J. Appl. Math. & Informatics Vol. **29**(2011), No. 5 - 6, pp. 1501 - 1509
Website: http://www.kcam.biz

BARRIER OPTION PRICING UNDER THE VASICEK MODEL OF THE SHORT RATE†

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ABSTRACT. In this study, assume that the stock price obeys the stochastic differential equation driven by mixed fractional Brownian motion, and the short rate follows the Vasicek model. Then, the Black-Scholes partial differential equation is held by using fractional Ito formula. Finally, the pricing formulae of the barrier option are obtained by partial differential equation theory. The results of Black-Scholes model are generalized.

AMS Mathematics Subject Classification : 60H10, 90A06.
Key words and phrases : barrier option pricing, Vasicek model, Black-Scholes model, fractional Brownian motion.

1. Introduction

In this article, we focus our analysis on the pricing of financial contracts with barriers under the Vasicek model of the short rate in mixed fractional Brownian motion environment. Barrier options are among the most popularly exotic options traded in financial markets. A barrier option offers the holder a payoff like that of a vanilla option, contingent on whether or not the underlying asset price process crosses some level(s)-called the barrier(s)-before or at the maturity date. The closed form formulae and replication strategies for barrier options are given by Peter Carr [1]. Analytical formulae using the method of images in the case of one barrier applied continuously are presented in Ref.[2]. Using reflection principle in Brownian motions, the solution in general as summation of an infinite number of normal distribution functions for standard double barrier options, and in many non-trivial cases the solution consists of only finite terms are expressed by Sanfelici [3]. For more information , a detailed comprehensive guide of option pricing formulae is that of Haug [4]. Recently, fractional Brownian motion has

Received November 13, 2010. Revised March 4, 2011. Accepted March 10, 2011.
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†This work was supported by the National Natural Science Foundation of China under Grant No.70471057 and graduate starting seed fund of Northwestern Polytechnical University No.Z2011073.

been considered to replace Brownian motion in the usual financial models as it has better behaved tails and exhibits long-term dependence while remaining Gaussian. For details about the stochastic analysis theory of fractional Brownian motion, see Ref.[5,6]. The fractional Brownian motion is applied in finance, such as Ref.[7,8,9]. However, all the above option pricing studies assume that the risk-free rate or the short rate is constant during the life of the option. Hence, in this study, we incorporate its stochastic nature into our option valuation model. Specifically, we use the following stochastic process(see Eq.(1)), first proposed by Vasicek, to depict its dynamics and derive explicit pricing formulae for Barrier option on a stock. The paper is organized as follows: In Section 2, we treat the Black-Scholes model that the short rate obeys the Vasicek model. In Section 3, we derive the formula for the price of a riskless zero-coupon bond paying \$1 at maturity based on Eq.(2). In Section 4, the pricing formulas for Barrier options on a stock are obtained. Section 5 contains conclusions.

2. The model

Firstly, we assume that the short rate of the market satisfied the Vasicek model

$$dr_t = \theta(\mu_r - r_t)dt + \sigma_{r1} \diamond dW_{H_1}(t) + \sigma_{r2} \diamond dW_{H_2}(t), \tag{1}$$

where r_t is the short-term interest rate. θ is the mean-reversion speed. μ_r is the long-term interest rate. σ_{r1} and σ_{r2} are the instantaneous volatility. $W_{H_1}(t)$ and $W_{H_2}(t)$ are the fractional Brownian motion with Hurst parameter H_1, H_2. $\int_0^t f(s) \diamond dW_{H_1}$ is Wick-Ito-Skorohod integral not Rieman-Stieltjes integral,see Ref.[5,6]. Secondly, there are zero-coupon bond and stock in this market. Let B_t be the price of a riskless zero-coupon bond paying 1 \$ at time T.

$$dB(t, r_t) = r_t B_t dt + \sigma_{b1} B(t, r_t) \diamond dW_{H_1} + \sigma_{b2} B(t, r_t) \diamond dW_{H_2},\ B(T, r_T) = 1. \tag{2}$$

And, the dynamics of the stock price process takes the following form

$$dS_t = \mu S_t dt + \sigma_1 S_t \diamond dW_{H_1}(t) + \sigma_2 S_t \diamond dW_{H_2}(t), \tag{3}$$

where μ is expectation return rate which is time-dependent. Constant σ_1 and σ_2 are volatility of the stock.

