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Barrier Option Pricing with Model Averaging Methods under Local Volatility Models

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Abstract. In this paper, we propose a method to provide the distribution of option price under local volatility model when market-provided implied volatility data are given. The local volatility model is one of the most widely used smile-consistent models. In local volatility model, the volatility is a deterministic function of the random stock price. Before estimating local volatility surface (LVS), we need to estimate implied volatility surfaces (IVS) from market data. To do this we use local polynomial smoothing method. Then we apply the Dupire formula to estimate the resulting LVS. However, the result is dependent on the bandwidth of kernel function employed in local polynomial smoothing method and to solve this problem, the proposed method in this paper makes use of model averaging approach by means of bandwidth priors, and then produces a robust local volatility surface estimation with a confidence interval. After constructing LVS, we price barrier option with the LVS estimation through Monte Carlo simulation. To show the merits of our proposed method, we have conducted experiments on simulated and market data which are relevant to KOSPI200 call equity linked warrants (ELWs.) We could show by these experiments that the results of the proposed method are quite reasonable and acceptable when compared to the previous works.

Keywords: Local Volatility, Barrier Option Price, Model Averaging, Monte Carlo Method

1. INTRODUCTION

1.1 Overview

Since Black-Scholes' proposition of the European option pricing formulae, there have been a lot of methods to value them. They range from numerical techniques such as Monte Carlo simulation and lattice methods to nonparametric pricing methods such as neural networks to Kernel-based methods (Garcia *et al.*, 2000; Gencay *et al.*, 2001; Han *et al.*, 2008; Han *et al.*, 2009; Hutchinson *et al.*, 1994). As popular exotic option, barrier options also have been studied for pricing them. Merton gave the analytic forms for a down-and-out call

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option for the first time, Rubinstein and Reiner provided the formulas for all 8 types of barrier options, and Haug gave the set of general formulae provided by Rubinstein and Reiner (Merton, 1973; Rubinstein *et al.*, 1991; Haug, 2007). These analytic forms assumed the Black-Scholes framework in continuous time, which is different in reality, because the asset price can be observed in discrete time. Therefore, Broadie *et al.* proposed the adjustment to the barrier value in order to explain discrete observation (Broadie *et al.*, 1997). Differently from the aforementioned analytic formulae, like most other-path dependent options, barrier options can be price via numerical techniques. For example, Derman *et al.* and Hull applied a binomial tree to pricing the barrier options by using the concepts of an inner barrier and an outer barrier in order to solve the problem that the true barrier is not the same with that implied by the tree due to the up-down movements of the underlying price in the tree (Derman *et al.*, 1995; Hull, 2009). Additionally, Kamrad and Ritchken provided the modified trinomial tree method to value the options, which gives better results than the binomial tree (Kamrad *et al.*, 1991). Furthermore, an explicit finite difference method was used for pricing barrier options, showing it is quite accurate in valuing them, and an implicit finite difference method and Crank-Nicolson method were proposed to attain better accuracy and stable convergence than the explicit method (Boyle *et al.*, 1998; Hull, 2009).

Under Black-Scholes framework, the parameters in the framework are unequivocally observable, except the volatility. The volatility is assumed to be a crucial constant parameter. In order to obtain it, the historical volatility was proposed and several methods to calculate it has been developed (Parkinson, 1980; Rogers *et al.*, 1991; Rogers *et al.*, 1994). The option prices using the historical volatility mismatch with the real market option prices since the assumptions of the Black-Scholes model do not satisfy the real market properties. Therefore, instead of the historical volatility, the implied volatility surface (IVS) attained from inverting the Black-Scholes formula is used for consistent valuation of the derivatives with the financial markets. Since the option prices evaluated by Black-Scholes formula are equal to the real market option prices with implied volatility, modeling the IVS directly becomes a major concern recently. There are several methods to formulate the implied volatility surface (Cont *et al.*, 2002; Dumas *et al.*, 1998; Konstantinidi *et al.*, 2008). Especially, local volatility models that were originally studied by Dupire, Derman *et al.*, and Rubinstein and inserted into highly efficient pricing models by Andersen *et al.* and Dempster *et al.* are very much dependent on an estimate of the IVS (Andersen *et al.*, 1997; Dempster *et al.*, 2000; Derman *et al.*, 1994; Dupire, 1994; Fengler, 2005; Rubinstein, 1994; Yang *et al.*, 2010). The estimate should be free of arbitrage. Otherwise, negative transition probabilities or negative volatilities might be produced, which can make the algorithm solving the generalized Black-Scholes partial differential equation not to converge. In our previous work, we provided the confidence interval information of implied and local volatility surface using model averaging method (Kim *et al.*, 2009). Since local volatility models are known to be useful to price path-dependent exotic options like barrier options, in this paper, we aim to provide reasonable confidence interval of barrier option prices.

The organization of this paper is as follows. In Section 1.2, we review the local volatility model while barrier option is described in Section 1.3 with simple pricing formulae. Then we present the whole methodology employed in this paper in Section 2. In Section 3, we give description of the simulated and ELW market data we used for experiment. In Section 4, the experimental results are presented and we conclude the paper in Section 5.

1.2 Local Volatility Model

In traditional Black-Scholes model, volatility is assumed to be constant over the life of an option. If an option's price is known, its implied volatility can be derived by inputting all the known factors into an option pricing model. The Black-Scholes option pricing formula is as follows:

$$C_{BS}(S,\, E,\, r,\, q,\, T,\, t,\, \sigma) = Se^{q(T-t)}N(d_1) - Ee^{-r(T-t)}N(d_2) \quad (1)$$

where

$$d_1 = \frac{\log(S/E) + (r - q + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = d_1 - \sigma\sqrt{T-t}$$

Implied volatility $\Sigma_{E,\tau}$ is defined as

$$C_{market} = C_{BS}(S,\, E,\, r,\, q,\, T,\, t,\, \Sigma_{E,\tau}) \quad (2)$$

However, in many empirical tests, implied volatility shows smile patterns, that is, ATM options have lower implied volatility than ITM and OTM options which is contrary to the constant assumption. Over the years, various models have been developed to accommodate volatility smile. The most widely used model is the local volatility model proposed by Dupire (1994). Local volatility model is a smile consistent volatility model which treats volatility as a function of the current asset level St and of time t.

$$\frac{dS_t}{S_t} = (r-q)dt + \sigma(S_t,\, t)dW_{t,} \quad S_0 \geq 0, \quad (3)$$

Where the notation stays as BS model except that $\sigma(S_t, t)$ is deterministic. Local volatility models are useful to price the path-dependent options like the barrier options.

