■ 論 文 ■

Development of Probability Theory based Dynamic Travel Time Models

확률론적 이론에 기초한 동적 통행시간 모형 정립

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목 차

Key Words : 동적 통행시간 모형, FIFO, 확률밀도함수, 지수분포, 교통흐름
Dynamic Travel Time, FIFO, Probability Density Function, Exponential Distribution, Traffic Flow

요 약

이 논문은 확률론적인 방법을 이용하여 동적 통행시간(dynamic travel time) 모형을 도출한다. 동적 통행시간 모형은 차량의 통행시간은 도로 공간상에서의 교통흐름 분포에 따라, 또는 통행구간 출발점에서 시간에 대한 교통흐름의 분포에 따라 결정된다고 가정하여 얻어진다. 이 모형들에서 교통흐름의 분포가 차량의 통행시간에 미치는 정도를 나타내는 확률밀도함수(probability density function)는 여러 가지 형태의 도입될 수 있으나 지수분포를 따른다고 가정한다.

This paper discusses models for estimating dynamic travel times based on probability theory. The dynamic travel time models proposed in the paper are formulated assuming that the travel time of a vehicle depends on the distribution of the traffic stream condition with respect to the location along a road when the subject vehicle enters the starting point of a travel distance or with respect to the time at the starting point of a travel distance. The models also assume that the dynamic traffic flow can be represented as an exponential distribution function among other types of probability density functions.

Ⅰ. Introduction

Dynamic travel times are essential for the study of traffic because they are a critical issue in transportation engineering. In static traffic, travel times are not an issue because they are constant over time in traversing a given distance, and they are linearly proportional to travel distance. Under dynamic traffic conditions, travel times in traversing a given distance vary over time and are not linearly proportional to travel distance because the state of the traffic stream condition, which is assumed to determine the vehicles' travel times, varies over both location and time.

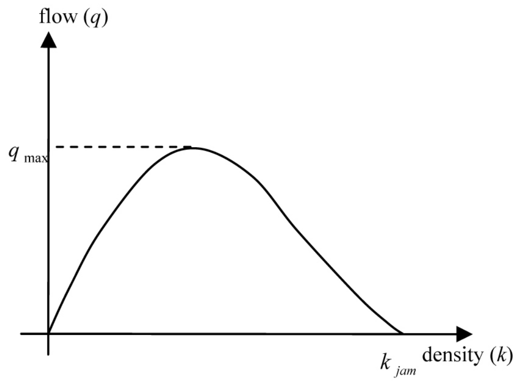
This paper derives dynamic travel time models through a probability approach. In this paper, dynamic travel time models are obtained by considering the distribution of the traffic stream condition with respect to either location along a road when the subject vehicle enters the starting point of a travel distance or with respect to time at the starting point of a travel distance.

1. A Simplified Kinematic Wave Theory

The most widely used macroscopic traffic flow model, which is known as the LWR model, was proposed by Lighthill and Whitman (1955) and Richards (1956). The key point of the LWR modelis that there is a functional relation between flow q and density k. The LWR model can be described by two conditions: the conservation equation and function relating flow q and density k as

$$\frac{\partial k}{\partial t}+\frac{\partial q}{\partial x}=0, \quad \text{and} \quad q=f(k(x,t),x) \qquad (1)$$

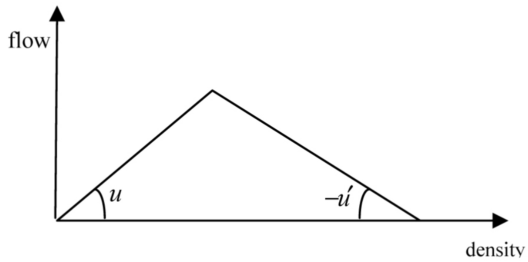
where x and t represent location and time, respectively and f is a function relating flow q and density k, which is referred to as 'a flow-density curve' (see ⟨Figure 1⟩).