3. Explicit pricing formulae of zero-coupon bond under the vasicek model

To solve the value of $B(t, r_t)$, by Eq. (1) and fractional Ito formula, we have

$$\begin{aligned} dB(t, r_t) =& \frac{\partial B(t, r_t)}{\partial t} dt + \frac{\partial B(t, r_t)}{\partial r_t} \diamond dr_t \\ &+ H_1 \sigma_{r1}^2 t^{2H_1 - 1} \frac{\partial^2 B(t, r_t)}{\partial r_t^2} dt + H_2 \sigma_{r2}^2 t^{2H_2 - 1} \frac{\partial^2 B(t, r_t)}{\partial r_t^2} dt \end{aligned}$$

$$
\begin{aligned}
=&\frac{\partial B(t,r_t)}{\partial t}dt+\theta(\mu_r-r_t)\frac{\partial B(t,r_t)}{\partial r_t}dt\\
&+\sigma_{r1}\frac{\partial B(t,r_t)}{\partial r_t}\diamond dW_{H_1}(t)+\sigma_{r2}\frac{\partial B(t,r_t)}{\partial r_t}\diamond dW_{H_2}(t)\\
&+H_1\sigma_{r1}^2t^{2H_1-1}\frac{\partial^2 B(t,r_t)}{\partial r_t^2}dt+H_2\sigma_{r2}^2t^{2H_2-1}\frac{\partial^2 B(t,r_t)}{\partial r_t^2}dt.
\end{aligned}
$$

Compare to Eq.(2), we obtain

$$
\begin{aligned}
&\frac{\partial B(t,r_t)}{\partial t}+\theta(\mu_r-r_t)\frac{\partial B(t,r_t)}{\partial r_t}+H_1\sigma_{r1}^2t^{2H_1-1}\frac{\partial^2 B(t,r_t)}{\partial r_t^2}\\
&\qquad+H_2\sigma_{r2}^2t^{2H_2-1}\frac{\partial^2 B(t,r_t)}{\partial r_t^2}-r_tB(t,r_t)=0.
\end{aligned}
$$

Then the value of zero-coupon bond at time t satisfied

$$
\begin{cases}
\frac{\partial B(t,x)}{\partial t}+\theta(\mu_r-x)\frac{\partial B(t,x)}{\partial x}+H_1\sigma_{r1}^2t^{2H_1-1}\frac{\partial^2 B(t,x)}{\partial x^2}\\
+H_2\sigma_{r2}^2t^{2H_2-1}\frac{\partial^2 B(t,x)}{\partial x^2}-xB(t,x)=0,\\
B(T,x)=1.
\end{cases}
\tag{4}
$$

Assume that $B(t,x)=\exp\{A_1(t)+xA_2(t)\}$, $A_1(T)=0, A_2(T)=0$, so that

$$
\begin{aligned}
&\frac{\partial B(t,r_t)}{\partial t}=A_1'(t)B(t,x)+xA_2'(t)B(t,x),\\
&\frac{\partial B(t,x)}{\partial x}=A_2(t)B(t,x),\\
&\frac{\partial^2 B(t,x)}{\partial x^2}=A_2(t)^2B(t,x).
\end{aligned}
\tag{5}
$$

Compare Eq.(4) and Eq.(5), then

$$
\begin{cases}
\theta A_2(t)-A_2'(t)+1=0,\\
A_1'(t)+\theta\mu_rA_2(t)+(H_1\sigma_{r1}^2t^{2H_1-1}+H_2\sigma_{r2}^2t^{2H_2-1})A_2(t)^2=0,\\
A_1(T)=0, A_2(T)=0.
\end{cases}
$$

Then we conclude that

$$
\begin{aligned}
A_1(t)=&-\mu_r(T-t)-\mu_r(1-e^{\theta(T-t)})\\
&-\int_t^T(H_1\sigma_{r1}^2s^{2H_1-1}+H_2\sigma_{r2}^2s^{2H_2-1})A_2(s)^2ds,\\
A_2(t)=&\frac{1-\theta\exp\{-\theta(T-t)\}}{\theta}.
\end{aligned}
$$

So that, the explicit solution of Eq.(4) is given by the following theorem.

