Journal of the Korean Institute of Industrial Engineers
Vol. 37, No. 4, pp. 358-366, December 2011.

Multi-Criteria ABC Inventory Classification Using the Cross-Efficiency Method in DEA

Jaehun Park[1] · Hyerim Bae[1†] · Sungmook Lim[2]

[1]Department of Industrial Engineering, Pusan National University, Busan, 609-735, Korea
[2]School of Business Administration, Korea University, Chungnam, 339-700, Korea

DEA의 교차효율성을 활용한 다기준 ABC 재고 분류 방법 연구

박재훈[1] · 배혜림[1] · 임성묵[2]

[1]부산대학교 산업공학과 / [2]고려대학교 경영학부

Multi-criteria ABC inventory classification, which aims to classify inventory items by considering more than one criterion, is one of the most widely employed techniques for inventory control. The weighted linear optimization (WLO) model proposed by Ramanathan (2006) solves the problem of multi-criteria ABC inventory classification by generating a set of criterion weights for each inventory item and assigning a normalized score to the item for ABC analysis. However, the WLO model has some limitations. First, many inventory items can share the same optimal score, which can hinder a precise classification of inventory items. Second, the model allows too much flexibility in weighting multiple criteria; each item is allowed to choose its own weights so that it can maximize its score. As a result, if an item dominates the others in terms of a certain criterion, it may be classified into a higher class regardless of other criteria by assigning an overwhelming weight to the criterion. Consequently, an item with a high value in an unimportant criterion and low values in others may be inappropriately classified as class A, leading to an inaccurate classification of inventory items. To overcome these shortcomings, we extend the WLO model by using the cross-efficiency method in data envelopment analysis. We claim that the proposed model can provide a more reasonable and accurate classification of inventory items by mitigating the adverse effect of flexibility in the choice of weights and yielding a unique ordering of inventory items.

Keywords: ABC Inventory Classification, Multiple Criteria Analysis, Cross Efficiency, Data Envelopment analysis

1. Introduction

ABC analysis is an inventory categorization technique, where inventory items are classified into groups of different importance based on the Pareto principle. It provides a mechanism for identifying different categories of inventory items that require different levels of management and control effort. When a typical single-criterion ABC analysis is carried out, inventory items are valued by their unit price multiplied by the annual demand. The results are ranked and then grouped typically into three categories, which are called ABC codes. More specifically, "class A" inventory will typically contain items that account for 80% of the total value but only make up 20% of the total items. "Class B" in-

This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MEST)(No.2010-0027309).
† Corresponding author : Professor Hyerim Bae, Department of Industrial Engineering, Pusan National University, 30-san, Jangjeon-dong, Geumjong-gu, Busan 609-735, E-mail : hrbae@pusan.ac.kr
Received September 10, 2011; Revision Received October 14, 2011; Accepted November 10, 2011.

ventory will have approximately 15% of the total value and 30% of the total items, and "class C" inventory will account for the remaining 5% of the total value and 50% of the total items. The designation of these classes is arbitrary and the number of classes may be increased depending on the extent to which an organization wants to differentiate control efforts. Guvenir and Erel (1998) suggested that tightest management control and individual demand forecasts should be made for class A items, least control for class C items, and moderate control effort for class B items. Silver and Peterson (1985) suggested alternative inventory control policies for different inventory classes.

The traditional method of classifying inventory items using annual use value (annual demand multiplied by unit price) as the only criterion is based on the assumption that inventory items being classified are similar in nature and the major difference among them is in their annual use values (Ramanathan, 2006). Organizations, however, usually have to manage numerous inventory items, which are not necessarily homogeneous. Thus, it has been recognized that the traditional ABC analysis may not be able to provide a good practical inventory classification scheme (Guvenir and Erel, 1998). For a more practical ABC analysis, it is needed to consider various important criteria such as inventory cost, part criticality, lead time, substitutability, and number of requests in a year, rather than considering only annual use value. To address that requirement, multi-criteria ABC inventory classification problems have been studied in the literature. Multi-criteria ABC inventory classification, usually requiring complex computational tools, considers more than one criterion for categorizing inventory items into groups of different importance.

