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Fuzzy r-Minimal 구조에서 Fuzzy Weak r-*M* Continuity의 특성 연구

Remarks on Fuzzy Weak r-*M* Continuity on Fuzzy r-Minimal Structures

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요 약
본 논문에서는 일반화 된 fuzzy r-minimal 열린 집합들을 이용하여 fuzzy 약 r-M 연속함수의 특성을 조사한다.

Abstract
In this paper, we study characterizations of fuzzy weakly r-M continuous function on r-minimal structures in terms of generalized fuzzy r-minimal open sets.

Key Words : fuzzy r-minimal structure, fuzzy weakly r-M continuous, fuzzy r-minimal semiopen, fuzzy r-minimal preopen, r-minimal regular open, r-minimal β-open.

1. 서 론

The concept of fuzzy set was introduced by Zadeh [12]. Chang [2] defined fuzzy topological spaces using fuzzy sets. In [3], Chattopadhyay, Hazra and Samanta introduced a smooth fuzzy topological space which is a generalization of fuzzy topological space. In [11], Yoo et al. introduced the concept of fuzzy r-minimal space which is an extension of the smooth fuzzy topological space. The concepts of fuzzy r-open sets, fuzzy r-semiopen sets, fuzzy r-preopen sets, fuzzy r-β-open sets and fuzzy r-regular open sets were introduced in [1, 4, 5, 6], which are kinds of fuzzy r-minimal structures. The concept of fuzzy r-M continuity was also introduced and investigated in [11]. In [8], the author introduced and studied the concept of fuzzy weak r-M continuity which is a generalization of fuzzy r-M continuity. In this paper, we study characterizations of fuzzy weakly r-M continuous function on r-minimal structures in terms of generalized fuzzy r-minimal open sets [7, 9]: fuzzy r-minimal semiopen sets, fuzzy r-preopen sets, fuzzy r-minimal β-open sets and fuzzy r-minimal regular open sets.

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2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member A of I^X is called a *fuzzy set* of X. By $\tilde{0}$ and $\tilde{1}$, we denote constant maps on X with value 0 and 1, respectively. For any $A \in I^X$, A^c denotes the complement $\tilde{1}-A$. All other notations are standard notations of fuzzy set theory.

A *fuzzy point* x_α in X is a fuzzy set x_α is defined as follows

$$x_\alpha(y) = \begin{cases} \alpha, & \text{if } y = x, \\ 0, & \text{if } y \neq x. \end{cases}$$

A fuzzy point x_α is said to belong to a fuzzy set A in X, denoted by $x_\alpha \in A$, if $\alpha \le A(x)$ for $x \in X$.

A fuzzy set A in X is the union of all fuzzy points which belong to A.

Let $f: X \to Y$ be a mapping and $A \in I^X$ and $B \in I^Y$. Then $f(A)$ is a fuzzy set in Y, defined by

$$f(A)(y) = \begin{cases} A(z)_{z \in f^{-1}(y)}, & \text{if } f^{-1}(y) \neq \varnothing, \\ 0, & \text{otherwise}, \end{cases}$$

for $y \in Y$ and $f^{-1}(B)$ is a fuzzy set in X, defined by $f^{-1}(B)(x) = B(f(x))$, $x \in X$.

A *fuzzy topology* [3, 4] on X is a map $T: I^X \to I$ which satisfies the following properties:

(1) $T(\tilde{0})=T(\tilde{1})=1$.
(2) $T(A_1 \cap A_2) \geq T(A_1) \wedge T(A_2)$ for $A_1,\ A_2 \in I^X$.
(3) $T(\cup A_i) \geq \wedge T(A_i)$ for $A_i \in I^X$.

The pair (X,T) is alled a *fuzzy topological space*.
And $A \in I^X$ is said to be *fuzzy r-open* (resp., *fuzzy r-closed*) if $T(A) \geq r$ (resp., $T(A^c) \geq$ r).

