A Novel Recursive Algorithm for Efficient ZF-OSIC Detection in a V-BLAST System

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Abstract

To reduce the implementation complexity of the Vertical Bell Labs layered space-time (V-BLAST) systems with respect to the zero-forcing (ZF) criterion, a computationally efficient recursive algorithm is proposed. A fast implementation of the proposed algorithm is developed and its complexity is analyzed in detail. The proposed algorithm matches the ZF-OSIC detection well, and its three significant advantages can be demonstrated by analyses and simulations. Firstly, its speedups over the conventional ZF-OSIC with norm-based ordering, the original fast recursive algorithm (FRA) and the fastest known algorithm (FKA) in the number of flops are 1.58, 2.33 and 1.22, respectively. Secondly, a much simpler implementation than FRA and FKA can be expected. Finally, the storage requirements are lower than those of FRA and FKA. These advantages make the proposed algorithm more efficient and practical.

Keywords: Multiple-input--multiple-output (MIMO) systems, V-BLAST, FRA, ZF, successive interference cancellation (SIC)

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1. Introduction

High spectral efficiencies can be achieved in the multiple-input multiple-output (MIMO) wireless communication systems in a rich scattering environment without additional power and bandwidth [1]. Vertical Bell Labs layered space-time (V-BLAST) is considered as a very promising architecture for implementation of the MIMO systems [2][3]. In a V-BLAST architecture, nulling filters based on zero-forcing (ZF) and minimum mean-square error (MMSE) criterion are often combined with ordered successive interference cancellation (OSIC) to generate ZF-OSIC and MMSE-OSIC detectors, respectively [4]. The high implementation complexity of the original V-BLAST, however, makes it difficult to be implemented in real-time systems [5].

To reduce the implementation complexity of the original V-BLAST detection algorithm, two main categories of simplified algorithms have been proposed based on the squared root algorithm (SRA) [6] and the fast recursive algorithm (FRA) [5], respectively. Moreover, the improved algorithms based on SRA and FRA were proposed in [7][8][9][10] and [11][12][13][14][15][16], respectively, to reduce their complexities further. The fast recursive algoirthms in [11] consider memory movement as a criteria for algorithm efficiency, and the algorithm I in [11] has a simple implementation without matrix permutations. The improved algorithms in [12][13][14][15] reduce the complexity of the original FRA to various degrees. The Algorithm II in [15], which took memory into consideration, reduced the number of matrix required to be stored so that the memory can be saved. The algorithm in [16] focuses on the application of FRA to G-STBC systems. Some results for FRA and its improved algorithms are given in [17].

Most of the proposed algorithms focus on the MMSE-OSIC detectors. In some scenarios, however, ZF-OSIC is also an important alternative, especially when simplicity, cost effectiveness and ease of implementation are the primary considerations. The contribution of this paper is to propose an improved ZF-OSIC recursive algorithm, which is based on two propositions developed to match the specific structure of ZF-OSIC detector. Fast implementation of the proposed algorithm can be achieved and its computational complexity is analyzed and simulated. The computational complexity, memory transfer requirements, and storage requirements of the proposed algorithm are compared with those of the existing recursive algorithms. Conclusions are made according to the theoretical analyses and simulations. The proposed algorithm has mainly three advantages. Firstly, compared with the conventional ZF-OSIC with norm-based ordering, the well known FRA and the fastest known algorithm (FKA), the proposed algorithm reduces the computational complexity by a factor of 1.58, 2.33 and 1.22 in terms of flops, respectively. Secondly, no permutations of the matrix columns and rows are required in this algorithm, which guaranties its simpler implementation than the permutation-based FRA and its improved algorithms. Finally, the memories for implementation of the proposed algorithm can also be saved. Comparisons with the algorithm with post-SNR optimal ordering in terms of bit-error-rate (BER) is also provided. Numerical results demonstrate the near-optimal performance of the proposed algorithm with practical number of antennas, which guaranties both the fairness of the above complexity comparisons and the numerical stability of the proposed algorithm.

The remainder of this paper is organized as follows. The system model and notation are introduced in Section 2. The proposed algorithm, its fast implementation and comparisons are presented in Section 3 and 4, respectively. Finally, we conclude this paper in Section 5.

2. System Model

In what follows, matrices and vectors are represented by uppercase and lowercase boldface symbols, respectively. Scalars are represented by italic symbols. Notations $(\cdot)^H$, $(\cdot)^T$ and $(\cdot)^{-1}$ denote the Hermitian transpose, transpose and matrix inversion, respectively. $\mathbf{I}_N$ and $\mathbf{0}_N$ denote the $N \times N$ identity matrix and zero matrix, respectively.

Consider a ZF-OSIC V-BLAST system with M transmitting antennas and N ($\geq M$) receiving antennas in a flat-fading and rich-scattering environment. The $N \times M$ complex channel matrix $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_M]$ has a full column rank with statistically independent entries. The transmitted symbol vector is $\mathbf{s}(k) = [s_1(k), s_2(k), \ldots, s_M(k)]^T$ with covariance $E\{\mathbf{s}(k)\mathbf{s}^H(k)\} = \sigma_s^2 \mathbf{I}_M$. Then the received symbol vector at time instant k is given by

$$\mathbf{r}(k) = \mathbf{H}\mathbf{s}(k) + \mathbf{w}(k), \tag{1}$$

where $\mathbf{w}(k)$ denotes the $N \times 1$ complex Gaussian noise vector with zero mean and covariance $\sigma_w^2 \mathbf{I}_N$.