1.3 Barrier Options under Black-Scholes models

Barrier options are the most actively traded exotic options in the OTC derivatives market, often structured in accordance with the investor's risk appetite. Hence, understanding how barrier options are priced is essential for both issuers and investors.

Barrier option is called path-dependent since the payoff depends on the underlying's crossing or reaching pre-specified barrier level before expiration. Barrier option is categorized into two types from the viewpoint of payoff situation. One of them is 'out' option, whose payoff occurs only if underlying asset price has never been above the barrier before expiration and the other one is 'in' option, which become valid after underlying asset price hits the barrier before expiration. Barrier option can be characterized also from the position of the barrier relative to the initial value of underlying. If the barrier is higher than the initial value of underlying, the barrier option is called 'up' option and 'down' option otherwise.

In this paper, simple four barrier options will be covered, such as up-and-out, up-and-in, down-and-out, and down-and-in. Payoffs given underlying price for different types of barrier option are shown in Figure 1.

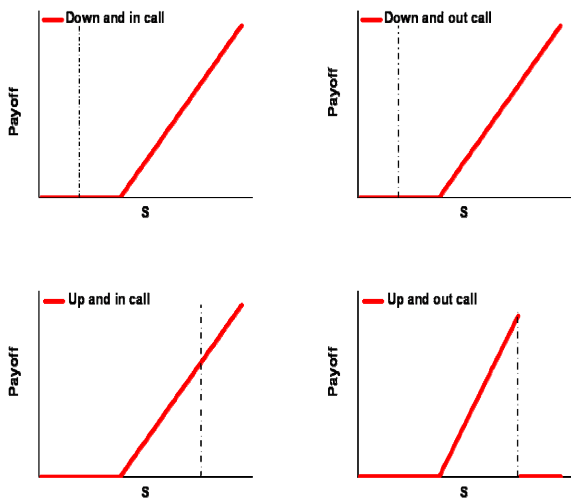

Figure 1. Payoff of different barrier options.

Merton priced a down-and-out call option in his seminal paper for the first time. The classic paper providing analytic pricing formulas for barrier options was done by Reiner and Rubinstein. They proposed simple pricing formulae, which are explained generally in Haug (2007). We omit the pricing formulae in this paper because they are already well-known. The option formulae are very sensitive to the volatility of the underlying, and thus it is highly dangerous to use constant volatility to price barrier options (Wilmott, 2000). Thus, we will price barrier option under local volatility model using Monte Carlo method. Then, we will compare with the prices from these formulae above.

2. PROPOSED METHODOLOGY

In this section, we explain our method for estimating barrier option prices with confidence interval. The method consists of five steps, which are explained below. The overall flow of the process is provided in Figure 2. After Step 1~Step 5 are repeated N times, where N is the number of different bandwidth values.

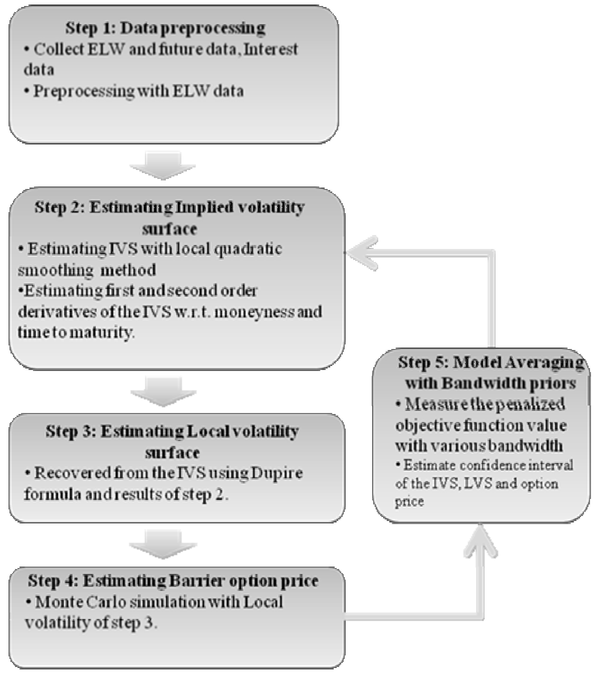

Figure 2. The overall flow of the process.

We preprocess the raw market data with three exclusionary criteria. First, the options with zero trading volume were removed in the experiment since they are meaningless. Second, we eliminated options with fewer than ten days or more than two hundred days to expiration. The short-term options have relatively small time premiums hence the estimation of volatility is extremely sensitive to measurement errors. The long-term options are illiquid. Therefore, including these options would disturb the cross-sectional fit. Finally, we eliminated options with extreme implied volatility values like over than one hundred percent. We regarded these data as outliers.

We assume no dividend effects. We use KORIBOR (KORea Inter-Bank Offered Rate) as risk-free rates. The KORIBOR is published for 10 maturities (1 and 2 weeks, and 1, 2, 3, 4, 5, 6, 9 and 12 months). They are linearly interpolated to approximate the riskless interest rate for the option's time to maturity.

The no-arbitrage price of the underlying index in a frictionless market without dividends is given by

$$S_t = \exp(-r_{T_F,t}(T_F - t))F_t, \tag{4}$$

where S_t and F_t denote the spot and the future price respectively, T_F the maturity date of the futures contract, and $r_{T,t}$ the interest rate with maturity T-t. With this notation, we can use simple moneyness measure

$$\kappa = \frac{K}{S_t}, \tag{5}$$

where K is the strike index of an option. An option is called at-the-money (ATM) if the strike price is the same as the spot price of the underlying security, i.e. moneyness is one. ATM option has no intrinsic value but only time value. An in-the-money (ITM) option has positive intrinsic value as well as time value. An option is called ITM when the strike price is below the spot price (for call option) or above the spot price(for put option). An out-of-money (OTM) option has no intrinsic value. An option is OTM when the strike price is above the spot price of the underlying security (for call option) or below the spot price (for put option).