Definition 2.1 ([11]). Let X be a nonempty set and $r \in (0,1]=I_0$. A fuzzy family $M: I^X \to I$ on X is said to have a *fuzzy r-minimal structure* if the family

$$M_r=\{A \in I^X : M(A) \geq r\}$$

contains $\tilde{0}$ and $\tilde{1}$.
Then the (X,M) is called a *fuzzy r-minimal space* (simply *r-FMS*). Every member of M_r is called a *fuzzy r-minimal open set*. A fuzzy set A is called a *fuzzy r-minimal closed set* if the complement of A (simply, A^c) is a fuzzy r-minimal open set.
Let (X,M) be an *r-FMS's* and $r \in (0,1]=I_0$. The fuzzy r-minimal closure and the fuzzy r-minimal interior of A [11], denoted by mC(A,r) and mI$(A,\ r)$, respectively, are defined as

$$\text{mC}(A,\ r) = \cap \{B \in I^X:\ B^c \in M_r \ \text{ and } \ A \subseteq B\};$$
$$\text{mI}(A,r)= \cup \{B \in I^X:\ B \in M_r \ \text{ and } \ B \subseteq A\}.$$

Theorem 2.2 ([11]). Let (X,M) be an *r-FMS* and $A,B \in I^X$. Then the following properties hold:
(1) mI$(A,r) \subseteq A$ and if A is a fuzzy r-minimal open set, then mI$(A,r)=A$.
(2) $A \subseteq$ mC(A,r) and if A is a fuzzy r-minimal closed set, then mC$(A,r)=A$.
(3) If $A \subseteq B$, then mI$(A,r) \subseteq$ mI(B,r) and mC$(A,r) \subseteq$ mC(B,r).
(4) mI$(A \cap B,r) \subseteq$ mI$(A,r) \cap$ mI(B,r) and mC$(A,r) \cup$ mC$(B,r) \subseteq$ mC$(A \cup B,r)$.
(5) mI(mI$(A,r),r)$=mI(A,r) and mC(mC$(A,r),r)$= mC(A,r).
(6) $\tilde{1}$-mC(A,r)=mI$(\tilde{1}-A,r)$ and $\tilde{1}$-mI(A,r)=mC$(\tilde{1}-A,r)$.

Let (X,M_X) and (Y,M_Y) be two *r-FMS's.* Then a function $f: X \to Y$ is said to be
(1) *fuzzy r-M continuous* [11] if for every $A \in M_Y$, $f^{-1}(A)$ is in M_X;
(2) *fuzzy weakly r-M continuous* [8] if for each fuzzy point x_α in X and each fuzzy r-minimal open set V containing $f(x_\alpha)$, there exists a fuzzy r-minimal open set G containing x_α such that $f(G) \subseteq$ mC$(V,\ r)$.

3. Characterizations of Fuzzy Weakly r-M Continuous Functions

Theorem 3.1 ([8]). Let $f: X \to Y$ be a function between *r-FMS's* (X,M_X) and (Y,M_Y). Then the following statements are equivalent:
(1) f is fuzzy weakly r-M continuous.
(2) $f^{-1}(V) \subseteq$ mI$(f^{-1}$(mC$(V,r)),r)$ for each fuzzy r-minimal open set V in Y.
(3) mC$(f^{-1}$(mI$(B,r)),r) \subseteq f^{-1}(B)$ for each fuzzy r-minimal closed set B in Y.
(4) mC$(f^{-1}(V),r) \subseteq f^{-1}$(mC$(V,r))$ for each fuzzy r-minimal open set V in Y.

Let X be a nonempty set and $M: I^X \to I$ a fuzzy family on X. The fuzzy family M is said to have the property (U) [11] if for $A_i \in M$ $(i \in J)$,

$$M(\cup A_i) \geq \wedge M(A_i).$$

Theorem 3.2 ([11]). Let (X,M) be an *r-FMS* with the property (U). Then
(1) mI$(A,r)=A$ if and only if A is fuzzy r-minimal open for $A \in I^X$.
(2) mC$(A,r)=A$ if and only if A is fuzzy r-minimal closed for $A \in I^X$.

Lemma 3.3 ([8]). Let $f: X \to Y$ be a function between *r-FMS's* (X,M_X) and (Y,M_Y). If M_X have property (U), then the following statements are equivalent:
(1) f is fuzzy weakly r-M continuous.
(2) mC$(f^{-1}$(mI$(F,r)),r) \subseteq f^{-1}(F)$ for each fuzzy r-minimal closed set F in Y.
(3) mC$(f^{-1}$(mI(mC$(B,r),r)),r) \subseteq f^{-1}$(mC$(B,r))$ for each $B \in I^Y$.
(4) f^{-1}(mI$(B,r)) \subseteq$ mI$(f^{-1}$(mC(mI$(B$,r$),r)),r)$ for each $B \in I^Y$.
(5) mC$(f^{-1}(V),r) \subseteq f^{-1}$(mC$(V,r))$ for a fuzzy r-minimal open set V in Y.