The objective of V-BLAST is to recover the symbol vector $\mathbf{s}(k)$ from the observations $\mathbf{r}(k)$ and the estimate $\mathbf{H}$. In the original V-BLAST system, the channel matrix $\mathbf{H}$ can be estimated by exploiting the training sequence in the preamble transmitted in each frame. In this study, we will not make the distinction between the realistic channel and its estimate.

3. Improved ZF-OSIC Recursive Algorithm

In this section, we propose an improved algorithm for ZF-OSIC architecture. The algorithm exploits the structure of ZF detector and is more efficient than the original FRA.

The column norm-based optimal ordering is considered in this algorithm to reduce the ordering complexity.The column norms $\|\mathbf{h}_m\|, m = 1, 2, \ldots, M$, are calculated and then sorted in an ascending order, generating index vector $\mathbf{t}(k) = [t_1, t_2, \ldots, t_M]$. Denote the detection ordering vector by $\mathbf{f}(k) = [p_1, p_2, \ldots, p_M]$, specifying the order in which components of the transmitted symbol vector $\mathbf{s}(k)$ are extracted. Based on the criteria that the strongest signal is detected first, the relationship between the index vector and the detection ordering vector can be simply expressed as $p_m = t_{M-m+1}, m = 1, 2, \ldots, M$, i.e., in the m-th iteration, $s_{t_{M-m+1}}(k)$ is the symbol to be detected.

Based on this index vector, a permuted channel matrix is defined as

$$\bar{\mathbf{H}} = \left[\mathbf{h}_{t_1}, \mathbf{h}_{t_2}, \ldots, \mathbf{h}_{t_M}\right], \tag{2}$$

where $\mathbf{h}_{t_m}$ is the t_m th column of the original channel matrix $\mathbf{H}$ in (1). Correspondingly, the symbol vector is permuted as

$$\bar{\mathbf{s}}(k) = [s_{t_1}(k), s_{t_2}(k), \ldots, s_{t_M}(k)]^T, \tag{3}$$

where $s_{t_m}(k)$ is the t_m th element of original transmitted symbol $\mathbf{s}(k)$ in (1). It is necessary to point out that the permutations of the channel matrix and the symbol vector are just steps needed for the following derivation and they are not involved in the proposed algorithm.

Define a $m \times m$ recursive matrix

$$\mathbf{\Psi}_m = \bar{\mathbf{H}}_m^H \bar{\mathbf{H}}_m, \tag{4}$$

where $m = 1, 2, \ldots, M$, and $N \times m$ matrix $\bar{\mathbf{H}}_m$ denotes the first m columns of $\bar{\mathbf{H}}$ in (2). Let $\bar{\mathbf{s}}_m(k)$ be the first m elements in $\bar{\mathbf{s}}(k)$ in (3), i.e., $\bar{\mathbf{s}}_m(k) = [s_{t_1}(k), s_{t_2}(k), \ldots, s_{t_m}(k)]^T$.

During the m th iteration, the ZF estimation of $\bar{\mathbf{s}}_{M-m+1}(k)$ is given by [3]

$$\bar{\mathbf{y}}_{M-m+1}(k) = \mathbf{\Psi}_{M-m+1}^{-1} \bar{\mathbf{H}}_{M-m+1}^H \mathbf{r}_{M-m+1}(k), \tag{5}$$

and

$$\hat{s}_{t_{M-m+1}}(k) = Q[y_{t_{M-m+1}}(k)], \tag{6}$$

where $\bar{\mathbf{y}}_{M-m+1}(k) = [y_{t_1}(k), y_{t_2}(k), \ldots, y_{t_{M-m+1}}(k)]^T$ is the estimate of $\bar{\mathbf{s}}_{M-m+1}(k)$, and $\hat{s}_{t_{M-m+1}}(k)$ is the quantized estimate signal of $s_{t_{M-m+1}}(k)$, and $Q[\cdot]$ denotes the quantization operation. $\mathbf{r}_{M-m+1}(k)$ denotes the updated received signal after the former $m-1$ cancellations, and

$$\mathbf{r}_{M-m+1}(k) = \mathbf{r}_{M-m+2}(k) - \hat{s}_{t_{M-m+2}}(k)\mathbf{h}_{t_{M-m+2}}. \tag{7}$$

Before introducing the new algorithm, two propositions are given as follows.

Proposition 1: If an $N \times N$ recursive matrix $\mathbf{P}_m$, $m = 1, 2, \ldots, M$, is defined as

$$\mathbf{P}_m = \mathbf{I}_N - \bar{\mathbf{H}}_m \mathbf{\Psi}_m^{-1} \bar{\mathbf{H}}_m^H, \tag{8}$$

where $\mathbf{P}_0 = \mathbf{I}_N$, and $\bar{\mathbf{H}}_m$ is of full-column rank, then

$$\mathbf{P}_{m+1} = \mathbf{P}_m - \frac{\mathbf{P}_m \mathbf{h}_{t_{m+1}} \mathbf{h}_{t_{m+1}}^H \mathbf{P}_m}{\mathbf{h}_{t_{m+1}}^H \mathbf{P}_m \mathbf{h}_{t_{m+1}}}, m = 0, 1, \ldots, (M-1), \tag{9}$$

