\omega\int_0^t Nu(s)dsdt-(\frac{t}{\omega}-\frac{1}{2})\int_0^\omega Nu(s)ds.$$

It is obvious that QN and $K_p(I-Q)N$ are continuous, and using the Arzela-Ascoli, it is easy to show that $K_p(I-Q)N(\overline{\Omega})$ is compact for any open bounded set $\Omega \subset X$. Furthermore, $QN(\overline{\Omega}$ is bounded. Hence, N is $L-$compact on $\overline{\Omega}$ for any open bounded set $\Omega \subset X$. Corresponding to the equation $Lx = \lambda Nx, \lambda \in (0, 1)$, we have

$$\begin{array}{l} u_1'(t) = \lambda(b_1(t) - b_{11}(t)p_{11}(t)\exp(\theta_{11}u_1(t-\tau_{11})) - b_{12}(t)p_{12}(t)\exp(\theta_{12}u_2(t-\tau_{12}))), \\ u_2'(t) = \lambda(b_2(t) - b_{21}(t)p_{21}(t)\exp(\theta_{21}u_1(t-\tau_{21})) - b_{22}(t)p_{22}(t)\exp(\theta_{22}u_2(t-\tau_{22}))). \end{array} \tag{3.10}$$

Suppose that $u(t) \in X$ is a solution of (3.10) for a certain $\lambda \in (0, 1)$. Integrating (3.10) from 0 to ω, we obtain

$$\int_0^\omega [b_{11}(t)p_{11}(t)\exp(\theta_{11}u_1(t-\tau_{11})) + b_{12}(t)p_{12}(t)\exp(\theta_{12}u_2(t-\tau_{12})]dt = \omega\overline{b_1} \tag{3.11}$$

and

$$\int_0^\omega [b_{21}(t)p_{21}(t)\exp(\theta_{21}u_1(t-\tau_{21})) + b_{22}(t)p_{22}(t)\exp(\theta_{22}u_2(t-\tau_{22})]dt = \omega\overline{b_2} \tag{3.12}$$

From (3.10),(3.11) and (3.12), we have

$$\begin{array}{l} \int_0^\omega |u_1'(t)|dt \\ \leq \lambda[\int_0^\omega b_1(t)dt + \int_0^\omega [b_{11}(t)p_{11}(t)\exp(\theta_{11}u_1(t-\tau_{11})) \\ \quad +b_{12}(t)p_{12}(t)\exp(\theta_{12}u_2(t-\tau_{12})]dt] \ < 2\omega\overline{b_1} \end{array} \tag{3.13}$$

$$\begin{array}{l} \int_0^\omega |u_2'(t)|dt \\ \leq \lambda[\int_0^\omega b_2(t)dt + \int_0^\omega [b_{21}(t)p_{21}(t)\exp(\theta_{21}u_1(t-\tau_{21})) \\ \quad +b_{22}(t)p_{22}(t)\exp(\theta_{22}u_2(t-\tau_{22})]dt] \ < 2\omega\overline{b_2} \end{array} \tag{3.14}$$

Since $u(t) \in X$, there exist $\xi_i, \zeta_i \in [0, \omega], i = 1, 2$ such that

$$u_i(\xi_i) = \min_{t\in[0,\omega]} u_i(t), u_i(\zeta_i) = \max_{t\in[0,\omega]} u_i(t).$$

From (3.11) and (3.12), we have

$$\begin{array}{l} \omega\overline{b_i} \geq \int_0^\omega b_{ii}(t)p_{ii}(t)\exp(\theta_{ii}u_i(t-\tau_{ii}))dt = \\ \int_{-\tau_{ii}}^{\omega-\tau_{ii}} b_{ii}(s+\tau_{ii})p_{ii}(s+\tau_{ii})\exp(\theta_{ii}u_i(s))ds \geq \omega\overline{b_{ii}p_{ii}}\exp(\theta_{ii}u_i(\xi_i)), i = 1, 2. \end{array}$$

Moreover

$$u_i(\xi_i) \leq \frac{1}{\theta_{ii}} \ln[\frac{\overline{b_i}}{\overline{b_{ii}p_{ii}}}], i = 1, 2.$$