Definition 3.4. Let (X,M_X) be an *r-FMS's* and $A \in I^X$. Then a fuzzy set A is said to be
(1) *fuzzy r-minimal semiopen* [7] if $A \subseteq$ mC(mI$(A,r),r)$;
(2) *fuzzy r-minimal preopen* [9] if $A \subseteq$ mI(mC$(A,r),r)$;
(3) *fuzzy r-minimal β-open* [9] if $A \subseteq$ mC(mI(mC$(A,r),r),r)$.
(4) *fuzzy r-minimal regular open* [9] (resp., *fuzzy r-minimal regular closed* if A=mI(mC$(A,r),r)$ (resp., A=mC(mI$(A,r),r)$).

A fuzzy set A is called a *fuzzy r-minimal semiclosed* (resp., *fuzzy r-minimal preclosed, fuzzy r-minimal β-closed*) set if the complement of A is a fuzzy r-minimal semiopen (resp., fuzzy r-minimal preopen, fuzzy r-minimal β-open) set.

Theorem 3.5. Let $f: X \to Y$ be a function between r-FMS's (X, M_X) and (Y, M_Y). If M_Y has the property (U), then f is fuzzy weakly r-M continuous if and only if $mC(f^{-1}(mI(mC(B,r),r)),r) \subseteq f^{-1}(mC(B,r))$ for each fuzzy r-minimal semiopen set B in Y.

Proof. Suppose f is fuzzy weakly r-M continuous and let B be a fuzzy r-minimal semiopen set in Y. Then $B \subseteq mC(mI(B,r),r)$ and by the property (U), $mC(mI(B,r),r)$ is fuzzy r-minimal closed. So by Lemma 3.3 (3) and $mC(B,r)=mC(mI(B,r),r)$, we have $mC(f^{-1}(mI(mC(B,r),r)),r)=mC(f^{-1}(mI(mC(mI(B,r),r),r)),r) \subseteq f^{-1}(mC(mI(B,r),r)) \subseteq f^{-1}(mC(B,r))$. Hence $mC(f^{-1}(mI(mC(B,r),r)),r) \subseteq f^{-1}(mC(B,r))$.

For the converse, let V be a fuzzy r-minimal open set in Y. Then V is also r-minimal semiopen and since $V \subseteq mI(mC(V,r),r)$, by hypothesis, $mC(f^{-1}(V),r) \subseteq mC(f^{-1}(mI(mC(V,r),r)),r) \subseteq f^{-1}(mC(V,r))$. Hence by Lemma 3.3 (4), f is fuzzy weakly r-M continuous.

Theorem 3.6. Let $f: X \to Y$ be a function between r-FMS's (X, M_X) and (Y, M_Y). If M_Y has the property (U), then f is fuzzy weakly r-M continuous if and only if $mC(f^{-1}(mI(mC(B,r),r)),r) \subseteq f^{-1}(mC(B,r))$ for each fuzzy r-minimal preopen set B in Y.

Proof. Let f be a fuzzy weakly r-M continuous function and B be a fuzzy r-minimal preopen set in Y. From the property (U), $mC(mI(B,r),r)$ is fuzzy r-minimal closed and so $mC(B,r)=mC(mI(B,r),r)$. From $B \subseteq mC(mI(B,r),r)$, we have

$$\begin{aligned} & mC(f^{-1}(mI(mC(B,r),r)),r) \\ & \quad = mC(f^{-1}(mI(mC(mI(B,r),r),r)),r) \\ & \quad \subseteq f^{-1}(mC(mI(B,r),r)) \\ & \quad \subseteq f^{-1}(mC(B,r)). \end{aligned}$$

Hence we have the result.

For the converse, let V be a fuzzy r-minimal open set in Y. Then by the property (U), V is also r-minimal preopen, and since $V \subseteq mI(mC(V,r),r)$, it follows $mC(f^{-1}(V),r) \subseteq mC(f^{-1}(mI(mC(V,r),r)),r) \subseteq f^{-1}(mC(V,r))$. So $mC(f^{-1}(V),r) \subseteq f^{-1}(mC(V,r))$ Hence by Lemma 3.3, f is fuzzy weakly r-M continuous.

Theorem 3.7. Let $f: X \to Y$ be a function between r-FMS's (X, M_X) and (Y, M_Y). If M_Y has the property (U), then the following statements are equivalent:

(1) f is fuzzy weakly r-M continuous.

