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Development of Centering Method for Automatic Generation of a Quadrilateral Mesh

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Abstract – A new method has been developed in this paper for automatic quadrilateral mesh generation for a two-dimensional domain. The method is named 'centering method' because it centers a point at the domain and then divides it into sub-domains using cutting lines from the center point. Each of the cutting lines is selected based on the criterion using the angles between the boundary of the domain and the cutting line. The decomposition of the domain into sub-domains is repeated until every subdomain has four or six nodes. Pre-defined splitters are used to divide six-node domains into quadrilateral elements depending on their configuration and presence on the boundary of the initial domain. Arbitrary domains are meshed as examples to verify the robustness of the new method.

***Keywords*: centering method, decomposition, quadrilateral mesh generation, six-node domain splitter.**

1. Introduction

Quadrilateral mesh generation is required for finite element method (FEM). FEM is one of the most powerful methods applied to solve engineering problems and uses finite elements for analysis. Mesh generation is the process to divide a domain of interest into finite elements. Triangular or quadrilateral elements are commonly used for analysis of a two-dimensional (2D) domain. While quadrilateral mesh is better than triangular mesh in analysis, it is more difficult to generate automatically because of its shape.

Algorithms for mesh generation in a 2D domain should produce a mesh of a good quality. The good quality includes the aspect ratio of an element and its internal angles. For example, square-like quadrilateral elements are considered the best because of a good aspect ratio, approximately 1.0, of two corresponding sides and approximately 90^o of the internal angle. However, it is very difficult to make square-like elements for the entire area of an arbitrary 2D domain.

Research has been carried out to develop methods for automatic quadrilateral mesh generation for an arbitrary 2D domain for the last few decades [1-14]. The four representative methods, shown in Fig. 1, are the grid-based method [1-3], the triangle merging method [4-6], the ideal splitting line method [7-9], the paving method [10-12], and the offsetting method [13]. These methods can generate a quadrilateral mesh at an arbitrary 2D domain. The grid-based meshing method covers the domain with a square net and adjusts the elements on the boundary of the domain into quadrilateral elements as shown in Fig. 1(a). The merging method combines two or more triangular elements into a quadrilateral element as shown in Fig. 1(b). The splitting line method in Fig. 1(c) divides the domain of analysis into two subdomains and continues this decomposition process until all subdomains have four or six nodes remaining. The paving method and the offsetting method generate an element by offsetting the boundary of the domain inward as shown in Fig. 1(d). As it can generate good-quality elements on the boundary, the paving method has been adapted to the commercial FEA pre-processor named PATRAN [15].

Each of the four methods has a drawback. The grid-based method is likely to produce bad-quality elements of large internal angles around the boundary as shown in Fig. 1(a). The merging method requires a triangular

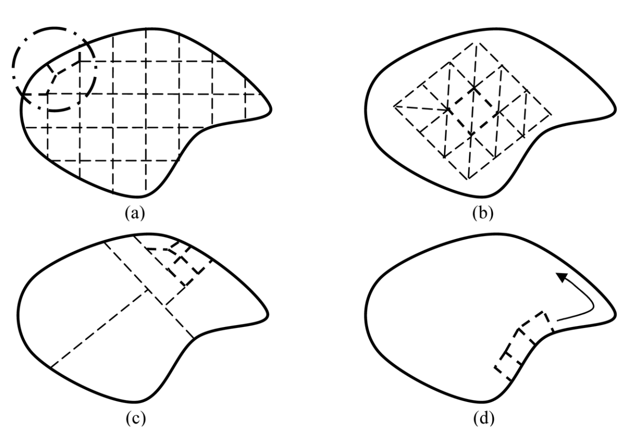

Fig. 1. Methods of 2D quadrilateral mesh generation. (a) grid-based method, (b) triangle merging method, (c) ideal splitting line method, and (d) paving and offsetting methods.

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meshing algorithm and its mesh quality depends on the quality of the triangular mesh and the merging algorithm used. The splitting line method may not produce good elements on the boundary, as shown in Fig. 1(c), since it does not use the geometry contour of the domain unlike the paving method. However, it can produce the smooth transition of element size because it regularly distributes nodes on the splitting line which passes through the domain and then, becomes the boundary of the subdomain [7]. The paving method has the disadvantage that elements grow dramatically larger if the boundary is convex or smaller if the boundary is concave [10]. In addition, an inappropriate mesh transition may occur at the local area of the domain where the large elements from a convex boundary encounter the small ones from a concave boundary.

A new method, named centering method, has been developed in this research to generate a quadrilateral mesh for an arbitrary 2D domain. It centers a point at the domain, makes cutting lines from the center point to the boundary and then, decomposes the domain into subdomains with the cutting lines. The cutting lines are selected in order to generate good-quality elements, like squares, on the boundary. The criterion for the selection is that they need to make the angle of approximately 90° to the boundary. The decomposition process continues until all subdomains have four or six nodes. The six-node domains are converted into quadrilateral elements using pre-defined six-node loop closures.

The centering method is designed to overcome the drawbacks of the splitting line method and the paving method. It is intended to generate good elements of the internal angles of 90° on the boundary of a 2D domain and to prevent elements from growing dramatically larger or smaller due to convexity or concavity of the boundary. The criterion using the right angle between the boundary and the cutting lines is to satisfy the first objective of good-quality elements on the boundary of a domain. The regular distribution of nodes on the cutting lines helps to accomplish the second objective of the gradual growth and smooth mesh transition.