Theorem 1. *The price of a riskless zero-coupon bond at time t can be written as:*

$$B(t, r_t) = \exp\{A_1(t) + r_t A_2(t)\}, B(T, r_T) = 1. \tag{6}$$

When $\theta = 0, \sigma_{r1} = 0, \sigma_{r2} = 0$, we have $dr_t = 0$,then $r_t = r$. And, the Eq.(6) can be changed as follows

$$B(t, r_t) = \exp\{r(T - t)\}.$$

4. Explicit pricing formulae for barrier options

In what follows we introduce some relevant derivatives of two stocks, and show how to obtain the formulae for the value of these derivatives. Let

$$D_1(t) = H_1\sigma_{b1}^2 B(t, r_t)^2 + H_1\sigma_{b1}^2 B(t, r_t)^2, D_2(t) = H_1\sigma_1^2 t^{2H_1-1} + H_2\sigma_2^2 t^{2H_2-1},$$

$$D_3(t) = H_1\sigma_{b1}\sigma_2 t^{2H_1-1} + H_2\sigma_{b1}\sigma_2 t^{2H_2-1}, D(t) = D_1(t) + D_2(t) - 2D_3(t).$$

In this study, we assume that there is no transaction cost, margin requirement and tax; all securities are divisible; security trading is continuous and borrowing, and short-selling is permitted without restriction; there is no dividend payout over the life of the option; all investors can borrow or lend at the same short rate. Further, we consider the down and out barrier call option with payoff. Denoted by T the maturity of the options, by K their strike, and by $HB(t, r_t)$ their barrier level, one can write the following arbitrage-free pricing formulae for the up and out call option:

$$(S_T - B(T, r_T)K)^+ I_{\{S_t > HB(t,r_t)\}},$$

where H is the constant, $B(t, r_t)$ is the zero-coupon bond of Eq.(6). According to our assumption , there are zero-coupon bond and stock in this market(no bank deposit), the strike price K at time T must be consider as K units zero-coupon bond, since $(S_T - B(T, r_T)K)^+ I_{\{S_t > HB(t,r_t)\}}$ can be written as

$$(S_T - B(T, r_T)K)^+ I_{\{S_t > HB(t,r_t)\}} = (S_T - K)^+ I_{\{S_t > HB(t,r_t)\}}, \tag{7}$$

where T is maturity date, K is exercise price.

Let $C = C(S_t, B(t, r_t), t, K)$ be the call price which is a function of the stock price S_t, the riskless zero-coupon bond price$B(t, r_t)$ at the time t. By Ito's lemma, the change in the call price over an infinitesimal time dt satisfies the following stochastic differential equation:

$$\begin{aligned} dC =& \frac{\partial C}{\partial t} dt + \frac{\partial C}{\partial B(t, r_t)} \diamond dB(t, r_t) + D_1(t) \frac{\partial^2 C}{\partial B(t, r_t)^2} dt \\ &+ \frac{\partial C}{\partial S_t} \diamond dS_t + D_2(t) S_t^2 \frac{\partial^2 C}{\partial S_t^2} dt + 2D_3(t) \frac{\partial^2 C}{\partial S_t \partial B(t, r_t)} dt \\ =& [\frac{\partial C}{\partial t} + D_1(t) \frac{\partial^2 C}{\partial B(t, r_t)^2} + D_2(t) S_t^2 \frac{\partial^2 C}{\partial S_t^2} + 2D_3(t) \frac{\partial^2 C}{\partial S_t \partial B(t, r_t)}] dt \\ &+ \frac{\partial C}{\partial B(t, r_t)} \diamond dB(t, r_t) + \frac{\partial C}{\partial S_t} \diamond dS_t. \end{aligned} \tag{8}$$

Now we form a hedge portfolio consisting of the stock, the riskless bond and the call. Let θ_t^0 be the number of shares of the bond, θ_t^1 be the number of the stock, and θ_t^2 be the number of the call. The self-finance hedge is formed such that the value (say, H) of the hedge portfolio is zero. That is

$$H = \theta_t^0 B(t, r_t) + \theta_t^1 S_t + \theta_t^2 C = 0.$$

Hence, we have

$$dH = \theta_t^0 dB(t, r_t) + \theta_t^1 dS_t + \theta_t^2 dc = 0. \tag{9}$$

Substituting Eq.(8) into Eq.(9) and grouping, Eq.(9) becomes

$$\begin{aligned} dH =& \theta_t^2 [\frac{\partial C}{\partial t} + D_1(t)\frac{\partial^2 C}{\partial B(t,r_t)^2} + D_2(t)S_t^2\frac{\partial^2 C}{\partial S_t^2} + 2D_3(t)\frac{\partial^2 C}{\partial S_t \partial B(t,r_t)}]dt \\ &+ [\theta_t^2 \frac{\partial C}{\partial S_t} + \theta_t^1] \diamond dS_t + [\frac{\partial C}{\partial B(t,r_t)} + \theta_t^0] \diamond dB(t,r_t). \end{aligned} \tag{10}$$