(2) $mC(f^{-1}(B),r) \subseteq f^{-1}(mC(B,r))$ for each fuzzy r-minimal preopen set B in Y.

(3) $f^{-1}(B) \subseteq mI(f^{-1}(mC(B,r)),r)$ for each fuzzy r-minimal preopen set B in Y.

Proof. (1) $\Rightarrow$ (2) Let f be a fuzzy weakly r-M continuous function and B be a fuzzy r-minimal preopen set in Y. Suppose $x_\alpha \not\in f^{-1}(mC(B,r))$; then there exists a fuzzy r-minimal open set V containing $f(x_\alpha)$ such that $V \cap B = \varnothing$. Since B is a fuzzy r-minimal preopen set, $B \cap mC(V,r) = \varnothing$. From fuzzy weak r-M continuity and $f(x_\alpha) \in V$, there exists a fuzzy r-minimal open set G containing x_α such that $f(G) \subseteq mC(V,r)$. Then since $B \cap mC(V,r) = \varnothing$, $B \cap f(G) = \varnothing$ and so $f^{-1}(B) \cap G = \varnothing$. This implies $x_\alpha \not\in mC(f^{-1}(B),r)$. Consequently, $mC(f^{-1}(B),r) \subseteq f^{-1}(mC(B,r))$.

(2) $\Rightarrow$ (3) For a fuzzy r-minimal preopen set B in Y, from (2), it follows

$$\begin{aligned} f^{-1}(B) & \subseteq f^{-1}(mI(mC(B,r),r)) \\ & = \tilde{1} - f^{-1}(mC(\tilde{1} - mC(B,r),r)) \\ & \subseteq \tilde{1} - mC(f^{-1}(\tilde{1} - mC(B,r)),r) \\ & = mI(f^{-1}(mC(B,r)),r). \end{aligned}$$

(3) $\Rightarrow$ (1) Let V be a fuzzy r-minimal open set in Y. Then by the property (U), V is also r-minimal preopen. Hence by Theorem 3.2 and Lemma 3.3, f is fuzzy weakly r-M continuous.

Theorem 3.8. Let $f: X \to Y$ be a function between r-FMS's (X, M_X) and (Y, M_Y). If M_Y has the property (U), then f is fuzzy weakly r-M continuous if and only if $mC(f^{-1}(mI(K,r)),r) \subseteq f^{-1}(K)$ for each fuzzy r-minimal regular closed set K in Y.

Proof. Let f be a fuzzy weakly r-M continuous function and let K be a fuzzy r-minimal regular closed set in Y. Since $K=mC(mI(K,r),r)$ is fuzzy r-minimal closed, by Theorem 3.2 and Lemma 3.3, we have $mC(f^{-1}(mI(K,r)),r) \subseteq f^{-1}(K)$.

For the converse, let V be a fuzzy r-minimal open set in Y. Then since $mC(V,r)=mC(mI(mC(V,r),r),r)$, $mC(V,r)$ is r-minimal regular closed. By hypothesis, $mC(f^{-1}(V,r)) \subseteq mC(f^{-1}(mI(mC(V,r),r)),r) \subseteq f^{-1}(mC(V,r))$, that is, $mC(f^{-1}(V,r)) \subseteq f^{-1}(mC(V,r))$. Hence by Lemma 3.3, f is fuzzy weakly r-M continuous.

Theorem 3.9. Let $f: X \to Y$ be a function between r-FMS's (X, M_X) and (Y, M_Y). If M_Y has the property (U), then f is fuzzy weakly r-M continuous if and only if $mC(f^{-1}(mI(mC(B,r),r)),r) \subseteq f^{-1}(mC(B,r))$ for each fuzzy r-minimal β-open set B in Y.

Proof. If f is a fuzzy weakly r-M continuous function, then by Lemma 3.3, easily the result is obtained.

For the converse, let B be a fuzzy r-minimal open set in Y. Then B is also a fuzzy r-minimal β-open set. From $B \subseteq mC(mI(mC(B,r),r),r)$ and $mC(B,r)=$

mC(mI(mC(B,r),r),r), it follows mC($f^{-1}(B)$,r)$\subseteq$mC(f^{-1}(mC(mI(mC(B,r),r),r)),r)$\subseteq f^{-1}$(mC(B,r)). Hence f is fuzzy weakly r-M continuous.

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