Proof: According to the definition of $\mathbf{P}_m$ in (8), it follows that

$$\mathbf{P}_{m+1} = \mathbf{I}_N - \bar{\mathbf{H}}_{m+1} \mathbf{\Psi}_{m+1}^{-1} \bar{\mathbf{H}}_{m+1}^H, \tag{10}$$

where

$$\bar{\mathbf{H}}_{m+1} = \left[\bar{\mathbf{H}}_m \quad \mathbf{h}_{t_{m+1}} \right], \tag{11}$$

and

$$\boldsymbol{\Psi}_{m+1} = \bar{\mathbf{H}}_{m+1}^H \bar{\mathbf{H}}_{m+1} = \begin{bmatrix} \boldsymbol{\Psi}_m & \bar{\mathbf{H}}_m^H \mathbf{h}_{t_{m+1}} \\ \mathbf{h}_{t_{m+1}}^H \bar{\mathbf{H}}_m & \mathbf{h}_{t_{m+1}}^H \mathbf{h}_{t_{m+1}} \end{bmatrix}. \tag{12}$$

Using the formula in [18] for inversion of block matrices, $\boldsymbol{\Psi}_{m+1}^{-1}$ can be computed as

$$\boldsymbol{\Psi}_{m+1}^{-1} = \begin{bmatrix} \boldsymbol{\Psi}_m^{-1} + \frac{1}{\beta_m}\mathbf{b}_m\mathbf{b}_m^H & \frac{1}{\beta_m}\mathbf{b}_m \\ \frac{1}{\beta_m}\mathbf{b}_m^H & \frac{1}{\beta_m} \end{bmatrix}, \tag{13}$$

where

$$\mathbf{b}_m = -\boldsymbol{\Psi}_m^{-1}\bar{\mathbf{H}}_m^H \mathbf{h}_{t_{m+1}}, \tag{14}$$

$$\beta_m = \mathbf{h}_{t_{m+1}}^H \mathbf{P}_m \mathbf{h}_{t_{m+1}}. \tag{15}$$

Substituting (11) and (13) into (10), and noting that $\mathbf{P}_m = \mathbf{I}_N - \bar{\mathbf{H}}_m \boldsymbol{\Psi}_m^{-1} \bar{\mathbf{H}}_m^H$, we can obtain the required result

$$\mathbf{P}_{m+1} = \mathbf{P}_m - \frac{\mathbf{P}_m \mathbf{h}_{t_{m+1}} \mathbf{h}_{t_{m+1}}^H \mathbf{P}_m}{\mathbf{h}_{t_{m+1}}^H \mathbf{P}_m \mathbf{h}_{t_{m+1}}}, \tag{16}$$

for $m = 0,1,\ldots,(M-1)$ with $\mathbf{P}_0 = \mathbf{I}_N$.

Proposition 2: If the columns of $\bar{\mathbf{H}}$, i.e., $\{\mathbf{h}_{t_i}, i = 1,2,\ldots,M\}$ are mutually linearly independent, then the recursive matrix $\mathbf{P}_m, m = 1,2,\ldots,M$, defined in (8) has the following properties:

(a) $\mathbf{P}_m^H = \mathbf{P}_m, (\mathbf{P}_m)^2 = \mathbf{P}_m$;
(b) $\mathbf{P}_m\mathbf{P}_k = \mathbf{P}_k\mathbf{P}_m = \mathbf{P}_m$, $1 \le k \le m$;
(c) $\mathbf{P}_m\mathbf{h}_{t_k} = \mathbf{0}_{N\times 1}$, $1 \le k \le m$.

Proof:
(a) follows directly from the definition of $\mathbf{P}_m$.
(b) If $m = k$, the result can be directly obtained by using property (a). If $m = k+1$, it can be seen that

$$\mathbf{P}_m = \mathbf{P}_{k+1} = \mathbf{P}_k - \frac{\mathbf{P}_k \mathbf{h}_{t_{k+1}} \mathbf{h}_{t_{k+1}}^H \mathbf{P}_k}{\mathbf{h}_{t_{k+1}}^H \mathbf{P}_k \mathbf{h}_{t_{k+1}}}, \tag{17}$$

Then both sides of (17) are multiplied by $\mathbf{P}_k$, we get

$$\mathbf{P}_m\mathbf{P}_k = \mathbf{P}_{k+1}\mathbf{P}_k = \mathbf{P}_k\mathbf{P}_k - \frac{\mathbf{P}_k\mathbf{h}_{t_{k+1}}\mathbf{h}_{t_{k+1}}^H\mathbf{P}_k}{\mathbf{h}_{t_{k+1}}^H\mathbf{P}_k\mathbf{h}_{t_{k+1}}}\mathbf{P}_k, \tag{18}$$

which, from (9) and (a), shows

$$\mathbf{P}_{k+1}\mathbf{P}_k = \mathbf{P}_{k+1}. \tag{19}$$

It can be similarly proved that $\mathbf{P}_k\mathbf{P}_{k+1} = \mathbf{P}_{k+1}$. If $m \geq k+2$, from (19), we have

$$\mathbf{P}_m\mathbf{P}_k = \mathbf{P}_m\mathbf{P}_{m-1}\ldots\mathbf{P}_{k+1}\mathbf{P}_k = \mathbf{P}_m. \tag{20}$$