For the past 20 years, various methods for multi-criteria ABC inventory classification have been developed. A cross-tabulate matrix methodology was proposed by Flores and Whybark (1986, 1987) for bi-criteria inventory classification. Essentially their approach is to use the standard (single-criterion) ABC classification with each of two criteria, and then combine the two single-criterion groupings through the use of a joint-criteria matrix. Though this approach is a step forward in multi-criteria ABC classification, it has two limitations: it becomes too complicated when the number of criteria exceeds two, and it is applicable only when the weights on different criteria can be assumed to be equal. To overcome the drawbacks of the cross-tabulate matrix methodology, a multivariate technique of cluster analysis, which is an approach of grouping items with similar nature together, was proposed. A solution procedure combining clustering analysis and operations constraints for inventory classification was proposed by Flores *et al.* (1992). Guvenir and Erel (1998) and Partovi and Anandarajan (2002) suggested the use of meta-heuristics such as genetic algorithms and artificial neural networks for multi-criteria inventory classification. However, these meta-heuristics are complicated, requiring much computation time, and are difficult for inventory managers to understand. Partovi and Hopton (1994) and Gajpal *et al.* (1994) proposed multi-criteria decision-marking tools for multi-criteria ABC inventory analysis, one of which is the analytic hierarchy process (AHP). The advantage of the AHP approach is that it can deal with various relevant qualitative and quantitative criteria for classifying inventory items, and does not need massive accounting and measurement systems. However, it requires a subjective judgment when making pair-wise comparisons of criteria. To remedy these drawbacks of the existing methods, Ramanathan (2006) developed a data envelopment analysis (DEA)-like model, called the weighted linear optimization (WLO) model, which generates a set of criterion weights for each inventory item and gives a normalized score (called inventory score) to the item for a subsequent ABC analysis. This differs from other methods such as those based on the AHP in which weights are specified exogenously. In contrast, the WLO model determines the weights endogenously. The basic concept of the WLO model is similar to the concept of DEA (Charnes *et al.*, 1978). The weights in the WLO model are generated by a DEA-like linear optimization to avoid the subjectivity of the weight assignments. This model is simple, easy to understand, and very flexible since it can easily incorporate additional information from decision makers for inventory classification. See Ng (2007).

Although the WLO model has many advantages, the following limitations prevent effective inventory classification :

1) The WLO model is based on DEA and many items can share the same optimal inventory score. Since ABC classification is carried out based on the optimal inventory scores of items, such ties among items can make it difficult to precisely classify items.
2) Second, the model allows too much flexibility in weighting multiple criteria; each item is allowed to choose its own weights so that it can maximize its score. As a result, if an item dominates the others in terms of a certain criterion, it may be classified into a higher class regardless of other criteria by assigning an overwhelming weight to the criterion. Thus, an item with a high value in an unimportant criterion may be inappropriately classified as class A.

To address these problems with the WLO model, we propose a DEA-based multi-criteria ABC inventory model which employs cross-efficiency evaluation in DEA. This model is an extension of the WLO model and incorporates cross-efficiency evaluations into the WLO model to provide a finer classification (or ranking) of inventory items. Since it was first proposed by Sexton *et al*. (1986), the cross-efficiency evaluation has been considered as a powerful extension of DEA. Over the last few years, numerous subsequent developments have been made and its use has proliferated (Doyle and Green, 1994). The proposed model uses the pairwise efficiency game formulation of cross-efficiency evaluation, suggested by Talluri (2000), where inventory items are pairwisely compared an inventory item under evaluation selects optimal weights that maximize its inventory score and at the same time minimize the inventory score of each competitor, in turn. Therefore, the optimal weights of an inventory item under evaluation may vary depending on the competing item being evaluated. In this way, an item under evaluation can involve multiple strategies (optimal weights), in which it emphasizes its strengths, as well as the weaknesses of a specific competing item. By changing the item under evaluation, the formulation is rerun n-1 times, which results in exactly n-1 inventory scores for each item in addition to the optimal score obtained from the WLO model, where n is the number of inventory items. The mean of these scores can be utilized as an index to rank inventory items and identify more important ones.

This paper is organized as follows. Section 2 provides a brief overview of multi-criteria ABC inventory classification. Section 3 discusses our proposed method, followed by an empirical study in Section 4. Finally, Section 5 summarizes our work.