Eq(10) implies that $\theta_t^2 \frac{\partial C}{\partial S_t} + \theta_t^1 = 0$, $\frac{\partial C}{\partial B(t,r_t)} + \theta_t^0 = 0$, and

$$\frac{\partial C}{\partial t} + D_1(t)B(t,r_t)^2\frac{\partial^2 C}{\partial B(t,r_t)^2} + D_2(t)S_t^2\frac{\partial^2 C}{\partial S_t^2} + 2D_3(t)S_t B(t,r_t)\frac{\partial^2 C}{\partial S_t \partial B(t,r_t)} = 0.$$

Hence, the following theorem can be obtained.

Theorem 2. *The price of European call option with payoff*$(S_T - B(T, r_T)K)^+$ *must satisfy*

$$\begin{cases} \frac{\partial C}{\partial t} + D_1(t)y^2\frac{\partial^2 C}{\partial y^2} + D_2(t)x^2\frac{\partial^2 C}{\partial x^2} + 2D_3(t)xy\frac{\partial^2 C}{\partial x \partial y} = 0, & \\ C(T,x,y) = (x - Ky)^+, & x > Hy, \\ C(t, Hy, y) = 0, & t > 0. \end{cases} \tag{11}$$

Let

$$\xi = \frac{x}{y}, F(t,\xi) = \frac{C}{y}, \tag{12}$$

we get

$$C_x = \frac{\partial F}{\partial \xi}, C_y = F - \xi\frac{\partial F}{\partial \xi}, C_{xx} = \frac{1}{y}\frac{\partial^2 F}{\partial \xi^2}, C_{xy} = -\frac{\xi}{y}\frac{\partial^2 F}{\partial \xi^2}, C_{yy} = \frac{\xi^2}{y^2}\frac{\partial^2 F}{\partial \xi^2}. \tag{13}$$

Substituting Eqs.(13) into Eq.(11), reduces to

$$\frac{\partial F}{\partial t} + D(t)\xi^2\frac{\partial^2 F}{\partial \xi^2} = 0, F(T,\xi) = (\xi - K)^+, F(t,H) = 0, \xi > H, t > 0. \tag{14}$$

Denoting

$$s = \int_t^T D(\tau)d\tau, F(t,\xi) = U(s,\xi), \tag{15}$$

we have

$$\frac{ds}{dt} = -D(t), \frac{\partial F}{\partial t} = \frac{\partial U}{\partial s}\frac{ds}{dt} = -D(t)\frac{\partial U}{\partial s}.$$

According to Eqs.(15), Eq.(14) becomes

$$\frac{\partial U}{\partial s}(s,\xi) = \frac{\partial^2 U}{\partial \xi^2}(s,\xi)\xi^2, U(0,\xi) = (\xi - K)^+, U(s,H) = 0, \xi > H, s > 0. \quad (16)$$

Let

$$z = \ln\frac{\xi}{H}, U(s,\xi) = H\exp\{\frac{1}{2}z - \frac{1}{4}s\}V(s,z), \quad (17)$$

Eq.(16) can be changed as follows

$$\begin{aligned} &\frac{\partial V}{\partial s} = \frac{\partial^2 V}{\partial z^2}, \\ &V(0,z) = V_0(z) = e^{-\frac{1}{2}z}(e^z - \frac{K}{H})^+, z \le 0, \\ &V(s,0) = 0, z > 0, s > 0 \end{aligned} \quad (18)$$