(c) Setting $m = k$, and substituting (9), it follows that

$$\mathbf{P}_m\mathbf{h}_{t_m} = \mathbf{P}_{m-1}\mathbf{h}_{t_m} - \frac{\mathbf{P}_{m-1}\mathbf{h}_{t_m}\mathbf{h}_{t_m}^H\mathbf{P}_{m-1}\mathbf{h}_{t_m}}{\mathbf{h}_{t_m}^H\mathbf{P}_{m-1}\mathbf{h}_{t_m}} = \mathbf{P}_{m-1}\mathbf{h}_{t_m} - \mathbf{P}_{m-1}\mathbf{h}_{t_m} = \mathbf{0}_{N\times 1}. \tag{21}$$

For $m \geq k+1$, using (19) and (21), it follows that

$$\mathbf{P}_m\mathbf{h}_{t_k} = \mathbf{P}_m\mathbf{P}_{m-1}\ldots\mathbf{P}_{k+1}\mathbf{P}_k\mathbf{h}_{t_k} = \mathbf{0}_{N\times 1}. \tag{22}$$

With the two propositions established above, the proposed algorithm can be derived now.

In the m-th iteration, $s_{t_{M-m+1}}(k)$ is the symbol to be detected. First multiplying both sides of (5) by $\bar{\mathbf{H}}_{M-m+1}$, we get

$$\bar{\mathbf{H}}_{M-m+1}\bar{\mathbf{y}}_{M-m+1}(k) = \bar{\mathbf{H}}_{M-m+1}\mathbf{\Psi}_{M-m+1}^{-1}\bar{\mathbf{H}}_{M-m+1}^H\mathbf{r}_{M-m+1}(k). \tag{23}$$

Using (8), (23) is written as

$$\bar{\mathbf{H}}_{M-m+1}\bar{\mathbf{y}}_{M-m+1}(k) = (\mathbf{I}_N - \mathbf{P}_{M-m+1})\mathbf{r}_{M-m+1}(k) = \mathbf{r}_{M-m+1}(k) - \mathbf{P}_{M-m+1}\mathbf{r}_{M-m+1}(k), \tag{24}$$

where $\mathbf{P}_{M-m+1}$ can be computed recursively by using (9). Then expanding $\bar{\mathbf{H}}_{M-m+1}\bar{\mathbf{y}}_{M-m+1}(k)$ in (24):

$$y_{t_1}(k)\mathbf{h}_{t_1} + y_{t_2}(k)\mathbf{h}_{t_2} + \cdots + y_{t_{M-m+1}}(k)\mathbf{h}_{t_{M-m+1}} = \mathbf{r}_{M-m+1}(k) - \mathbf{P}_{M-m+1}\mathbf{r}_{M-m+1}(k). \tag{25}$$

Define $\mathbf{g}_j = \mathbf{P}_{j-1}\mathbf{h}_{t_j}$. Multiplying both sides of (25) by $\mathbf{g}_j^H$ and using proposition 2, we have

$$\sum_{i=j}^{M-m+1} y_{t_i}(k)\mathbf{g}_j^H\mathbf{h}_{t_i} = \mathbf{g}_j^H\mathbf{r}_{M-m+1}(k),\ j = 1,\ldots,M-m+1. \tag{26}$$

Setting $j = M-m+1$, then from (26), $y_{t_{M-m+1}}(k)$ can be expressed as

$$y_{t_{M-m+1}}(k) = \frac{\mathbf{g}_{M-m+1}^H \mathbf{r}_{M-m+1}(k)}{\mathbf{g}_{M-m+1}^H \mathbf{h}_{t_{M-m+1}}}, m = 1, 2, \ldots, M, \tag{27}$$

where the received symbol vector $\mathbf{r}_{M-m+1}(k)$ has been updated by (7). Recall from (5)(6) that $y_{t_{M-m+1}}(k)$ is the estimate of the symbol $s_{t_{M-m+1}}(k)$, and its quantized estimate $\hat{s}_{t_{M-m+1}}(k)$ is obtained using (6).

Based on the derivation above, the proposed algorithm is summarized in **Table 1**.

Table 1. The improved ZF-OSIC algorithm

Initialization phase:	a) Compute $\|\mathbf{h}_m\|$, for $m = 1, 2, \ldots, M$. b) Sort $\|\mathbf{h}_m\|$, $m = 1, 2, \ldots, M$, in an ascending order, generating index vector $\mathbf{t}(k) = [t_1, t_2, \ldots, t_M]$. c) Compute $\mathbf{P}_{M-m+1}$, for $m = M, M-1, \ldots, 2$ according to (9) with initial condition $\mathbf{P}_0 = \mathbf{I}_N$.
Recursion phase:	for $m = 1, 2, \ldots, M$ a) Compute $y_{t_{M-m+1}}(k)$, the estimate of the transmitted symbol, according to (27). b) Quantize $y_{t_{M-m+1}}(k)$ to get $\hat{s}_{t_{M-m+1}}(k) = Q[y_{t_{M-m+1}}(k)]$. Judgement: If $m = M$, output decoded symbol vector $[\hat{s}_{t_M}(k), \hat{s}_{t_{M-1}}(k), \ldots, \hat{s}_{t_1}(k)]^T$; otherwise continue. c) Update the received symbol vector by $\mathbf{r}_{M-m}(k) = \mathbf{r}_{M-m+1}(k) - \hat{s}_{t_{M-m+1}}(k)\mathbf{h}_{t_{M-m+1}}$.
Solution:	The estimate of the transmitted signal $s_{t_{M-m+1}}$ is $\hat{s}_{t_{M-m+1}}$, $m = 1, 2, \ldots, M$.