2. The Weighted Linear Optimization (WLO) Model for Multi-Criteria Inventory Classification

In this section, we describe the WLO method proposed by Ramanathan (2006), which is a model for solving multi-criteria ABC inventory classification problems. Using the mechanism of DEA, the WLO model automatically generates a set of criterion weights for an item being evaluated and assigns a normalized score to the item for ABC analysis.

DEA is a non-parametric linear programming based technique for measuring relative efficiencies of a homogeneous set of decision making units (DMUs) with multiple inputs and outputs. Model (1) below is the input-oriented CCR model in multiplier form Charnes *et al*. (1978), a basic DEA model used to determine the efficiency score of DMU k:

$$\begin{aligned} & Max\ \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \\ & s.t.\ \ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1\,;\, j=1,\cdots,n \\ & \qquad u_r,\, v_i \ge 0\,;\, r=1,\cdots,s\,;\, i=1,\cdots,m \end{aligned} \tag{1}$$

where u_r is the weight given to the r-th output of DMU k, v_i is the weight given to the i-th input of DMU k, n is the number of DMUs, s is the number of outputs, m is the number of inputs, y_{rj} is the amount of the r-th output produced by DMU j, and x_{ij} is the amount of the i-th input used by DMU j.

Model (1) can be transformed into a linear model, as shown in Model (2). For more details on model development, refer to (Charnes *et al*., 1978).

$$\begin{aligned} & Max\ \sum_{r=1}^{s} u_{rk} y_{rk} \\ & s.t.\ \ \sum_{i=1}^{m} v_{ik} x_{ik} = 1 \\ & \qquad \sum_{r=1}^{s} u_{rk} y_{rj} \le \sum_{i=1}^{m} v_{ik} x_{ij};\, j=1,\cdots,n \\ & \qquad u_{rk},\, v_{ik} \ge 0\ \forall\, r,\, i \end{aligned} \tag{2}$$

When inventory items are viewed as DMUs, a multi-criteria inventory classification problem can be cast into a DEA problem. The performance of an item in terms of a criterion can be considered as one of its outputs, and the levels of inputs are set constant. Using the same symbols in Model (2), n inventory items have to be classified into A, B or C based on their performance in terms of s criteria. We denote the performance of the k-th item in terms of the r-th criterion by y_{rk} and assume that all of the criteria are positively related to the overall importance level of the item. The WLO model aggregates the multiple performance levels of an item with respect to different criteria into a single score, gl_k for the subsequent ABC inventory classification by using the following optimization model:

$$\begin{aligned} & gl_k = Max \sum_{r=1}^{s} u_{rk} y_{rk} \\ & s.t.\ \ \sum_{r=1}^{s} u_{rk} y_{rj} \le 1;\ j=1,\cdots,n \\ & \qquad u_{rk} \ge 0\ \forall\, r \end{aligned} \tag{3}$$

The weighted additive function, gl_k aggregates the performance of an inventory item in terms of different criteria and its optimal value is used as the (inventory score of the k-th item. The function is maximized under the condition that the weighted sum of the performance levels for each item, computed using the same set of weights, should be less than or equal to 1. Due to the assumptions made above, the function is positively related to the overall importance level of the item. Hence, the larger the optimal inventory score of an item is, the greater the chance should be that the item is classified into class A. To obtain the optimal inventory scores for all inventory items, Model (3) has to be solved repeatedly by altering the item being evaluated.

3. The Proposed Model

In this section, we describe our proposed model, which can deal with the aforementioned limitations of the WLO model for multi-criteria ABC inventory classification problems. The proposed model incorporates cross-efficiency evaluations into the WLO model to provide a finer classification (or ranking) of inventory items. As a result, the proposed model performs in a peer-evaluation mode while the WLO model works in a self-evaluation mode. Specifically, the proposed model uses the pair-wise efficiency game formulation of cross-efficiency suggested by Talluri (2000). In the model, a pair of inventory items is compared with each time. An inventory item under evaluation selects optimal weights that maximize its (inventory) efficiency score and at the same time minimize the inventory score of each competitor, in turn. Therefore, a single run of the model with an item under evaluation yields a set of multiple optimal scores; one is for the item itself, and the others are for the competing items. After a complete run of the model, each item will be given n optimal scores, where n is the number of inventory items; one is the efficiency score (in DEA sense) for the item itself, and the others are the item's cross-efficiency scores evaluated by the competing items.