2.2 Estimating Implied Volatility Surface

To estimate the LVS using Dupire formula, we need the derivatives of the IVS. Using bivariate local quadratic smoothing methods, we can achieve the derivatives of implied volatility function with respect to moneyness measure and time to maturity. Also, the bias problem visible in the sparse region is less present in local quadratic smoothing (Fengler, 2005; Kim *et al.*, 2009). Thus we use bivariate local quadratic smoothing to estimate the IVS. Local quadratic smoothing is the quadratic minimization problem employing kernel weights as:

$$\min_{\beta\in\mathbb{R}^6}\sum_{i=1}^{n}\{y_i-\beta_0-\beta_1(x_1^i-x_1)-\beta_2(x_2^i-x_2)-\beta_3(x_1^i-x_1)^2 \\ -\beta_4(x_1^i-x_1)(x_2^i-x_2)-\beta_5(x_2^i-x_2)^2\}^2K_h(x_i-x), \tag{6}$$

Where $\beta=(\beta_0,\cdots,\beta_5)^T$, denotes the vector of coefficients. In the experiment, we multiply the kernel weights by log trading volume of each data. Thus we can use matrix notations

$$X=\begin{pmatrix}1 & \kappa_1-\kappa & \tau_1-\tau & (\kappa_1-\kappa)^2 & (\kappa_1-\kappa)(\tau_1-\tau) & (\tau_1-\tau)^2\\ 1 & \kappa_2-\kappa & \tau_2-\tau & (\kappa_2-\kappa)^2 & (\kappa_2-\kappa)(\tau_2-\tau) & (\tau_2-\tau)^2\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 1 & \kappa_n-\kappa & \tau_n-\tau & (\kappa_n-\kappa)^2 & (\kappa_n-\kappa)(\tau_n-\tau) & (\tau_n-\tau)^2\end{pmatrix} \tag{7}$$

and $y=(y_1,\cdots,y_n)^T$, and finally

$$W=\begin{pmatrix}\log(V_1)K_h(x-x_1) & 0 & \cdots & 0\\ 0 & \log(V_2)K_h(x-x_2) & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \log(V_n)K_h(x-x_n)\end{pmatrix}, \tag{8}$$

where V is the trading volume of each data and

$$K_h(u)=\frac{1}{h}K(\frac{u}{h}). \tag{9}$$

We employed the Gaussian kernel with infinite support, which is given by:

$$K(u)=\frac{1}{\sqrt{2\pi}}e^{-u^2/2}. \tag{10}$$

In this paper, we use bivariate model, so we need two dimensional kernels. It is obtained by products of univariate kernels:

$$K(u_1,u_2)=K(u_1)K(u_2). \tag{11}$$

Then $\hat\beta$ can be obtained via

$$\hat\beta(x)=(X^TWX)^{-1}X^TWy. \tag{12}$$

An important advantage of local quadratic estimators is that the derivatives of regression function are obtained as byproduct of estimators.

$$\hat f_x(x)=\hat\beta_1(x),\cdots,\hat f_{xy}(x)=\hat\beta_4(x), \\ \hat f_{yy}(x)=2!\hat\beta_5(x). \tag{13}$$

2.3 Estimating Local Volatility Surface

Local volatility surface is recovered from implied volatility surface by Dupire formula (1994).

$$\sigma_{\kappa,\tau}^2(S_t,t)=\frac{\Sigma^2+2\Sigma\tau\dfrac{\partial\Sigma}{\partial\tau}}{1+2\kappa\sqrt{\tau}d_1\dfrac{\partial\Sigma}{\partial\kappa}+d_1d_2\kappa^2\tau(\dfrac{\partial\Sigma}{\partial\kappa})^2+\Sigma\tau\kappa^2\dfrac{\partial^2\Sigma}{\partial\kappa^2}}, \tag{14}$$

where d_1 and d_2 are interpreted as

$$d_1=-\ln(\kappa)/(\Sigma\sqrt{\tau})+0.5\Sigma\sqrt{\tau} \text{ and } d_2=d_1-\Sigma\sqrt{\tau} \tag{15}$$

The derivatives of the IVS are estimated as derivatives of local quadratic estimators which are used to smooth the IVS.

2.4 Estimating Barrier option prices

2.4.1 Monte Carlo Methods

Monte Carlo method, firstly suggested by John von Neumann and Stanislaw Ulam (1949) is the computational algorithm involving repeated random sampling to get results. This method is widely used in many areas, especially simulating physical and mathematical systems because this method just relies on calculating repeated random numbers. For this reason, Monte Carlo simulation is also very useful to solve problem without closed form solution. Typically, for measuring business risk or option price, input data has highly significant uncertainty, so those problems which is intractable by deterministic algorithm can be solved by using Monte Carlo simulation.

To briefly explain Monte Carlo simulation method used also in this paper for calculating barrier option prices, we deal with an example of the processes where the natural logarithm of the underlying asset is governed by geometric Brownian motion as:

$$S + dS = S\exp\left[\left(\mu - 0.5\sigma^2\right)dt + \sigma dz\right] \quad (16)$$

To simulate this process, we first change above equation as follows. The significant difference of following equation comparing with previous one is that this equation is based on discrete time intervals, Δt, instead of dt.

$$S + \Delta S = S\exp\left[\left(\mu - 0.5\sigma^2\right)\Delta t + \sigma\varepsilon_t\sqrt{\Delta t}\right] \quad (17)$$

In the Monte Carlo simulation, the difference on the underlying asset price is calculated in the chosen time interval Δt and ε_t, which is a random variable having zero mean and variance with 1. The result is not deterministic because ε_t is random variable, so the simulation is should be repeated many times and the final result is obtained by averaging individual results. Therefore, Monte Carlo simulation essentially needs intensive computer calculation. This is main disadvantage of this method.