4. Fast Implementation and Evaluation of Implementation Complexity

In this section, we evaluate the implementation complexity of the proposed algorithm and show that the algorithm can be efficiently implemented. The complexity of the algorithm is analyzed in detail and compared with the conventional ZF-OSIC algorithm with norm-based ordering and other fast recursive algorithms [5][11][12][13][14][15]. Since both the signal and the channel matrix are complex, all operations (additions, multiplications and divisions) refer to complex ones.

4.1 Fast Implementation of the Proposed Algorithm

A fast implementation of the proposed algorithm and its computational complexity are shown as follows.

1) In the *Initialization* phase:

- Computing $\|\mathbf{h}_m\|$, $m = 1, 2, \ldots, M$. MN multiplications and $M(N-1)$ additions are needed.
- $\mathbf{P}_{M-m+1}$ can be computed as follows. Recall that $\mathbf{g}_j = \mathbf{P}_{j-1}\mathbf{h}_{t_j}$, then from (9), $\mathbf{P}_{M-m+1}$ can be expressed as

$$\mathbf{P}_{M-m+1} = \mathbf{P}_{M-m} - \frac{\mathbf{g}_{M-m+1}\mathbf{g}_{M-m+1}^H}{\mathbf{h}_{t_{M-m+1}}^H \mathbf{g}_{M-m+1}}. \tag{28}$$

It takes N^2 and $2N$ multiplications to compute $\mathbf{g}_{M-m+1}$ and $\mathbf{g}_{M-m+1}^H/(\mathbf{h}_{t_{M-m+1}}^H \mathbf{g}_{M-m+1})$, respectively, and the number of additions needed are $N(N-1)$ and $N-1$, correspondingly. Note that the first and second terms on the right hand side of (28) are Hermitian matrices, which can be utilized to simplify the calculation. Only $\frac{1}{2}N(N+1)$ entries of $\mathbf{P}_{M-m+1}$ are required to be computed in each iteration, which needs $\frac{1}{2}N(N+1)$ multiplications and additions. Since the number of iterations is $M-1$, computing $\mathbf{P}_{M-m+1}$ requires totally

$$\sum_{m=M}^{2}[N^2+2N+\tfrac{1}{2}N(N+1)]=(\tfrac{3}{2}N^2+\tfrac{5}{2}N)(M-1) \tag{29}$$

multiplications and

$$\sum_{m=M}^{2}[N(N-1)+(N-1)+\tfrac{1}{2}N(N+1)]=(\tfrac{3}{2}N^2+\tfrac{1}{2}N-1)(M-1) \tag{30}$$

additions.

- Thus, the initialization takes totally $\frac{3}{2}N^2(M-1)+\frac{7}{2}NM-\frac{5}{2}N$ multiplications and $\frac{3}{2}(N^2M-N^2+MN)-\frac{1}{2}N-2M+1$ additions.

2) In the *Recursive* phase:

- For each iteration m, $m=1,2,\ldots,M$, we need to compute $y_{t_{M-m+1}}(k)$ by (27). Noting that $\mathbf{g}_{M-m+1}$ and $\mathbf{g}_{M-m+1}^H \mathbf{h}_{t_{M-m+1}}$ in (27) has been obtained when computing (28), only additional N multiplications and $N-1$ additions are needed for $y_{t_{M-m+1}}(k)$ in each iteration.
- The update of $\mathbf{r}_{M-m}(k)$ needs N multiplications and additions in each iteration.
- Hence, the total number of multiplications and that of additions for the recursion are $(2M-1)N$ and $2MN-M-N$, respectively.

Collecting the above results, the proposed algorithm requires the total number of multiplications

$$\frac{3}{2}MN^2+4MN-\frac{3}{2}(N-M)N-\frac{7}{2}N=\frac{3}{2}MN^2+O(MN+N^2) \tag{31}$$

and the total number of additions

$$\frac{3}{2}MN^2+2MN-\frac{3}{2}(N-M)N-3M-\frac{3}{2}N+1=\frac{3}{2}MN^2+O(MN+N^2)\,. \tag{32}$$

If $M=N$, then the proposed algorithm requires $\frac{3}{2}M^3+O(M^2)$ multiplications and additions, respectively. The fast implementation and the number of operations for each step are summarized in **Table 2**.

4.2 Comparison of Implementation Complexity

In this subsection, the complexity of the proposed algorithm is evaluated by comparison with some existing algorithms. For fairness, the MMSE-OSIC algorithms are simplified to ZF-OSIC ones to make sure that all of the algorithms are on the same level. Since the difference between ZF-OSIC and MMSE-OSIC is only related to the computation of $\mathbf{Q}$ in the initialization phase [5], the simplification can be easily done for a fast recursive algorithm.

We first compare the computational complexity of the proposed algorithm with conventional one that has the same ordering method. The fast algorithm of the conventional algorithm can be obtained using Sherman-Morrison formula [5], and its complexity in each stage is shown in **Table 3**. The complexity of the proposed algorithm in each stage is also provided for comparison. It can be observed that the number of multiplications and additions of the conventional algorithm are $\frac{5}{2}M^3+O(M^2)$ and $2M^3+O(M^2)$, respectively, when $M=N$. The speedups of the proposed algorithm over the conventional one in the number of multiplications and additions are $\frac{5}{2}/\frac{3}{2}\approx 1.67$ and $2/\frac{3}{2}\approx 1.33$, respectively. Since one complex multiplication and addition require six and two floating-point operations (in flops), respectively [5], the speedup in terms of flops is $[(\frac{5}{2}\times 6)+(2\times 2)]/[(\frac{3}{2}\times 6)+(\frac{3}{2}\times 2)]\approx 1.58$.