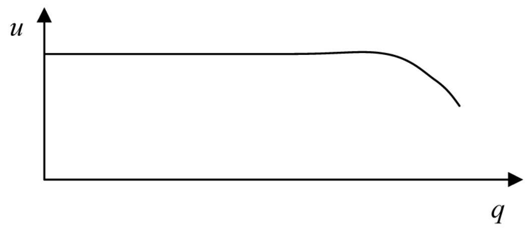
⟨Figure 1⟩ A flow-density curve

Newell (1993abc) simplified the LWR kinematic wave theory after identifying its serious deficiency, where the relation between flow q and density k is not valid under dynamic traffic conditions. At low density, a "desired speed" nearly independent of the average spacing is observed, and at high density, the relation between speed and density is unstable, resulting in "stop-and-go traffic." In order to avoid mathematical complications, he simplified the density-flow relation, which is typical of freeway traffic, to a triangular-shaped curve with only two wave speeds: one for free-flowing traffic (positive) and the other for congested traffic (negative). This method easily evaluates physical queue propagations. As shown in ⟨Figure 2⟩, the density-speed relation is drawn using only two wave speeds, a constant free-flow speed u for low density and a constant backward shockwave speed $-u'$ for high density.

The Highway Capacity Manual (2000) sponsored by the National Cooperative Highway Research Program (NCHRP) adapted a flow-speed relation at low density, as shown in ⟨Figure 3⟩, which is typical of multi-lane highway and basic freeway traffic, based on a



〈Figure 2〉 A simplified kinematic wave theory



〈Figure 3〉 Flow-speed relation on HCM

major data collection and analysis effort. Thepattern of the density-speed relation at low density is the same as the simplified kinematic wave theory proposed by Netwell (1993abc). In fact, speed u at low density is constant regardless of density u .

2. Whole-Link Travel Time Models

Friesz et al. (1993) introduced a whole-link travel time model, in which the link travel time of a vehicle entering a link at time t is taken as a linear function of the number of vehicles on a link at time t. This model has been widely used in mathematical programming models for dynamic traffic assignments, which simulate the interaction between a driver's route choice pattern and network performance. Friesz et al. (1993) employed the following linear link travel time model for the use of a dynamic traffic assignment in a network model:

$$\tau(t) = a + b \cdot n(t) \tag{2}$$

where $\tau(t)$ is the link travel time of a vehicle entering a link at time t, a and b are constants, and $n(t)$ is the number of vehicles on a link at time t. Equation (2) describes a fixed travel time 'a' plus a congestion related travel time delay '$b \cdot n(t)$.'

The non-linear version of whole-link travel time models has been proposed by Astarita (1995,1996), Wu et al. (1998), Xu et al. (1999), Zhu and Marcotte (2000), and Carey and McCartney (2002). Non-linear whole-link travel time models have a general form where link travel times are a function of the number of vehicles on a link, as given below:

$$\tau(t) = f(n(t)) \tag{3}$$

Furthermore, whole-link travel time models areexpressed as a function of whole-link variables, such as inflows, outflows, or the number of vehicles on a link (Ran et al., 1993). Carey et al. (2003) introducedanother whole-link travel time model where the travel time of a vehicle entering a link is taken as a function of a weighted average of the inflow rate when a user enters a link and the outflow rate when the same vehicle exits from a link. Various whole-link travel time models have been proposed. However, whole-link travel time modelscause unreliable results for estimating link travel times such as violation of 'first-in-first-out' (FIFO) discipline because as Daganzo (1995) argues, they have not taken into account the distribution of vehicles with respect to location along a link.

Ⅱ. Instantaneous Travel Times

The space-based instantaneous travel time is defined as the time a vehicletakes to traverse a distance, assuming that the traffic stream condition along a road does not change during its travel. The space-based instantaneous travel time has the following form:

$$\tau^{\sin}(x,t,l) = \int_{x}^{x+l} \frac{1}{u_e(x_v,t)} dx_v \qquad (4)$$

where $\tau^{\sin}(x,t,l)$ is the instantaneous travel time of a vehicle entering the starting point of a travel distance at location x and time t in traversing a distance l, $u(x_v,t)$ is the speed, x_v is the location variable, and t is the entry time when the subject vehicle enters the starting point x of a travel distance.

The space-based instantaneous travel time can be explained as follows. As an expected value is calculated by integrating the product of a variable and the probability density function (pdf), the space-based instantaneous travel time (expected value) is obtained by integrating the product of the traffic stream condition which is represented by a form of travel time $l/u(x_v,t)$ along a road and the pdf $f^{\sin}(x_v)$, which is defined as the degree to which the differential elements of the traffic stream condition $l/u(x_v,t)$ affect the travel time of a vehicle traversing a distance l. In the space-based instantaneous travel time model, the pdf $f^{\sin}(x_v)$ is uniformly distributed from the starting point of a travel distance x to its end $x+l$ as shown in equation (5):

$$f^{\sin}(x_v) = \begin{cases} 1/l & x \le x_v \le x+l \\ 0 & Otherwise \end{cases} \qquad (5)$$