2.4.2 Barrier option pricing

In quadratic smoothing method, we estimate local volatilities on the mesh points. To apply Monte Carlo simulation for pricing barrier options, we need to estimate the local volatilities at points between the mesh points when generated path locate in those positions. We use a two-dimensional interpolation method called bilinear interpolation. Bilinear interpolation is a two dimensional extension of linear interpolation with two variables on a grid. The purpose of interpolation is to estimate values for third value at given new two variables. This method can be easily understood by considering following schematic diagram Figure 3.

In this paper, two variables on the grid for bilinear interpolation are time and stock price. Values of four points at specific stock prices and time in the figure are V_1, V_2, V_3, V_4 respectively. The areas made by the four corners and interior new point are represented as A_1, A_2, A_3, A_4. The approximated value at new point is expressed by following equation.

$$\sum_{i=1}^{4} A_i V_i \Big/ \sum_{i=1}^{4} A_i \quad (18)$$

Through Monte Carlo simulation, we have 10,000 different generated paths. Then we can examine that which paths touch the barrier. We price the option value by computing average discounted payoff of all generated paths.

2.5 Model averaging with bandwidth Priors

We applied model averaging method to estimate barrier option prices. The estimated local volatility surfaces are varied with bandwidth h. Thus we need to choose one bandwidth. However, to find the best bandwidth is difficult and complicated. There are many criteria, such as the integrated squared error (ISE), the average squared error (ASE), and so on (Fengler, 2005). The penalizing approach is asymptotically equivalent to the bandwidth obtained by minimizing the ASE. In penalizing approaches, a weighted version of the resubstitution estimate is employed

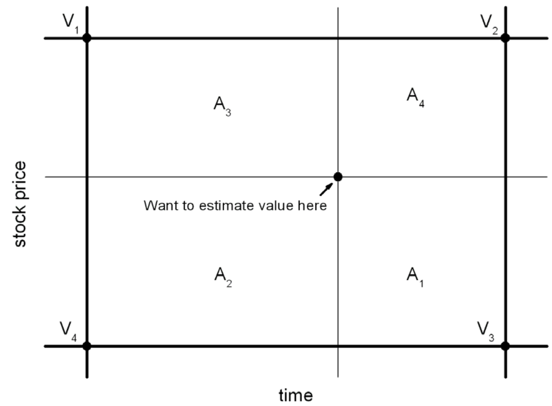

Figure 3. Bilinear interpolation.

$$G(h) = \frac{1}{n}\sum_{i=1}^{n}\{y_i - \hat{f}(x_i)\}^2 \tilde{w}(x_i)\Xi(\frac{1}{n}w_{i,n}(x_i)) \quad (19)$$

where $\tilde{w}(\cdot)$ is some weight function and $\Xi(\cdot)$ is the correction function.

Instead of finding the best bandwidth, we use model averaging method, which is simply explained in subsection 2.2 and subsection 2.3, suggested in Kim *et al.* (2009). The values of the above penalized objective function can be the measure of suitability of the model. Thus we assume the inverse values of penalized objective function as bandwidth priors. Then we can calculate the weighted means of IVS, LVS and barrier option prices. Using N different bandwidth values, we can form a 90% pointwise confidence band from the percentiles at each point: we find the N*0.05th largest and smallest values at each point. In the experiment, we used 200 different bandwidth values.

3. DATA DESCRIPTION

3.1 Simulated Data

To verify the effectiveness of the proposed method, we generated option data under the Heston Stochastic

Volatility model. The Heston Stochastic Volatility model assumes that volatility is a random process and the randomness of the variance process varies as the square root of variance.

$$\frac{dS_t}{S_t} = (r-q)dt + \sigma_t dW_t, \quad S_0 \geq 0, \tag{20}$$

with the (squared) volatility following the classical Cox-Ingersoll-Ross(CIR) process:

$$d\sigma_t^2 = \kappa(\eta - \sigma_t^2)dt + \theta\sigma_t d\tilde{W}_t, \quad \sigma_0 \geq 0, \tag{21}$$

where both W_t and $\tilde{W}_t$ are Wiener processes, and

$$Cov[dW_t \, d\tilde{W}_t] = \rho dt.$$

To generate the data from the Heston Stochastic Volatility model, we gave some specific values to the four parameters of Heston model, κ, η, θ and ρ. We generated the option prices for specific initial values, S_0 and σ_0, with variety strike prices and time to maturities. In this study, when generating simulated data, we used $r-q=0.1$, $\kappa=0.2$, $\eta=0.1$, $\theta=0.05$, $\rho=0.05$ for the four parameters, and $S_0=100, \sigma_0=0.2$ for the two initial values. We did not add any noise term to the data. To give reality, we pulled strike prices from a normal distribution with mean. Thus data were generated more in the ATM region and sparse in the far OTM and ATM region.

We compared the results of barrier option prices with confidence interval using the proposed method with Heston Stochastic Volatility model prices.

3.2 Real Market Data

An Equity-Linked Warrant (ELW) is a kind of option that gives the holder the right but not the obligation to buy or sell an underlying asset at a set price, on or before an expiry date.

For the application, we have used daily market data of KOSPI200 ELWs provided by the Korea Investors Service (KIS). Total number of observations is 182. We were also provided the trading volume information for each ELW. KOSPI200 futures contract and interest rate data in daily frequency are obtained from the Bank of Korea.

An overview of the data is given in Table 1. The statistics are in form of the IV data not in form of the price data. The distribution of the moneyness of ELW data appears in Figure 4. Solid line is for all observations, dashed line is for puts, and the dotted line is for calls only. All densities are shifted to the right. This may be due to the depressed market. Put and call densities are also shifted. This shows well the higher liquidity of ATM and OTM.