Table 2. Fast implementation of the proposed algorithm and the complexity of each step

Phase	Fast implementation of the proposed algorithm	Complexity
Initialization:	$\lVert\mathbf{h}_m\rVert$, $m=1,2,\ldots,M$	MN /* $M(N-1)$
	$\mathbf{t}(k)=[t_1,t_2,\ldots,t_M]\leftarrow\lVert\mathbf{h}_m\rVert$	-/-
	$\mathbf{P}_0=\mathbf{I}_N$. For $m=M,M-1,\ldots,2$,	
	a) $\mathbf{g}_{M-m+1}=\mathbf{P}_{M-m}\mathbf{h}_{t_{M-m+1}}$	$N^2/N(N-1)$
	b) $\eta_{M-m+1}=\mathbf{h}^H_{t_{M-m+1}}\mathbf{g}_{M-m+1}$	$N/N-1$
	c) $\mathbf{g}'_{M-m+1}=\mathbf{g}^H_{M-m+1}/\eta_{M-m+1}$	N /-
	d) $\mathbf{P}_{M-m+1}=\mathbf{P}_{M-m}-\mathbf{g}_{M-m+1}\mathbf{g}'_{M-m+1}$	$\frac{1}{2}N(N+1)/\frac{1}{2}N(N+1)$
Recursion:	for $m=1,2,\ldots,M$	
	a) $y_{t_{M-m+1}}(k)=\mathbf{g}'_{M-m+1}\mathbf{r}_{M-m+1}(k)$	$N/N-1$
	b) $\hat{s}_{t_{M-m+1}}(k)=Q[y_{t_{M-m+1}}(k)]$	-/-
	c) If $m=M$, output $[\hat{s}_{t_M}(k),\hat{s}_{t_{M-1}}(k),\ldots,\hat{s}_{t_1}(k)]^T$; otherwise continue	
	d) $\mathbf{r}_{M-m}(k)=\mathbf{r}_{M-m+1}(k)-\hat{s}_{t_{M-m+1}}(k)\mathbf{h}_{t_{M-m+1}}$	N/N
Solution:	The estimate of the transmitted signal $s_{t_{M-m+1}}$ is $\hat{s}_{t_{M-m+1}}$. The total complexity is $\frac{3}{2}MN^2+O(MN)/\frac{3}{2}MN^2+O(MN)$.	

*Notation X/Y means that the number of multiplications and additions are X and Y, respectively.

Table 3. Complexity comparison of the conventional algorithm with norm-based ordering and the proposed algorithm

	conventional algorithm	proposed algorithm
Ordering:	$MN/M(N-1)$	$MN/M(N-1)$
Nulling:	$2MN(M+1)/\frac{1}{2}MN(3M+1)$	$(\frac{3}{2}N^2+\frac{5}{2}N)(M-1)/(\frac{3}{2}N^2+\frac{1}{2}N-1)(M-1)$
Detection:	$\frac{1}{2}M(N+1)(M+1)/$ $\frac{1}{2}NM^2+\frac{1}{2}NM-M$	$MN/M(N-1)$
Quantilizing and cancellation:	$(M-1)N/(M-1)N$	$(M-1)N/(M-1)N$
Total	$\frac{5}{2}M^2N+\frac{9}{2}MN+\frac{1}{2}M^2+\frac{1}{2}M-N/$ $2M^2N+3MN-2M-N$	$\frac{3}{2}MN^2+4MN-\frac{3}{2}(N-M)N-\frac{7}{2}N/$ $\frac{3}{2}MN^2+2MN-\frac{3}{2}(N-M)N-3M-\frac{3}{2}N+1$

The computational complexity of the proposed algorithm is compared with that of the well known FRA [5] and FKA [15] built from the existing improvements on the former. The comparison is shown in **Table 4**. It can be obtained from **Table 4** that the complexity of FRA is $\frac{11}{3}M^3+O(M^2)$ multiplications and $3M^3+O(M^2)$ additions when $M=N$. Therefore, the speedups of the proposed algorithm over FRA in the number of multiplications and additions are $\frac{11}{3}/\frac{3}{2}\approx 2.44$ and $3/\frac{3}{2}=2$, respectively, and the speedup in terms of flops is 2.33. For FKA in [15], the number of multiplications and additions to obtain $\mathbf{Q}$ in the initialization phase is claimed to be $\frac{1}{2}M^3$, and actually it should be $\frac{5}{6}M^3$, since $\mathbf{T}_{(m-1)}\mathbf{v}_{(m-1)}$ is also required to be computed. Then the FKA has a complexity of $\frac{11}{6}M^3$ multiplications and additions, respectively. With similar calculations, we obtain that the speedup of the proposed algorithm over FKA in terms of flops is 1.22. It can be seen from **Table 4** that the complexity difference of the proposed algorithm and the other two lies in the recursive phase. For example, FRA needs $O(M^3)$ multiplications for recursions, but the proposed algorithm needs only $2MN$. The reason is that the proposed algorithm makes better use of the results generated in the initialization phase to save calculations.