Then

$$u_i(t) \leq u_i(\xi_i) + \int_0^\omega |u_i'(t)|dt \leq \frac{1}{\theta_{ii}} \ln[\frac{\overline{b_i}}{\overline{b_{ii}p_{ii}}}] + 2\omega\overline{b_i} \doteq R_i, i = 1, 2. \tag{3.15}$$

On the other hand, from (3.11) and (3.12), we obtain $\int_{-\tau_{ii}}^{\omega-\tau_{ii}} b_{ii}(s+\tau_{ii})p_{ii}(s+\tau_{ii})\exp(\theta_{ii}u_i(s))ds = \omega\overline{b_i} - \int_{-\tau_{ij}}^{\omega-\tau_{ij}} b_{ij}(s+\tau_{ij})p_{ij}(s+\tau_{ij})\exp(\theta_{ij}u_j(s))ds \ i \neq j, i = 1, 2, j = 1, 2$. Then we get

$$\overline{b_{ii}p_{ii}}\exp(\theta_{ii}u_i(\zeta_i)) \geq \overline{b_i} - \overline{b_{ij}p_{ij}}\exp(\theta_{ij}R_j), i \neq j, i = 1, 2, j = 1, 2.$$

which implies

$$u_i(\zeta_i) \geq \frac{1}{\theta_{ii}} \ln\{\frac{1}{\overline{b_{ii}p_{ii}}}(\overline{b_i} - \overline{b_{ij}p_{ij}} \exp[\frac{\theta_{ij}}{\theta_{ii}} \ln(\frac{\overline{b_i}}{\overline{b_{ii}p_{ii}}}) + 2\theta_{ij}\omega\overline{b_i}])\} \doteq S_i, \qquad (3.16)$$

where $i \neq j, i = 1, 2, j = 1, 2$. From (3.13),(3.14) and (3.16), we have

$$u_i(t) \geq u_i(\zeta_i) - \int_0^\omega |u_i'(t)|dt > S_i - 2\omega\overline{b_i}, i = 1, 2. \qquad (3.17)$$

which,together with (3.15)implies

$$\max_{t\in[0,\omega]} |u_i(t)| < \max\{|R_i|, |S_i - 2\omega\overline{b_i}|\} = F_i, i = 1, 2.$$

Clearly, $F_i, i = 1, 2$ are not dependent on the choice of λ. We can take sufficiently large M such that $M > F_1 + F_2$ and the solution of equation $(*)$ satisfies $M > |x_1^*| + |x_2^*|$.
Set $\Omega \doteq \{u = (u_1, u_2) \in X| \|u\| < M\}$. It is clear that Ω satisfies the requirement (a) in Lemma 2.2, When $u \in \partial\Omega \cap KerL = \partial\Omega \cap R^2$, u is a constant vector in R^2 with $\|u\| = M$. Then

$$QNu = \begin{pmatrix} \overline{b_1} - \overline{b_{11}p_{11}} \exp(\theta_{11}u_1) - \overline{b_{12}p_{12}} \exp(\theta_{12}u_2) \\ \overline{b_2} - \overline{b_{21}p_{21}} \exp(\theta_{21}u_1) - \overline{b_{22}p_{22}} \exp(\theta_{22}u_2) \end{pmatrix} \neq 0,$$

The requirement (b) in Lemma 2.2 is also satisfied. In view of the assumption in Theorem 3.2, it is easy prove that

$$deg\{JQNu, \Omega \cap KerL, 0\} \neq 0.$$

By now all the assumptions required in Lemma 2.2 hold. It follows by Lemma 2.2 that system (3.9) has a $\omega-$ periodic solution $u^*(t) = (u_1^*(t), u_2^*(t)) \in \overline{\Omega}$. By the change of $x_i^*(t) = \exp(u_i^*(t)), i = 1, 2$, we obtain that $x^*(t) = (x_1^*(t), x_2^*(t))$ is a positive $\omega-$ periodic solution of (2.2).
By Lemma 2.1 and $y_i^*(t) = \prod_{0<\tau_k<t} (1+h_i^k)x_i^*(t), i = 1, 2$, then system (1.1) has an $\omega-$ positive periodic solution $y^*(t) = (y_i^*(t), y_i^*(t))$. The proof is complete. □