<Figure 1> depicts the required steps of the proposed model. First, we calculate the optimal efficiency score of each inventory item by solving the WLO model with only the primary goal of maximizing its score. Second, the cross-efficiency scores of each item are calculated by incorporating the secondary goal. Third, for each item, the final score is computed, which is the average of its cross-efficiency scores. This final score of each item can be used as an approximate measure of its overall importance, where a higher score indicates higher importance. Finally, inventory items are categorized into classes A, B and C based on their importance indices.

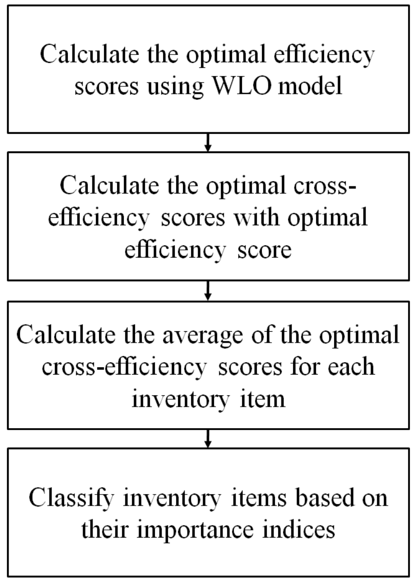

Figure 1. The steps of the proposed model

The dual problem to model (3) is shown as model (4) :

$$
\begin{aligned}
gl_k = Min &\sum_{j=1}^{n} \lambda_j \\
s.t. &\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rk}, r = 1, \cdots, s, \\
&\lambda \geq 0, j = 1, \cdots, n
\end{aligned} \tag{4}
$$

Model (4) seeks to find the optimal weights that minimize the sum of weights assigned to all the items under the conditions that the weighted sum of all the items is greater than or equal to item k. λ_j is the dual variable which is assigned to the j-th item. The larger the sum of weights of an item is, the greater the chance should be that the item is classified into class A. Note that the dual problem is more computationally easier than the primal one, considering the fact that the number of inventory items (n) is typically much more than the number of criteria(s).

The optimal cross-efficiency score of item p evaluated by item k is determined by solving the following Model (5)

$$
\begin{aligned}
pc_{pk} = Min &\sum_{r=1}^{s} u_{rk} y_{rp} \\
s.t. &\sum_{r=1}^{s} u_{rk} y_{rk} = gl_k \\
&\sum_{r=1}^{s} u_{rk} y_{rj} \leq 1; j = 1, \cdots, n \\
&u_{rk} \geq 0 \; \forall r
\end{aligned} \tag{5}
$$

where gl_k is the optimal efficiency score of item k determined by Model (4). Note that Model (5) seeks to find the optimal weights that maximize the score of item k as the primary goal and subsequently minimize the score of competing item p as a secondary goal. The first constraint is a blanket constraint that prevents the efficiency score of the target item from being either higher or lower than its optimal efficiency score. The second constraint is a normalization constraint that prevents the efficiency scores of all the items from exceeding a value of 1. For each item, this model is repeatedly solved by altering the competing item, resulting in n-1 optimal weights. Therefore, in the end, each item will have n-1 optimal cross-efficiency scores given by n-1 competing items along with its own optimal efficiency score. <Figure 2> depicts the process for calculating the optimal cross-efficiency scores and the importance index of each competing item.

The dual problem to Model (5) is shown as Model (6)

$$\begin{aligned} pc_{pk} = Max \sum_{j=1}^{n} \lambda_j + gl_k \theta_p \geq y_{rp},\ r=1,\cdots,s, \\ s.t. \sum_{j=1}^{n} \lambda_j y_{rj} + y_{rk}\theta_p \geq y_{rp},\ r=1,\cdots,s, \\ \lambda_j \geq 0,\ j=1,\cdots,n \\ \theta_p \geq 0,\ p=1,\cdots,n \\ gl_k \geq 0,\ k=1,\cdots,n \end{aligned} \tag{6}$$

where Θ_p is the dual variable which is assigned to the p-th competing item and λ_j is the dual variable which is assigned to the j-th item.