Eq.(18) is standard one-dimensional heat equation thus can be solved. According to the Fourier integral theorem, we have

$$\begin{aligned} V(s,z) &= \frac{1}{2\sqrt{\pi s}}\int_0^{+\infty} u_0(\tau)[e^{-\frac{(\tau-z)^2}{4s}} - e^{-\frac{(\tau+z)^2}{4s}}]d\tau \\ &= \frac{1}{2\sqrt{\pi s}}\int_0^{+\infty} e^{-\frac{1}{2}\tau}(e^\tau - \frac{K}{H})^+[e^{-\frac{(\tau-z)^2}{4s}} - e^{-\frac{(\tau+z)^2}{4s}}]d\tau = I_1 - I_2, \end{aligned}$$

where

$$\begin{aligned} I_1 =& \frac{1}{2\sqrt{\pi s}}\int_0^{+\infty} e^{-\frac{1}{2}\tau}(e^\tau - \frac{K}{H})^+ e^{-\frac{(z-\tau)^2}{4s}}d\tau \\ =& \frac{1}{2\sqrt{\pi s}}\int_{\ln\frac{K}{H}}^{+\infty} e^{\frac{1}{2}\tau}e^{-\frac{(z-\tau)^2}{4s}}d\tau - \frac{K}{H}\frac{1}{2\sqrt{\pi s}}\int_{\ln\frac{K}{H}}^{+\infty} e^{-\frac{1}{2}\tau}e^{-\frac{(z-\tau)^2}{4s}}d\tau \\ =& \exp\{\frac{s}{4} + \frac{z}{2}\}\frac{1}{2\sqrt{\pi s}}\int_{\ln\frac{K}{H}}^{+\infty} e^{-\frac{(\tau-s-z)^2}{4s}}d\tau - \frac{K}{H}\exp\{\frac{s}{4} - \frac{z}{2}\}\frac{1}{2\sqrt{\pi s}}\int_{\ln\frac{K}{H}}^{+\infty} e^{-\frac{(\tau-z+s)^2}{4s}}d\tau \\ =& \exp\{\frac{s}{4} + \frac{z}{2}\}\Phi(\frac{s+z-\ln\frac{K}{H}}{\sqrt{2s}}) - \frac{K}{H}\exp\{\frac{s}{4} - \frac{z}{2}\}\Phi(-\frac{z-s-\ln\frac{K}{H}}{\sqrt{2s}}), \end{aligned}$$

and

$$\begin{aligned} I_2 =& \frac{1}{2\sqrt{\pi s}}\int_0^{+\infty} e^{-\frac{1}{2}\tau}(e^\tau - \frac{K}{H})^+ e^{-\frac{(z+\tau)^2}{4s}}d\tau \\ =& \frac{1}{2\sqrt{\pi s}}\int_{\ln\frac{K}{H}}^{+\infty} e^{\frac{1}{2}\tau}e^{-\frac{(z+\tau)^2}{4s}}d\tau - \frac{K}{H}\frac{1}{2\sqrt{\pi s}}\int_{\ln\frac{K}{H}}^{+\infty} e^{-\frac{1}{2}\tau}e^{-\frac{(z+\tau)^2}{4s}}d\tau \end{aligned}$$

$$=\exp\{\frac{s}{4}-\frac{z}{2}\}\frac{1}{2\sqrt{\pi s}}\int_{\ln\frac{K}{H}}^{+\infty}e^{-\frac{(\tau-s+z)^2}{4s}}d\tau-\frac{K}{H}\exp\{\frac{s}{4}+\frac{z}{2}\}\frac{1}{2\sqrt{\pi s}}\int_{\ln\frac{K}{H}}^{+\infty}e^{-\frac{(\tau+z+s)^2}{4s}}d\tau$$

$$=\exp\{\frac{s}{4}-\frac{z}{2}\}\Phi(\frac{s-z-\ln\frac{K}{H}}{\sqrt{2s}})-\frac{K}{H}\exp\{\frac{s}{4}+\frac{z}{2}\}\Phi(-\frac{\ln\frac{K}{H}+z+s}{\sqrt{2s}}).$$

Such that, we have

$$\begin{aligned}V(s,z)=&\exp\{\frac{s}{4}+\frac{z}{2}\}\Phi(\frac{s+z-\ln\frac{K}{H}}{\sqrt{2s}})-\frac{K}{H}\exp\{\frac{s}{4}-\frac{z}{2}\}\Phi(-\frac{z-s-\ln\frac{K}{H}}{\sqrt{2s}})\\&-\exp\{\frac{s}{4}-\frac{z}{2}\}\Phi(\frac{s-z-\ln\frac{K}{H}}{\sqrt{2s}})-\frac{K}{H}\exp\{\frac{s}{4}+\frac{z}{2}\}\Phi(-\frac{\ln\frac{K}{H}+z+s}{\sqrt{2s}}).\end{aligned}$$

By the inverse transformation of Eq.(12), Eq.(15) and Eq.(17), the price of down and out call is hold.