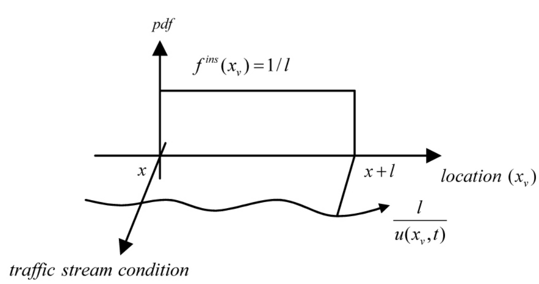
Integrating the product of the traffic stream condition $l/u(x_v,t)$ and the pdf $f^{\sin}(x_v)$ yields equation (4). The space-based instantaneous travel time model is graphically described in ⟨Figure 4⟩.

Ⅲ. Dynamic Travel Time

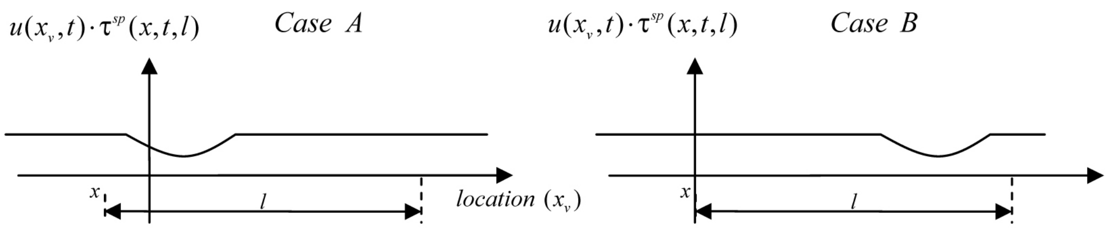
1. A Space-Based Travel Time Model

Let $\tau^{sp}(x,t,l)$ be the space-based travel time of a vehicle traveling at entry location x and entry time t in traversing a travel distance l. Note that the travel distance l can also be defined as "expected travel distance" that a vehicle can traverse during a given time $\tau^{sp}(x,t,l)$. The expected travel distance l is estimated as follows.

In order to estimate the expected travel distance l, the traffic stream condition along a road must be a form of travel distance as $u(x_v,t) \cdot \tau^{sp}(x,t,l)$. In other words, the form of the traffic stream condition should be equivalent to that of the estimated value. Regarding the pdf, a natural assumption is that it differs according to location along a road. This is illustrated in ⟨Figure 5⟩, which shows the traffic stream condition with respect to location x_v at entry time t. The travel time of a vehicle at entry location x and entry time t in Case A is likely to be longer than in Case B. The reasonis as follows. The downward fluctuation of the traffic stream condition in Case A is located closer to entry location x than in Case B. This downward fluctuation in Case A will impact the travel time of the subject vehicle more than for Case B



⟨Figure 4⟩ Graphical representation of space-based instantaneous travel time model



〈Figure 5〉 The impact of the traffic stream condition distribution versus location

during its travel of a distance l. Thus, the travel time in Case A must be longer than in Case B. This indicates that the pdf should differ with respect to location along a road.

Even though various types of pdf can be applied, this paper employs the non-linear pdf (exponential distribution) to consider dynamic traffic flow. Note that the parameter of the pdf with the exponential distribution should be automatically the inverse of expected travel distance as $1/l$. Then, the pdf is given as follows:

$$f^{sp}(x_v) = \begin{cases} \frac{1}{l} e^{-\frac{1}{l}(x_v - x)} & x_v \ge x \\ 0 & x_v < x \end{cases} \tag{6}$$

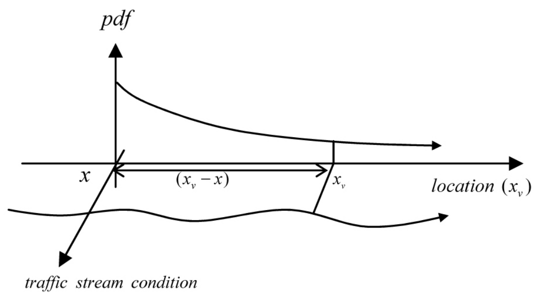
Integrating the product of the traffic stream condition $u_e(x_v,t)\cdot\tau^{sp}(x,t,l)$ and the pdf yields the "expected travel distance l" as

$$l = \int_x^{\infty} u(x_v,t)\cdot\tau^{sp}(x,t,l)\cdot\frac{1}{l}e^{-\frac{1}{l}(x_v-x)}dx_v \tag{7}$$