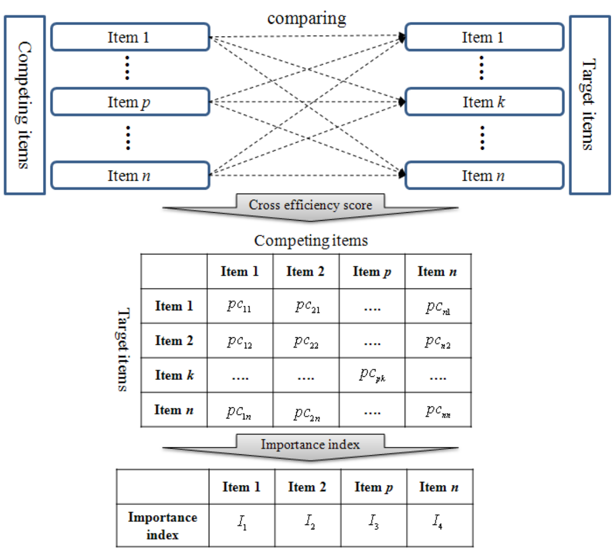

Figure 2. Process for calculating the optimal cross-efficiency scores and the importance index

All of the n scores that an item obtains are averaged to give the item's importance index. Specifically, I_i is the importance index of item i and is computed by the following Equation (7).

$$I_i = \frac{\sum_{j=1}^{n} pc_{ij}^{*}}{n},\ i=1,\cdots,n \tag{7}$$

Note that is the optimal cross-efficiency score of item i evaluated by item j.

We illustrate the proposed model with the example data given in <Table 1>. The data set consists of ten inventory items (from item 1 to item 10) that are evaluated in terms of four criteria : average unit cost, annual dollar usage, critical factor (1.00 for a very critical item, 0.01 for a non-critical item and 0.50 for a moderately critical item), and lead time. These criteria are referred from the experiment in the research of Ramanathan (2006), but the each data are randomly assigned based on the data in corresponded experiment.

First, the optimal efficiency scores for the five items are computed using the WLO model, and the results are shown in <Table 2>. Items 1, 3, 5, and 6 all have an efficiency score of one and are deemed as being of greater importance, whereas items 2, 4, 7, 8, 9, and 10 are considered to be of relatively lower importance since they have smaller efficiency scores. If we follow the classification scheme of the traditional ABC analysis, only the two items with the greatest importance should be classified into class A, three items should be classified into class B, and the remaining four items should be classified into class C. The top four items (1, 3, 5 and 6), however, have the same efficiency scores, which obstructs the traditional ABC classification scheme. This shows the first limitation of the WLO model, as explained in Section 1. Comparing between items 2 and 7, the annual dollar usage and critical factor of item 2 are higher than those of item 7. However, the optimal efficiency score of item 7 is higher than that of item 2 and this shows the second limitation of WLO model.

On the other hand, if we apply the proposed model to the same data set, we obtain the cross-efficiency table as shown in <Table 3>. Each cell contains the cross-efficiency score of the competing item in the corresponding column evaluated by the target item under evaluation in the corresponding row. For instance, the cross-efficiency score of competing item 2 evaluated by target item 1 is 0.57.

The average cross-efficiency scores are shown in the second row of <Table 4>, which will be used as a measure for each item's overall importance. The results in <Table 3> and <Table 4> reveal that the importance index of an item is lower than the item's optimal efficiency score. In other words, an item cannot have a cross efficiency score higher than its optimal efficiency score.

Table 1. Example data

Item no.	Average unit cost ($)	Annual dollar usage ($)	Critical factor	Lead time (Min)
1	71.21	34.40	0.01	7
2	58.45	467.60	0.50	4
3	40.82	163.28	1.00	3
4	19.80	79.20	0.01	2
5	86.50	103.80	1.00	7
6	71.20	854.40	1.00	4
7	78.40	313.60	0.01	4
8	51.68	103.36	0.01	6
9	14.66	703.68	0.48	4
10	72.00	216.00	0.46	5

Table 2. The optimal efficiency score of each inventory item

Item no.	1	2	3	4	5	6	7	8	9	10
gl_m	1.00	0.79	1.00	0.31	1.00	1.00	0.96	0.86	0.92	0.86