Table 4. Complexity comparison of the proposed algorithm, FRA [5] and FKA [15]

	FRA [5]	**FKA** [15]	**proposed algorithm**
Init.	$\frac{5}{2}MN(M+1)-M^2$ /* $2NM^2+NM-\frac{3}{2}M^2$	$\frac{1}{2}M^2N+\frac{5}{6}M^3+\frac{1}{2}M(N-M)$ / $\frac{1}{2}M^2N+\frac{5}{6}M^3+\frac{1}{2}M(N-3M)$	$\frac{3}{2}N^2(M-1)+\frac{7}{2}NM$ / $\frac{3}{2}(N^2M-N^2+MN)$
Rec.	$\frac{2}{3}M^3+\frac{1}{2}M^2(N+1)+\frac{3}{2}NM$ / $\frac{1}{2}M^3+\frac{1}{2}M^2(N-1)+\frac{3}{2}NM$	$\frac{1}{2}M^3+\frac{3}{2}M^2+MN$ / $\frac{1}{2}M^3+MN$	$2MN$ / $2MN$
Total	$\frac{2}{3}M^3+3NM^2-\frac{1}{2}M^2+4NM$ / $\frac{1}{2}M^3+\frac{5}{2}NM^2-2M^2+\frac{5}{2}NM$	$\frac{1}{2}M^2N+\frac{4}{3}M^3+\frac{3}{2}MN+M^2$/ $\frac{1}{2}M^2N+\frac{4}{3}M^3+\frac{3}{2}MN-\frac{3}{2}M^2$	$\frac{3}{2}MN^2+4MN-\frac{3}{2}(N-M)N$ / $\frac{3}{2}MN^2+2MN-\frac{3}{2}(N-M)N$

* For clarity, $O(M)$ is ignored for all algorithms.

For clarity, the complexity comparison of the proposed algorithm and other existing fast recursive algorithms [5][11][12][13][14][15] is shown in **Fig. 1** for practical number of transmitting/receiving antennas. It can be seen that the proposed algorithm outperforms the other alternatives in terms of computational complexity.

Beside the computational complexity, it is also important to look at other aspects of the implementation complexity, such as memory transfer requirements and storage requirements.

Memory transfer FRA [5] and its improved algorithms [12][13][14][15] require much memory transfer for matrix reordering in each iteration (permuting the columns and rows of the matrices $\mathbf{R},\mathbf{Q},\mathbf{H}$ and $\mathbf{f}(k)$). The memory transfer requires additional clock cycles, though its complexity is not counted in the previous algorithms. With a computational complexity of $3M^3$, the Algorithm I in [11] avoiding the memory transfer and tracking operations without additional implementation burden. From **Table 2**, we note that no permutations are required in the proposed algorithm so that a much simpler implementation can be achieved than the permutation-based FRA and its improved algorithms. Moreover, a speedup of 2 over the Algorithm I [11] in terms of computational complexity is achieved.

Another advantage of the proposed algorithm is its low storage requirements. In the original FRA and most of the improved FRA algorithms, the memories for storing $\mathbf{R},\mathbf{Q},\mathbf{H},\mathbf{f}(k)$ and

$\mathbf{r}(k)$ are needed, and, as mentioned above, the memories for permutations of $\mathbf{R}, \mathbf{Q}, \mathbf{H}$ and $\mathbf{f}(k)$ are required in each iteration [11][12][13][14]. To save memories, the improved algorithm II in [15] avoids calculating $\mathbf{R}$ so that the memories for storing $\mathbf{R}$ and the corresponding permutation operations can be saved. However, the speedup of the algorithm II is only 1.4 compared with FRA [15]. For the proposed algorithm, it can be seen from **Table 2** that only $\mathbf{P}, \mathbf{H}, \mathbf{t}(k)$ and $\mathbf{r}(k)$ are needed to be stored and no permutation operations are required. Therefore, the memories required for implementation of the proposed algorithm is close to those for the improved algorithm II when $M = N$. Furthermore, the proposed algorithm has a speedup of 1.67 over the improved algorithm II.

To our knowledge, the proposed algorithm is the first algorithm with advantages in computational complexity, memory transfer requirements and storage requirements.