Rearranging equation (7) yields

$$\tau^{sp}(x,t,l) = \int_x^{\infty}\frac{l}{u(x_v,t)}\cdot\frac{1}{l}e^{-\frac{1}{l}(x_v-x)}dx_v \tag{8}$$

In equation (7), the distribution of the traffic stream condition along a road was defined as $u(x_v,t)\cdot\tau^{sp}(x,t,l)$. Note that in equation (8), the traffic stream condition has been shifted to



〈Figure 6〉 Graphical representation of the space-based travel time model

$l/u(x_v,t)$, which is a form of travel time. The space-based travel time model is graphically described in 〈Figure 6〉.

2. A Time-Based Travel Time Model

Let $\tau^{ti}(x,t,l)$ be the time-based travel time of a vehicle at location x and time t in traversing a distance l. In the time-based travel time model, the traffic stream condition with respect to time t_v at the starting point of a travel distance x should be defined as a form of travel time as

$$\frac{l}{u(x,t_v)} \tag{9}$$

In the time-based travel time model, the pdf is assumed to be exponentially distributed with respect to the time difference $(t - t_v)$ between the entry time t when the subject vehicle enters the starting point of a travel distance and the time t_v of the traffic stream condition at the starting point of a travel distance. If the pdf is exponentially distributed, the parameter of the pdf should automatically be the inverse of the expected travel time $1/\tau^{ti}(x,t,l)$. As such, the pdf is given as follows:

$$f^{ti}(t_v) = \begin{cases} \frac{1}{\tau^{ti}(x,t,l)} \cdot e^{-\frac{1}{\tau^{ti}(x,t,l)}(t-t_v)} & t_v \le t \\ 0 & t_v > t \end{cases} \quad (10)$$

Integrating the product of the traffic stream condition and the pdf yields

$$\tau^{ti}(x,t,l) = \int_{-\infty}^{t} \frac{l}{u(x,t_v)} \cdot \frac{1}{\tau^{ti}(x,t,l)} e^{-\frac{1}{\tau^{ti}(x,t,l)}(t-t_v)} dt_v \quad (11)$$

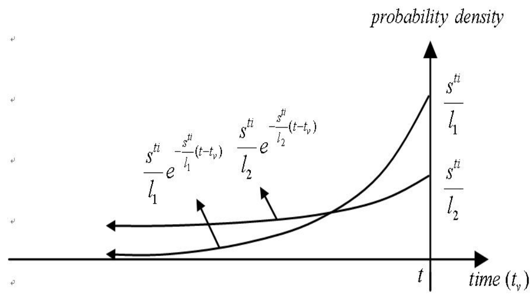
One of the methods used to solve equation (11) is the "trial-and-error method," which yields a satisfactory result by trying out various means or theories until the error is sufficiently reduced or eliminated.

Equation (11) also can be expressed as follows:

$$\tau^{ti}(x,t,l) = \int_{-\infty}^{t} \frac{l}{u(x,t_v)} \cdot \frac{s^{ti}(x,t,l)}{l} e^{-\frac{s^{ti}(x,t,l)}{l}(t-t_v)} dt_v \quad (12)$$

where $s^{ti}(x,t,l)$ is the average speed of a vehicle during the travel of a distance l.

The proposed model has an interesting characteristic: the travel distance l and the average speed $s^{ti}(x,t,l)$ consist of the parameter of the pdf. As shown in ⟨Figure 7⟩, the pdf with a travel distance l_1 that is shorter than the travel distance l_2 is steeper than the pdf with the travel distance l_2. This means that the



⟨Figure 7⟩ Understanding the probability density function

traffic stream conditionthat is just in the front of the subject vehicle is critical for the travel time of the subject vehicle if its travel distanceis short. However, the traffic stream condition that is far away from the subject vehicle can significantly affect the travel time of the subject vehicle if its travel distance is long.