Table 3. The cross-efficiency score of each inventory item

	Competing										
	Items	1	2	3	4	5	6	7	8	9	10
Target	1	***1.00***	0.57	0.43	0.29	1.00	0.57	0.57	0.86	0.57	0.71
	2	0.96	***0.79***	0.50	0.31	1.00	1.00	0.72	0.86	0.92	0.80
	3	0.01	0.60	***1.00***	0.01	1.00	1.00	0.01	0.01	0.48	0.46
	4	0.97	0.79	0.50	***0.31***	1.00	1.00	0.72	0.86	0.92	0.79
	5	1.00	0.79	1.00	0.31	***1.00***	1.00	0.96	0.97	0.92	0.86
	6	0.96	0.79	1.00	0.31	1.00	***1.00***	0.96	0.86	0.92	0.86
	7	0.81	0.77	0.50	0.24	1.00	1.00	***0.96***	0.61	0.33	0.86
	8	0.99	0.79	0.50	0.31	1.00	1.00	0.71	***0.86***	0.92	0.79
	9	0.96	0.79	0.49	0.31	1.00	1.00	0.71	0.86	***0.92***	0.79
	10	0.82	0.77	0.50	0.24	1.00	1.00	0.95	0.61	0.35	***0.86***

Table 4. The importance index of each item

Items	1	2	3	4	5	6	7	8	9	10
Importance index	0.85	0.75	0.64	0.27	1.00	0.96	0.73	0.74	0.72	0.78
Ranking	3	5	9	10	1	2	7	6	8	4
Classified into Class	B	B	C	C	A	A	C	C	C	B

Using the results obtained from the proposed model, ten inventory items can be ranked in terms of their overall importance. Item 5 ranks first because it has the greatest importance index value of 1.00, followed in order by items 6, 1, 10, 2, 8, 7, 9, 3, and 4, as shown in <Table 4>. Based on this finer ranking, items 5 and 6 are classified into class A, 1, 10 and 2 into class B, and 8, 7, 9, 3 and 4 into class C.

As observed in this example, the proposed model provides a more reasonable and accurate scheme for classifying inventory items since it eliminates the possibility of items choosing unrealistic factor weights and thus yields a unique ordering of items for effective inventory classification.

4. An Illustrative Example

To further demonstrate the usefulness of the proposed model, we apply the proposed model to a larger-scale example. The problem instance we use here was originally introduced by Flores *et al.* (1992), where the overall importance level of inventory items are evaluated in terms of four criteria: average unit cost ($), annual dollar usage ($), critical factor (1, 0.5 and 0.01 for very-critical, moderately critical and non-critical item), and lead time (min) (ranging from 1 to 7 weeks). For the purpose of a comparative study, we maintain the same distribution of class A, B and C items with that given by Ramanathan (2006), i.e., 10, 14 and 23 items in classes A, B and C, respectively. We assume all four criteria to be positively related to the overall importance of items. The importance index scores of all 47 inventory items are computed using both the WLO model and the proposed model. <Table 5> presents the data and the related results. With the WLO model, 15 items (13, 2, 9, 3, 29, 23, 21, 45, 34, 15, 1, 36, 24, 32 and 11) have the same efficiency score of one and are deemed as being of greater importance and can be classified into class A. However, the same efficiency score among many items possibly create the problem of insufficient differentiation among items and inaccurate ordering in classifying the items, which prevents an effective ABC classification. Note that more items will share the same efficiency score as the number of criteria increases, which is a typical property of the traditional DEA. In the result, the WLO model has critical problem for classifying items into class A. For example, if we classify the items for class A, it is difficult to precisely classify these 15 items into class A since only 10 out of these 15 items can be classified into class A. On the other hand, the proposed model can solve this problem by yielding a unique categorization of items.

Comparing the classification results between the WLO model and the proposed model, inventory item 1 is ranked as 1 by the WLO model with the highest annual dollar usage and a relatively low average unit cost and lead time. However, it is ranked as 17 in the proposed model since the proposed model takes into consideration not only the highest annual dollar usage but also its lower average unit cost and lead time when determining the item's importance level. Another example is provided by items 14 and 45. The average unit cost, annual dollar usage and critical factor of item 14 are higher than those of item 45. However, items 14 and 45 are ranked as 20 and 1, respectively, by the WLO model, for the same reason as in the case of item 1. On the contrary, the proposed model classified items 14 and 45 are ranked as 9 and 10, respectively. In this result, we emphasize that the proposed model help to increase the accuracy of item classification in the inventory management aspects.