Theorem 3. *When $H < K$, the price of down and out call option can be written as follow*

$$\begin{aligned}C_{down-and-out}(t,B(t,r_t),S_t)=&S_t\Phi(d_1)-KB(t,r_t)\Phi(d_2)\\&-HB(t,r_t)\Phi(d_3)+\frac{K}{H}S_t\Phi(d_4),\end{aligned}\qquad(19)$$

where,

$$d_1=-\frac{\ln S_t-\ln B(t,r_t)-\ln K+s}{\sqrt{2s}},\ d_2=-\frac{\ln S_t-\ln B(t,r_t)-\ln K-s}{\sqrt{2s}},$$

$$d_3=\frac{\ln B(t,r_t)-\ln S_t-\ln K+2\ln H+s}{\sqrt{2s}},\ d_4=\frac{\ln B(t,r_t)-\ln S_t-\ln K+2\ln H-s}{\sqrt{2s}}.$$

when $H > K$ the price of down and out call option is

$$C_{down-and-out}(t,B(t,r_t),S_t)=0.\qquad(20)$$

By the same way, other barrer options' formulae can be expressed as following:

Corollary 1. *When $H < K$ down and in call option is*

$$C_{down-and-in}(t,B(t,r_t),S_t)=HB(t,r_t)\Phi(d_3)-\frac{K}{H}S_t\Phi(d_4).\qquad(21)$$

Corollary 2. *When $H > K$ down and in call option is*

$$C_{down-and-in}(t,B(t,r_t),S_t)=S_t\Phi(d_1)-KB(t,r_t)\Phi(d_2).\qquad(22)$$

Corollary 3. *When $H > K$, up and in call option can be written as*

$$\begin{aligned}C_{up-and-in}(t,B(t,r_t),S_t)=&S_t\Phi(d_5)-KB(t,r_t)\Phi(d_6)+HB(t,r_t)[\Phi(d_3)\\&-\Phi(d_7)]-\frac{K}{H}S_t[\Phi(d_4)-\Phi(d_8)].\end{aligned}\qquad(23)$$

where

$$d_5=\frac{\ln B(t,r_t)-\ln S_t+\ln H-s}{\sqrt{2s}},\ d_6=\frac{\ln B(t,r_t)-\ln S_t+\ln H+s}{\sqrt{2s}},$$

$$d_7 = \frac{\ln S_t - \ln B(t, r_t) - \ln H - s}{\sqrt{2s}}, d_8 = \frac{\ln S_t - \ln B(t, r_t) - \ln H + s}{\sqrt{2s}}.$$

Corollary 4. *When $H < K$,up and in T call option can be written as*

$$C'_{up-and-in}(t, B(t, r_t), S_t) = S_t\Phi(d_1) - KB(t, r_t)\Phi(d_2). \tag{24}$$

Corollary 5. *When $H > K$,up and out call option can be written as*

$$\begin{aligned} &C_{up-and-out}(t, B(t, r_t), S_t) \\ &= S_t[\Phi(d_1) - \Phi(d_5)] - KB(t, r_t)[\Phi(d_2) - \Phi(d_6)] \\ &\quad - HB(t, r_t)[\Phi(d_3) - \Phi(d_7)] + \frac{K}{H}S_t[\Phi(d_4) - \Phi(d_8)]. \end{aligned} \tag{25}$$

Corollary 6. *When $H < K$, up and out call option can be written as*

$$C'_{up-and-out}(t, B(t, r_t), S_t) = 0. \tag{26}$$

5. Conclusion

In this paper, we derived a closed-form pricing formula for barrier options. Previous option pricing studies typically assume that the short rate is constant or time-function over the life of the option. And the stock is driven by standard Brownian motion. In reality, the short rate is evolving randomly through time, and the stock is driven by fractional Brownian motion. Our findings suggest that barrier options on a stock can be calculated when the short rate follows the Vasicek model in fractional Brownian motion environment. It is clear that the Eq.(19) - Eq.(26) are the generalization of the classical Black-Scholes model.

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