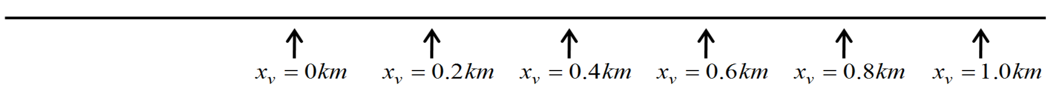
From the time-based travel time model (equation (11)), the travel time of a vehicle traveling a distance between any two locations on a road can also be obtained. Let $\tau^{ti}(x,t,l_1 \sim l_2)$ be the travel time of a vehicle at location x and time t traveling between the ends of two different travel distances l_1 and l_2, which begin from location x. The travel time $\tau^{ti}(x,t,l_1 \sim l_2)$ is simply obtained with the following:

$$\tau^{ti}(x,t,l_1 \sim l_2) = \tau^{ti}(x,t,l_2) - \tau^{ti}(x,t,l_1) \quad (13)$$

Ⅳ. Numerical Examples

1. Scenario 1

In this section, an illustrative numerical



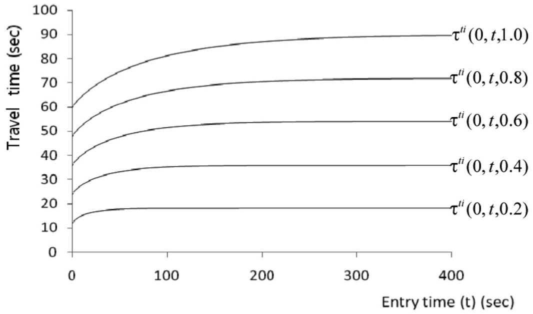
⟨Figure 8⟩ Layout of the case study site example is presented with the time-based travel time model (equation (11)). The objective is to demonstrate the properties of the time-based travel time model and validate its reasonability with a general traffic situation. Let traffic speeds at the starting point of a travel distance $x=0km$ be as follows:

$$u(0,t_v)=\begin{cases}60\ \ km/hr & for\ \ -\infty<t_v<0\ \ \sec\\ 40\ \ km/hr & for\ \ 0\le t_v<400\ \ \sec\end{cases}$$

The layout of the numerical example site is shown in ⟨Figure 8⟩.

From the time-based travel time model (equation (11)), the travel time of a vehicle entering the starting point of a travel distance $l=0.2\ km$ at entry location $x=0\ km$ and entry time t is calculated as

$$\tau^{ti}(0,t,0.2)=\int_{-\infty}^{t}\frac{0.2}{u(0,t_v)}\cdot\frac{1}{\tau^{ti}(0,t,0.2)}e^{-\frac{1}{\tau^{ti}(0,t,0.2)}(t-t_v)}dt_v$$
$$=\int_{-\infty}^{0}\frac{0.2}{60}\cdot\frac{1}{\tau^{ti}(0,t,0.2)}e^{-\frac{1}{\tau^{ti}(0,t,0.2)}(t-t_v)}dt_v+\int_{0}^{t}\frac{0.2}{40}\cdot\frac{1}{\tau^{ti}(0,t,0.2)}e^{-\frac{1}{\tau^{ti}(0,t,0.2)}(t-t_v)}dt_v$$
$$=\frac{0.2}{60}\cdot e^{-\frac{1}{\tau^{ti}(0,t,0.2)}(t-t_v)}\Big|_{-\infty}^{0}+\frac{0.2}{40}\cdot e^{-\frac{1}{\tau^{ti}(0,t,0.2)}(t-t_v)}\Big|_{0}^{t}$$
$$=\frac{0.2}{60}\cdot e^{-\frac{1}{\tau^{ti}(0,t,0.2)}t}+\frac{0.2}{40}-\frac{0.2}{40}\cdot e^{-\frac{1}{\tau^{ti}(0,t,0.2)}t} \qquad (14)$$



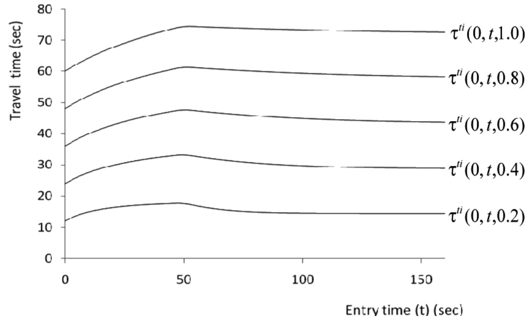
⟨Figure 9⟩ Travel times $\tau^{ti}(0,\ t,\ l)$ where l=0.2, 0.4, 0.6, 0.8 and 1.0

Equation (14) is solved by the "trial-and-error method." ⟨Figure 9⟩ shows the estimated profile of travel times with various travel distances versus entry time t when the subject vehicle enters the starting point of the travel distance. Results from the estimated travel times versus entry time t show an upward trend in mean travel time as the entry time t increases (the positions of vehicles in the queue move back) and the travel times stabilize slowly as travel distance is longer.