When we compare the uniqueness, many items can share the same inventory score of 1 in the WLO model. For instance, inventory items 13, 2, 9, 3, 29, 23, 21, 45, 34, 15, 1, 36, 24, 32 and 11 have the same optimal inventory score of 1, and items 18, 28 and 40, and items 31, 19, 39. 33, 37, 43 and 47, and items 38 and 16, and items 44, 26 and 46, and items 42 and 41 have the same scores of 0.857, 0.714, 0.500, 0.429 and 0.286, respectively. On the contrary, the proposed model yields a unique ordering of items. Therefore, we claim that the proposed model provides a more reasonable and appropriate index for multi-criteria ABC inventory classification as compared to the WLO model in classifying the items aspects.

5. Conclusion

In this paper, we have proposed a DEA-based model for multi-criteria ABC inventory classification problems which incorporates cross-evaluations into the WLO model developed by Ramanathan (2006). The proposed model enables a finer classification of inventory items and resolves the deficiencies of the WLO model.

Although ABC inventory classification has been widely used primarily due to its simplicity and effectiveness, these advantages are difficult to maintain when multiple criteria are involved due to practical considerations. The several methodologies that have been proposed in the literature to deal with multiple criteria in inventory classification are too complex to be used in practice. Although the WLO model is relatively simple and easy to manage, its effectiveness is rather flawed. In particular, its inability to provide a finer classification of items of higher importance becomes a significant problem considering the fact that the identification of class A items is critical to the success of ABC analysis. In this regard, the proposed model improves upon the WLO model and can be used as a simple and an effective scheme for inventory classification.

The proposed model provides a more reasonable and accurate ordering scheme in classifying the inventory items than the WLO model does since it yields a unique ordering of the inventory items and eliminates unrealistic factor weights. However, despite its several advantages, the proposed model also has the limitation of a long computational time, especially in the case of

Table 5. Results of the comparison between the WLO model and the proposed model

Item no.	Average unit cost	Annual dollar usage	Critical factor	Lead time	Score		Rank		ABC classification
					Proposed model	WLO model	Proposed model	WLO model	Proposed model
S13	86.5	1038	1	7	0.934	1.000	1	1	A
S02	210	5670	1	5	0.903	1.000	2	1	A
S09	73.4	2423.5	1	6	0.866	1.000	3	1	A
S03	23.8	5037.1	1	4	0.715	1.000	4	1	A
S18	49.5	594	0.5	6	0.691	0.857	5	16	A
S29	134.3	268.68	0.01	7	0.690	1.000	6	1	A
S23	86.5	432.5	1	4	0.684	1.000	7	1	A
S21	24.4	463.6	1	4	0.650	1.000	8	1	A
S14	110.4	883.2	0.5	5	0.642	0.782	9	20	A
S45	34.4	34.4	0.01	7	0.638	1.000	10	1	A
S34	7.07	190.9	0.01	7	0.628	1.000	11	1	B
S10	160.5	2407.5	0.5	4	0.609	0.781	12	21	B
S31	72	216	0.5	5	0.609	0.714	13	23	B
S19	47.5	570	0.5	5	0.602	0.714	14	23	B
S15	71.2	854.4	1	3	0.600	1.000	15	1	B
S12	20.9	1043.5	0.5	5	0.597	0.732	16	22	B
S01	49.9	5840.6	1	2	0.578	1.000	17	1	B
S28	78.4	313.6	0.01	6	0.576	0.857	18	16	B
S36	40.8	163.3	1	3	0.574	1.000	19	1	B
S24	33.2	398.4	1	3	0.573	1.000	20	1	B
S40	51.7	103.4	0.01	6	0.559	0.857	21	16	B
S22	65	455	0.5	4	0.522	0.593	22	32	B
S20	58.5	467.6	0.5	4	0.518	0.581	23	33	B
S32	53	212.1	1	2	0.501	1.000	24	1	B
S17	14.7	703.7	0.5	4	0.500	0.578	25	34	C
S11	5.1	1075.2	1	2	0.486	1.000	26	1	C
S05	58	3478.8	0.5	3	0.484	0.613	27	31	C
S39	59.6	119.2	0.01	5	0.475	0.714	28	23	C
S33	49.5	197.9	0.01	5	0.471	0.714	29	23	C
S37	30	150	0.01	5	0.461	0.714	30	23	C
S43	29.9	59.8	0.01	5	0.459	0.714	31	23	C
S06	31.2	2936.7	0.5	3	0.458	0.573	32	35	C
S07	28.2	2820	0.5	3	0.455	0.566	33	36	C
S47	8.5	25.4	0.01	5	0.449	0.714	34	23	C
S38	67.4	134.8	0.5	3	0.435	0.500	35	37	C
S08	55	2640	0.01	4	0.435	0.690	36	30	C
S16	45	810	0.5	3	0.431	0.500	37	37	C
S35	60.6	181.8	0.01	3	0.300	0.436	38	39	C
S44	48.3	48.3	0.01	3	0.291	0.429	39	40	C
S26	33.8	338.4	0.01	3	0.290	0.429	40	40	C
S46	28.8	28.8	0.01	3	0.282	0.429	41	40	C
S04	27.7	4769.6	0.01	1	0.210	0.817	42	19	C
S42	37.7	75.4	0.01	2	0.199	0.286	43	44	C
S41	19.8	79.2	0.01	2	0.190	0.286	44	44	C
S27	84	336.1	0.01	1	0.151	0.400	45	43	C
S30	56	224	0.01	1	0.129	0.267	46	46	C
S25	37.1	370.5	0.01	1	0.118	0.188	47	47	C