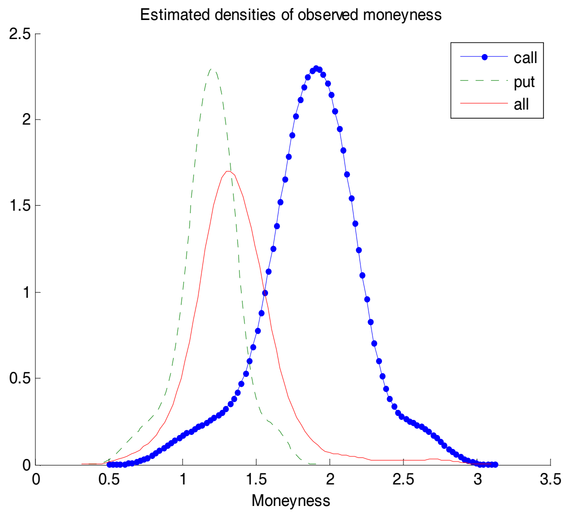

Figure 4. Nonparametrically estimated densities of observed moneyness for 20081106.

4. RESULTS

In this section, we represent the results of applying the above presented five-step method to estimate the barrier option prices with confidence interval. There are two types of data, generated data and real market data. We applied the proposed method to the both of simulated and real market data.

4.1 Experiment on Simulated Data

Using simulated data we estimated the implied volatility surface and local volatility surface by changing the bandwidth value. The initial stock price and volatility were given as 100, 0.2 respectively. We used 200 different bandwidth values. Thus, we had 200 different esti-

Table 1. Statistics of KOSPI200 index ELW data in form of the IV data.

Observation Date	Time to Expiry (days)	Min	Max	Mean	Standard Deviation	Total number of Observations Calls
20081106	35	0.5062	1.4367	0.6683	0.1673	62 45
	63	0.4370	0.8317	0.5962	0.0920	44 30
	98	0.3507	1.3706	0.6967	0.1895	48 20
	126	0.4490	1.0079	0.6907	0.1655	28 16

mators. In Figure 5, we present two surfaces, the implied volatility surface estimate and local volatility surface estimate. These are the weighted mean values of the fitted estimator. The circles present IVs implied from the generated data in left panel. Figure 6 shows some slices from the surface in the direction of the strike price. The two left panels and the two right panels in Figure 6 present the implied volatility smile and local volatility smile respectively. The upper panels are for time .107 and lower panels for 0.3. Both IV and LV have very narrow confidence interval since we do not added any noise term to the data. Using these local vola-

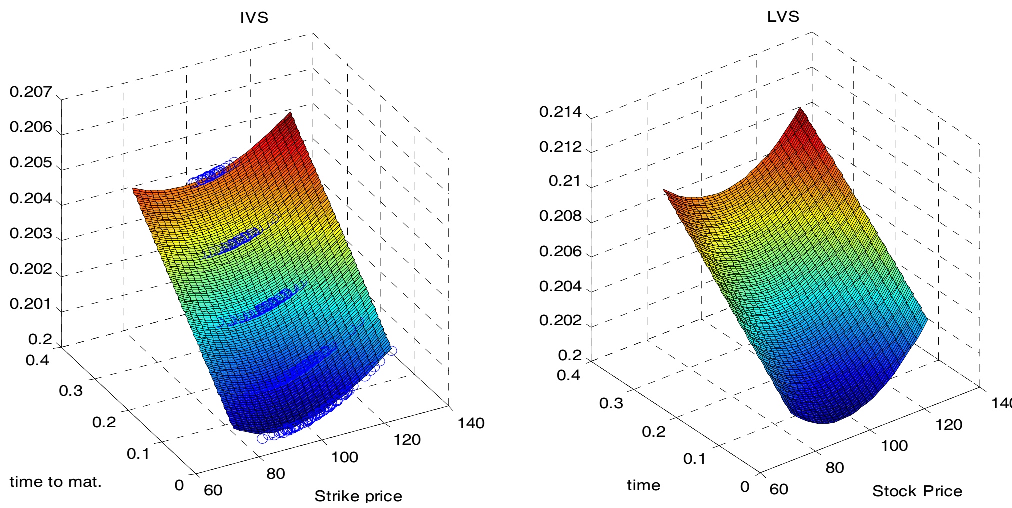

Figure 5. Left panel: IVS fit for simulated data; right panel: LVS fit for simulated data. The single circles denote IV data obtained by inverting BS formula separately for each observation.