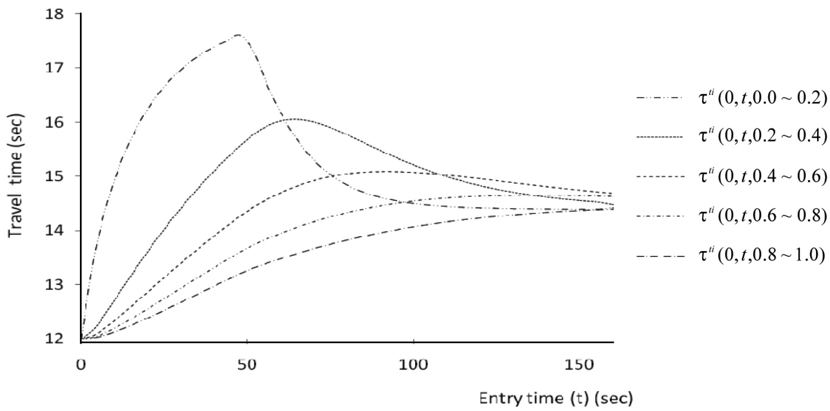
2. Scenario 2

One of the main purposes of the numerical example is to ensure that the model (equation (11)) holds under 'first-in-first-out' (FIFO) discipline. Under the FIFO discipline, vehicles must leave the end of a travel distance in the same order as entry order to the starting point of a travel distance. Accdording to Astarita (1996), the FIFO discipline is violated only if travel times decline rapidly, that is, if $d\tau^{ti}(x,t,l)/dt\le-1$. In order to generate a rapid decline in travel time, a rapid speed increase from 40 km/hr to 50 km/hr at $t_v=50$ sec is applied as

$$u(0,t_v)=\begin{cases}60\ \ km/hr & for\ \ -\infty<t_v<0\ \ \sec\\ 40\ \ km/hr & for\ \ 0\le t_v<50\ \ \sec\\ 50\ \ km/hr & for\ \ 50\le t_v<400\ \ \sec\end{cases}$$



⟨Figure 10⟩ Travel times $\tau^{ti}(0,\ t,\ l)$ where l=0.2, 0.4, 0.6, 0.8 and 1.0



〈Figure 11〉 Travel times τ^{ll} (0, t, l_1~l_2)

From equation (11), the travel times are obtained as shown in 〈Figure 10〉. The sharp increase in speed causes the travel time at entry time $t=50$sec to start to drop, as shown in 〈Figure 10〉, but the FIFO discipline is not violated $(d\tau^{ll}(0,t,l)/dt > -1)$.

From equation (13), the travel times between two locations on a road are obtained as shown in 〈Figure 11〉. An interesting feature is that the travel time $\tau^{ll}(0,t,0.8\sim1.0)$ at entry time $t=50$sec does not start to drop, as shown in 〈Figure 11〉. In addition, the resulting profile of the travel time $\tau^{ll}(0,t,0.8\sim1.0)$ is smoother than the travel time $\tau^{ll}(0,t,0.2\sim0.0)$. This result illustrates that vehicles disperses to areas with a higher traffic speed and lower density.

Ⅴ. Conclusions

This paper derived dynamic travel time models using a probability approach. Dynamic travel time models are obtained by assuming that the travel time of a vehicle depends on the distribution of the traffic stream condition with respect to location or time. From the case study, the models proposed in this paper were validatedwith very encouraging results. Themodels properly describe even the behavior of traffic dispersion.

The proposed model (equation (11)) possessesseveral interesting characteristics. First, the travel distance consists ofthe parameter of the pdf so that travel times in traversing any travel distance can be estimated. Furthermore, travel times in traveling between any two different locations on a road can also be obtained (see equation (13)). This allows for wide use of the models.

Second, the models do not have an unknown factor for geometric road conditions. In the models, it is assumed that the travel time isdetermined by the distribution of the traffic stream condition, l/u, where u is the speed that is determined by a geometric road condition and the weight of traffic. This means that the unknown factor for geometric road conditionsis already reflected in the traffic stream condition.

Finally, the models hold under the FIFO discipline. In order to consider dynamic traffic flow, this paper employs the non-linear pdf (exponential distribution), which induces the proposed models to hold under this discipline.

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