Theorem 3.5. *If the conditions of Theorem (3.2) are hold. Furthermore* $\tau_{11} = \tau_{22} = 0, \theta_{12} = \theta_{22}, \theta_{11} = \theta_{21}$ *and*$(b_{11}p_{11})^l - (p_{21}b_{21})^m > 0, (b_{22}p_{22})^l - (p_{12}b_{12})^m > 0$. *Then system (1.1) has a unique* $\omega-$*periodic solution* $(y_1^*(t), y_2^*(t))$ *which is globally asymptotically stable.*

Proof. Suppose $x^*(t) = (x_1^*(t), x_2^*(t))$ is a positive $\omega-$periodic solution of system (2.2). We need to show that $x^*(t)$ is globally asymptotically stable. Consider a Lyapunov functional $V(t)$ defined by

$$\begin{aligned} V(t) &= |\ln x_1(t) - \ln x_1^*(t)| + |\ln x_2(t) - \ln x_2^*(t)| + \\ &\int_{t-\tau_{12}}^t b_{12}(s+\tau_{12})p_{12}(s+\tau_{12})|(x_2(s))^{\theta_{22}} - (x_2^*(s))^{\theta_{22}}|ds + \\ &\int_{t-\tau_{21}}^t b_{21}(s+\tau_{21})p_{21}(s+\tau_{21})|(x_1(s))^{\theta_{11}} - (x_1^*(s))^{\theta_{11}}|ds, t \geq 0. \end{aligned}$$

By calculating and estimating the upper right derivative of $V(t)$ along the solution of (2.2), we have

$$D^+V(t) \leq -(b_{11}(t)p_{11}(t) - p_{21}(t+\tau_{21})b_{21}(t+\tau_{21}))|(x_1(t))^{\theta_{11}} - (x_1^*(t))^{\theta_{11}}|$$
$$-(b_{22}(t)p_{22}(t) - p_{12}(t+\tau_{12})b_{12}(t+\tau_{12}))|(x_2(t))^{\theta_{22}} - (x_2^*(t))^{\theta_{22}}|$$
$$\leq -((b_{11}p_{11})^l - (p_{21}b_{21})^m)|(x_1(t))^{\theta_{11}} - (x_1^*(t))^{\theta_{11}}|$$
$$-((b_{22}p_{22})^l - (p_{12}b_{12})^m)|(x_2(t))^{\theta_{22}} - (x_2^*(t))^{\theta_{22}}|, t \geq 0.$$

Then

$$D^+V(t) \leq -\mu(|(x_1(t))^{\theta_{11}} - (x_1^*(t))^{\theta_{11}}| + |(x_2(t))^{\theta_{22}} - (x_2^*(t))^{\theta_{22}}|) \tag{3.18}$$

where

$$\mu = \min\{(b_{11}p_{11})^l - (p_{21}b_{21})^m, (b_{22}p_{22})^l - (p_{12}b_{12})^m\}.$$

Integrating from 0 to t on both sides of (3.18) leads to:

$$V(t)+\mu\int_0^t(|(x_1(s))^{\theta_{11}} - (x_1^*(s))^{\theta_{11}}| + |(x_2(s))^{\theta_{22}} - (x_2^*(s))^{\theta_{22}}|)ds \leq V(0) < +\infty, t \geq 0, \tag{3.19}$$

then

$$\int_0^t(|(x_1(s))^{\theta_{11}} - (x_1^*(s))^{\theta_{11}}| + |(x_2(s))^{\theta_{22}} - (x_2^*(s))^{\theta_{22}}|)ds \leq \frac{V(0)}{\mu} < +\infty, t \geq 0$$

and hence

$$|(x_1(t))^{\theta_{11}} - (x_1^*(t))^{\theta_{11}}| + |(x_2(t))^{\theta_{22}} - (x_2^*(t))^{\theta_{22}}| \in L^1[0, +\infty).$$