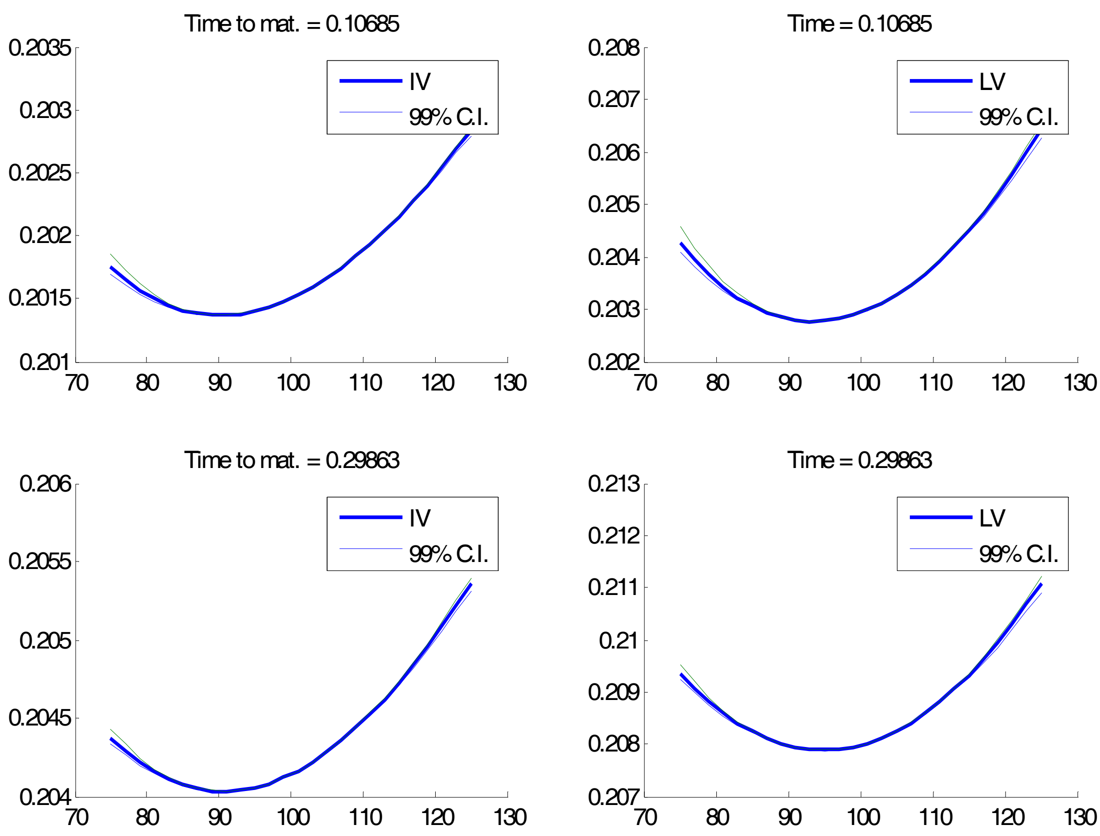

Figure 6. The implied volatility smile and local volatility smile with 99% confidence interval.

tilities, we priced barrier options through Monte Carlo simulation. We consistently averaged over 10,000 simulated paths. All options have a maturity of 0.35 year. An important issue for the path-dependent options like barrier options is the frequency at which the stock price is observed for purposes of determining whether the barrier level has been reached. We have assumed a discrete number of observations, namely at the close of each trading day. Moreover, we have assumed that a year consists of 252 trading days.

In order to check the accuracy of our experiment algorithm, we computed the barrier option price under the Heston model through Monte Carlo method using the same parameter values as used in the data generation. We assume that the Heston Stochastic Volatility model prices are true value. Thus the performance of the proposed method can be evaluated whether the Heston SV prices belong to the confidence interval of our estimate. In Figure 7 to Figure 10, we show prices for barrier options (as a strike price). The lower barrier was always taken equal to 80 and the upper barrier was 120. The thick line is mean value of 200 different estimators and thin dashed lines are confidence interval. The circled lines are the Heston model prices. In the cases of DIB,

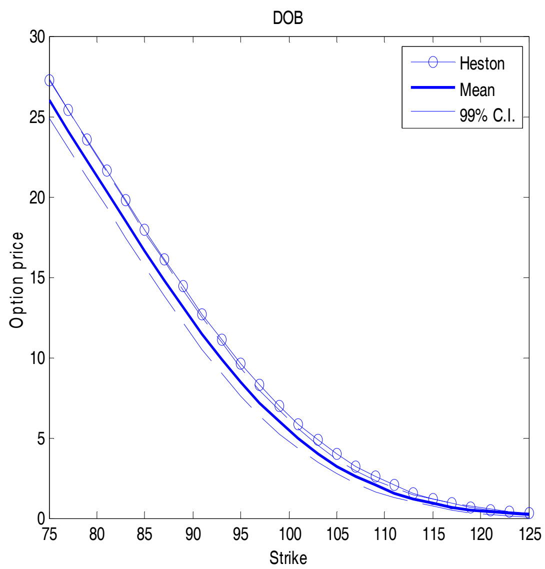

Figure 7. DOB prices with 99% C.I. At $S_0 = 100$, maturity = 0.35 year, and barrier = 80.

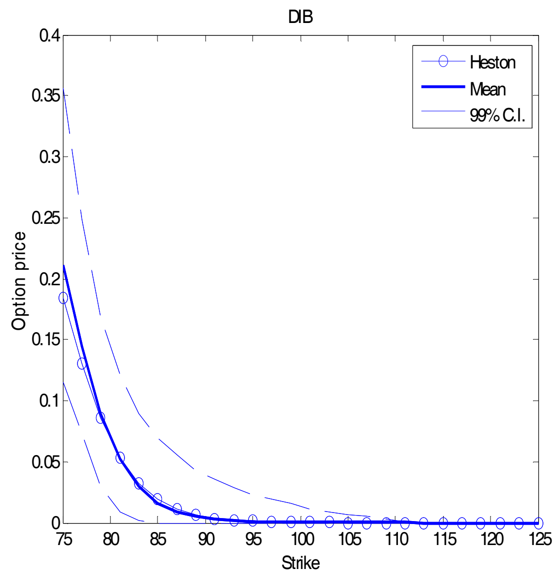

Figure 8. DIB prices with 99% C.I. At $S_0 = 100$, maturity = 0.35 year, and barrier = 80.

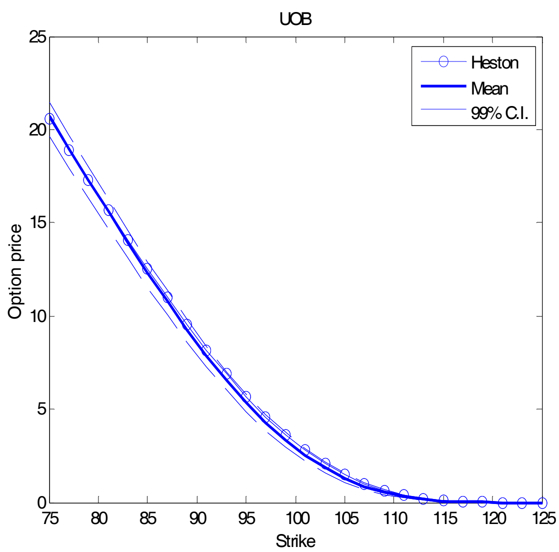

Figure 9. UOB prices with 99% C.I. At $S_0 = 100$, maturity = 0.35 year, and barrier = 120.