numerous inventory items since each inventory item has to be compared to all the remaining items. Further research will be needed to develop a new model to reduce the computational time in cross-evaluations.

References

Charnes, A., Cooper, W. W., and Rhodes, E. (1978), Measuring the efficiency of decision making units, *Journal of Operating Research*, **2**(6), 429-444.

Doyle, J. and Green, R. (1994), Efficiency a Cross-efficiency in DEA : Derivations, Meanings and Uses, *Journal of Operational Research Society*, **45**(5), 567-578.

Flores, B. E. and Whybark, D. C. (1986), Multiple criteria ABC analysis, *International Journal of Operations and Production Management*, **6**(3), 38-46.

Flores, B. E. and Whybark, D. C. (1987), Implementing multiple criteria ABC analysis, *Journal of Operations Management*, **7**(1/2), 79-84.

Flores, B. E., Olson, D. L., and Dorai, V. K. (1992), Management of multi criteria inventory classification, *Mathematical and Computer Modelling*, **16**(12), 71-82.

Gajpal, P. P., Ganesh, L. S., and Rajendran, C. (1994), Criticality analysis of spare parts using the analytic hierarchy process, *International Journal of Production Economics*, **35** (1/3), 293-297.

Guvenir, H. A. and Erel, E. (1998), Multi criteria inventory classification using a genetic algorithm, *European Journal of Operational Research*, **105**(1), 29-37.

Ng, W. L. (2007), A simple classifier for multiple criteria ABC analysis, *European Journal of Operational Research,* **177** (1), 344-353.

Partovi, F. Y. and Anandarajan, M. (2002), Classifying inventory using an artificial neural network approach, *Computers and Industrial Engineering*, **41**(4), 389-404.

Partovi, F. Y. and Hopton, W. E. (1994), The analytic hierarchy process as applied to two types of inventory problems, *Production and Inventory Management Journal*, **35**(1), 13-19.

Ramanathan, R. (2006), ABC inventory classification with multiple-criteria using weighted linear optimization, *Computers and Operations Research*, **33**(3), 695-700.

Sexton, T. R., Silkman, R. H., and Hogan, A. J. (1986), Data Envelopment Analysis: critique and extensions. In Measuring efficiency : An Assessment of Data Envelopment Analysis. San Francisco : Jossey-Bass, 73-105.

Silver, E. A. and Peterson, R. (1985), Decision system for inventory management and production planning, New York : Wiley.

Talluri, S. (2000), A benchmarking method for business-process reengineering and improvement, *The International Journal of Flexible Manufacturing System*, **12**(4), 291-304.