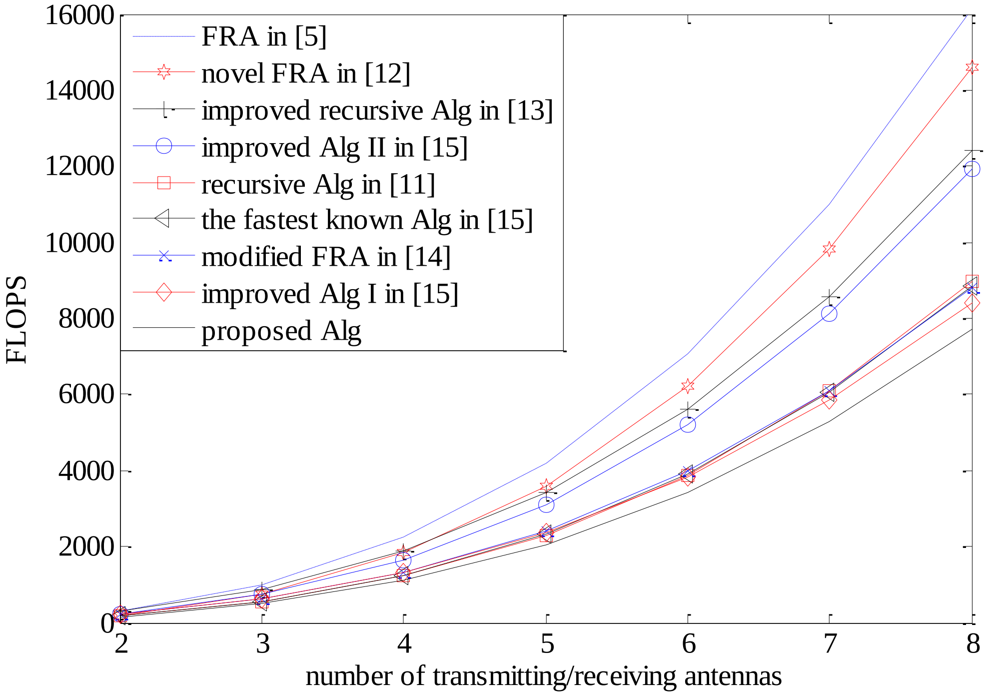

Fig. 1. Comparison of computational complexities of the proposed algorithm and the algorithms in [5] [11][12][13][14][15]

4.3 BER Performance

In **Fig. 2**, the BER performance of the proposed algorithm is compared with the optimal post-SNR based ZF-OSIC algorithm [3] and FRA under practical scenarios. The wireless channel is assumed to be quasi-static. Taking into account the limited size and power of the mobile station, we set the number of antennas: 2 for mobile station and 2 to 4 for base station. BPSK modulation is adopted. Obviously, the performance of the proposed algorithm is close to that of FRA and the optimal algorithm. This result indicates that the performance loss due to norm-based ordering is trivial under practical number of antennas. It also demonstrates the fairness of the abovementioned complexity comparison and the numerical stability of the proposed algorithm.

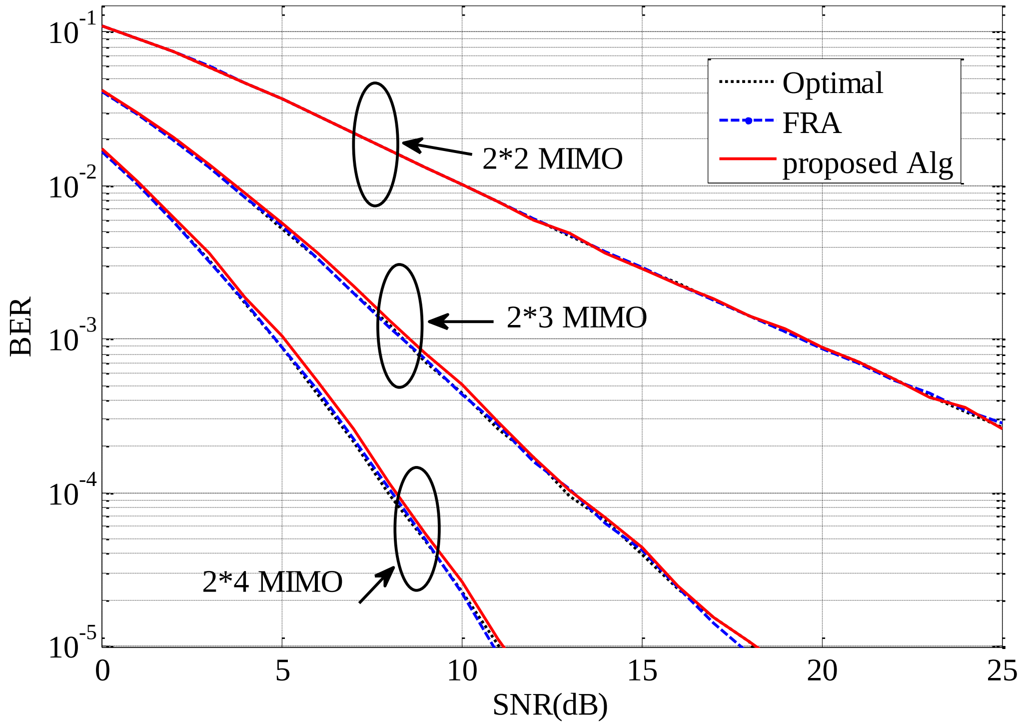

Fig. 2. Comparisons of BER performance among the proposed algoirthm, FRA and the optimal algorithm

5. Conclusion

In this paper, an improved ZF-OSIC recursive algorithm has been proposed for the V-BLAST system. A fast implementation of the proposed algorithm is developed and its computational complexity is given. Three advantages of the proposed algorithm are analyzed. Firstly, compared with the conventional norm-based ZF-OSIC algorithm, the well known FRA and FKA, the proposed algorithm reduces the complexity by a factor of 1.58, 2.33 and 1.22 in flops, respectively. Secondly, a much simpler implementation can be achieved than both FRA and its improved algorithms based on permutations. Finally, memories for implementation of the proposed algorithm can be saved. Compared with the existing algorithm for saving memories, the proposed algorithm has a speedup of 1.67 in flops with similar storage requirements. Near-optimal BER performance under practical scenarios guaranties the fairness of the above comparison. Due to good match to the ZF-OSIC detection, the proposed algorithm appears a competitive candidate for implementation of ZF-OSIC architecture.

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