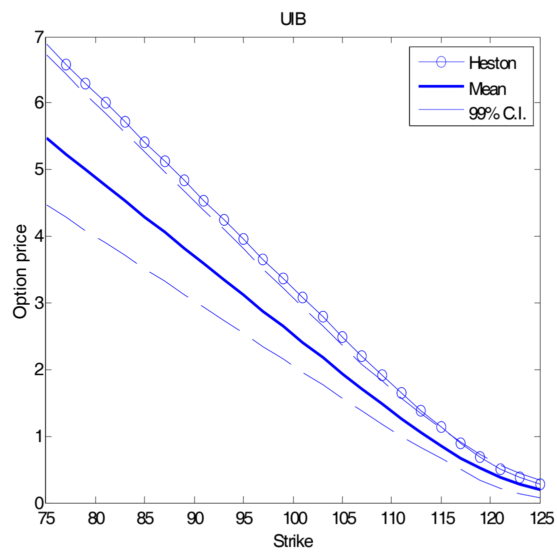

Figure 10. UIB prices with 99% C.I. At $S_0 = 100$, maturity = 0.35 year, and barrier = 120.

UOB, the Heston prices are included within the confidence interval.

4.2 Experiment on Real Market Data

Using KOSPI200 ELW data described in the previous section, we now estimate the four kinds of barrier options. Figure 11 presents two surfaces, IVS and LVS for the ELW data. The confidence intervals are broader than those of simulated data. The estimated values are fluctuated with the change of bandwidth since the real data have noise terms. These are represented well in Figure 12. In Figure 13 to Figure 16, we show prices for all barrier options (as a percentage of the spot). The

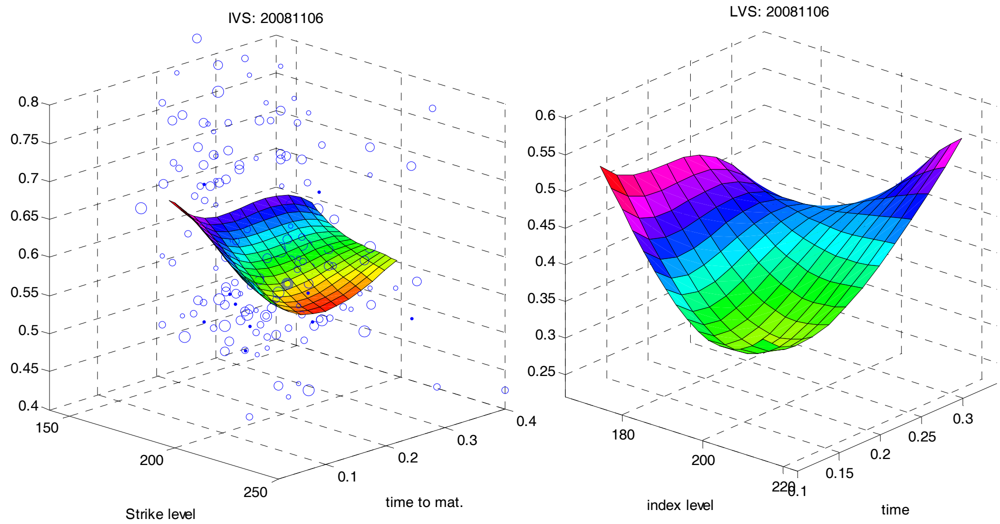

Figure 11. Left panel: IVS fit for 20081106; right panel: LVS fit for 20081106. The single circles denote IV data obatained by inverting the BS formula separately for each observation.

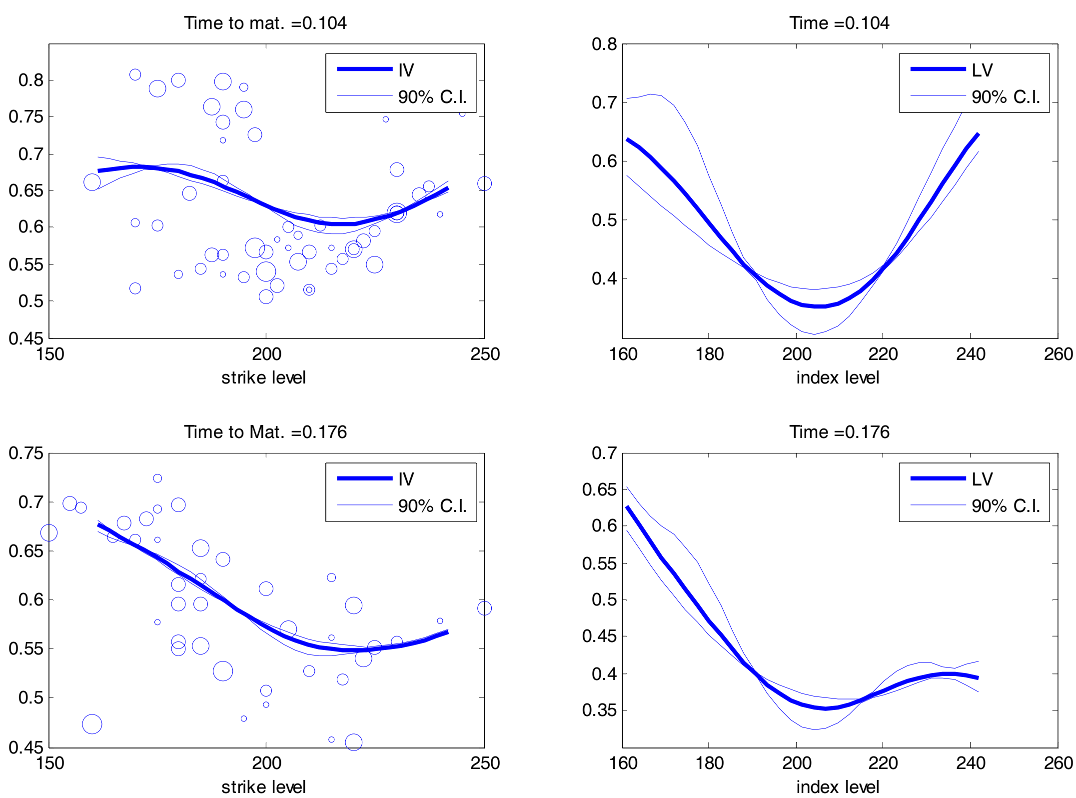

Figure 12. The implied volatility smile and local volatility smile with 90% confidence interval.

strike K was always taken equal to the spot S_0. The maturity is 0.35 year. Unlike the simulated data experiment, we do not know the true prices and thus we cannot evaluate the performance. Instead we computed the prices using the barrier option pricing formulae explained in Haug (2007). The volatility σ in the formulae is selected to agree well with the identity DIB+DOB = vanilla call = UIB+UOB.