In view of the definition of $V(t)$ and (3.19), by Barbalat's lemma [17], we have

$$\lim_{t\to+\infty}(|(x_1(s))^{\theta_{11}} - (x_1^*(s))^{\theta_{11}}| + |(x_2(s))^{\theta_{22}} - (x_2^*(s))^{\theta_{22}}|) = 0.$$

Hence

$$\lim_{t\to+\infty}|(x_1(s))^{\theta_{11}} - (x_1^*(s))^{\theta_{11}}| = 0, \lim_{t\to+\infty}|(x_2(s))^{\theta_{22}} - (x_2^*(s))^{\theta_{22}}| = 0.$$

Further,

$$\lim_{t\to+\infty}|(x_1(s) - x_1^*(s)| = 0, \lim_{t\to+\infty}|(x_2(s) - x_2^*(s)| = 0. \tag{3.20}$$

By Lemma 2.1, we know $y_i(t) = \prod_{0<\tau_k<t}(1+h_i^k)x_i(t), y_i^*(t) = \prod_{0<\tau_k<t}(1+h_i^k)x_i^*(t), i = 1, 2$ are solution of system (1.1).
By the hypotheses (H_2) and (3.20),we obtain

$$\lim_{t\to+\infty}|(y_1(s) - y_1^*(s)| = 0, \lim_{t\to+\infty}|(y_2(s) - y_2^*(s)| = 0.$$

This completes the proof of Theorem 3.5. □

For system (1.3), we obtain the following results using the method of system (1.1).

Theorem 3.6. *For system (1.3), we assume that $(H_1) - (H_3)$ hold and $P_1 > 0, Q_1 > 0$. If t is large enough, then $K_1 \leq y_1(t) \leq K_2, N_1 \leq y_2(t) \leq N_2$, Where*

$$K_1 = m_1(\frac{P_1}{b_{11}^m M_1})\exp((P_1 - K_2)\tau^*), K_2 = M_1(\frac{b_1^m}{b_{11}^l m_1})\exp(b_1^m\tau^*)$$

$$N_1 = m_2(\frac{Q_1}{b_{22}^m M_2})\exp((Q_1 - N_2)\tau^*), N_2 = M_2(\frac{b_2^m}{b_{22}^l m_2})\exp(b_2^m \tau^*)$$

where

$$P_1 = b_1^l - b_{12}^m N_2, Q_1 = b_2^l - b_{21}^m K_2$$

Theorem 3.7. *If* $(H_1) - (H_3)$ *hold and satisfies* $\tau_{11} = \tau_{22} = 0, \overline{b_{11}p_{11}} \cdot \overline{b_{22}p_{22}} > \overline{b_{12}p_{12}} \cdot \overline{b_{21}p_{21}}, \overline{b_i} - \overline{b_{ij}p_{ij}} \exp[\ln(\frac{\overline{b_i}}{\overline{b_{ii}p_{ii}}}) + 2\omega \overline{b_i}] > 0, i \neq j, i = 1,2, j = 1,2,$ $(b_{11}p_{11})^l - (p_{21}b_{21})^m > 0$ *and* $(b_{22}p_{22})^l - (p_{12}b_{12})^m > 0$.
Then system (1.3) has a positive ω-periodic solution which is globally asymptotically stable.

4. Conclusion

In this paper, we propose a periodic two species Gilpin-Ayala competition system with constant delay and impulsive perturbations. Our results indicate that under the appropriate linear periodic impulsive perturbations, the impulsive competition system with delay argument equation (1.1) remain the behave of global attractivity of nonimpulsive system (2.2). By using the method of coincidence degree and constructing suitable Lyapunov functional, we obtain sufficient conditions for existence and global asympotic stability of positive periodic solutions of system (1.1). From Theorems 3.1 to 3.7, we can see that the time delays and pulse have effect on the permanence, existence and global asympotic stability of positive periodic solutions.

We expect a similar technique to work in higher-dimensional systems with delay and impulsive perturbations. We leave this investigation for further work.

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