As mentioned above, using constant volatility for pricing barrier options is unreliable because it is very sensitive to the volatility. The prices from local volatility model and the prices from BS model (constant volatility) are similar in the cases of DOB and DIB prices. However, the discrepancy is larger in UIB and UOB prices. Thus we can verify the above facts, that is, smile-inconsistent models such as BS model lead to result different from smile-consistent result such as local volatility models.

5. CONCLUSION

In this paper, we introduced a model averaging method to estimate the barrier option prices with confidence interval. The method is composed of five steps. In each step, we get different implied volatility surface, local volatility surface and barrier option prices by repeatedly changing the bandwidth. Applying model aver-

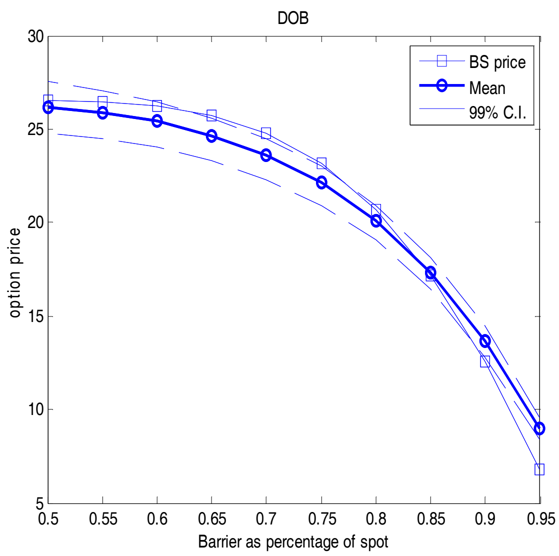

Figure 13. DOB prices. At K = S_o, maturity = 0.35 year.

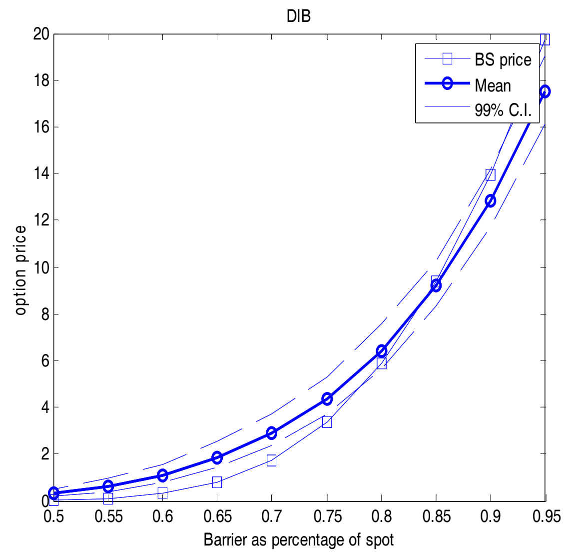

Figure 14. DIB prices. At K = S_o, maturity = 0.35 year.

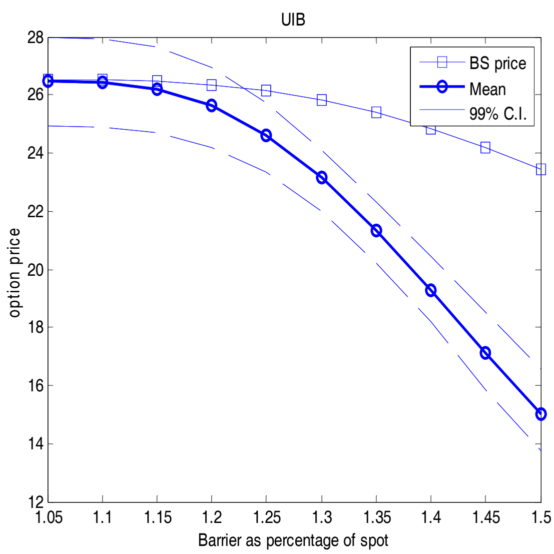

Figure 15. UIB prices. At K = S_o, maturity = 0.35 year.

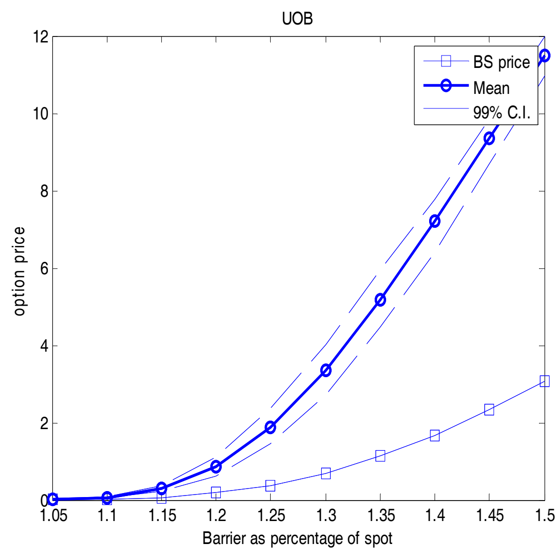

Figure 16. UOB prices. At K = S_o, maturity = 0.35 year.

aging method we can estimate the volatility surface and option prices with confidence interval. There are some pricing methods with local volatility. Among them we used Monte Carlo method. For the further research, more reliable and enhanced estimation techniques, e.g. kernel-based techniques as in Jung *et al.* (2010), Kim *et al.* (2007), Lee *et al.* (2006), Lee *et al.* (2006), Lee *et al.* (2007), Lee *et al.* (2009), or Lee *et al.* (2010) needs to be further investigated with suitable modifications or extensions. Also, we can use FDM to price barrier options. Applying the local volatility surface from the proposed method to dynamic-hedging problems will be an important issue.

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