2. Overview of the new method

2.1 Definition of the domain boundary and generation of boundary nodes

Looping algorithm [7] is used for the centering method to define the contour of the boundary of an arbitrary 2D domain as shown in Fig. 2. It has the advantage that the domain is defined with continuous lines regardless of presence of holes inside. One cutting line can decompose the domain into two subdomains as shown in Fig. 2(b) and then the two subdomains are separately defined in a loop. For consistency, the loop is directed counterclockwise. As the centering method is a domain decomposition method using cutting lines, boundary definition with a loop is appropriate.

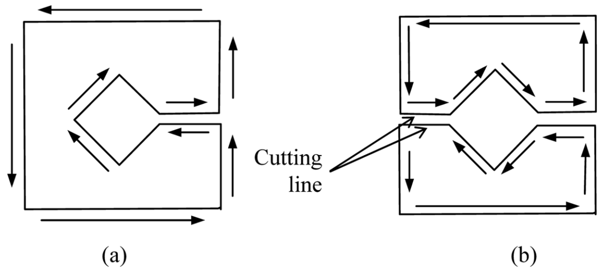


Fig. 2. Domain definition with continuous lines. (a) one domain in one loop, and (b) two subdomains and their definition.

2.2 Overview of the centering method

Fig. 3 illustrates the overview of the centering method developed in this research for 2D quadrilateral mesh generation. Boundary nodes are generated as shown in Fig. 3(a) and the domain is divided at a convex boundary as shown in Fig. 3(b). If there is a convex boundary of a domain, it may lead to location of the center point out of the domain, for example, in Fig. 2(a). The division of the domain at the convex boundary can prevent the wrong location of the center point as shown Fig. 3(b). The two subdomains in Fig. 3(b) do not have any convexity on their boundary. A point is centered at each subdomain as shown in Fig. 3(c) and the cutting lines are selected and then, divide each domain into subdomains as shown in Fig. 3(d). The method continues the division process, as shown in Fig. 3(e), until all subdomains have 4 or 6 nodes. The mesh generation is completed in Fig. 3(f) when all the subdomains of 6 nodes are divided into quadrilateral elements using the pre-defined splitters for 6-node loops.

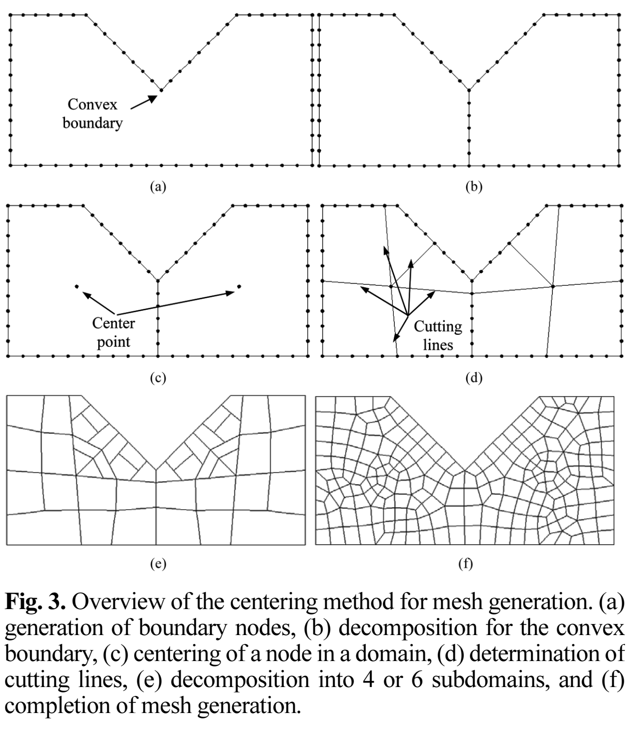


Fig. 3. Overview of the centering method for mesh generation. (a) generation of boundary nodes, (b) decomposition for the convex boundary, (c) centering of a node in a domain, (d) determination of cutting lines, (e) decomposition into 4 or 6 subdomains, and (f) completion of mesh generation.

3. Algorithm of the centering method

3.1 Centering of a point at a domain and determination of cutting lines

The method developed in this research centers a point at a domain of analysis and then, selects cutting lines starting from the center point to the boundary of the domain. The point is centered with the following equation;

$$(X_{CP}, Y_{CP}) = \frac{1}{N}\sum_{k=1}^{N}(X_{BN}^{k}, Y_{BN}^{k}) \tag{1}$$

where (X_{CP}, Y_{CP}) is the coordinate of the center point, N is the number of the boundary nodes of the domain, and (X_{BN}^{k}, Y_{BN}^{k}) is the coordinate of each boundary node. Eq. (1) using the boundary nodes centers a point at the domain as shown in Fig. 3(c). The coordinate of the center point is determined with that of the boundary nodes.

Selection of cutting lines is based on the angles between the domain boundary and a candidate to a best cutting line. Cutting line index (*CLI*) in Eq. (2) is used to determine the best cutting lines that can make boundary elements rectangular;

$$CLI = \left|\frac{\pi}{2} - \theta_i\right| + \left|\frac{\pi}{2} - \theta_j\right| \tag{2}$$

where θ_i and θ_j are the angles between a candidate to a cutting line and the boundary as shown in Fig. 4. If the boundary angles, θ_i and θ_j, are 85°, respectively, *CLI* is 0.175. If *CLI* of a candidate to a best cutting line is lower than 0.175, for example, for this research, the candidate is selected. The selection process continues until all cutting lines are identified with their *CLI* lower than 0.175. The number of the cutting lines selected should be the same with the number of the subdomains the cutting lines divide the domain into. For example, 5 cutting lines were selected in Fig. 3(d) and they divided the domain into 5 subdomains. At least, two cutting lines should be taken to divide the domain because one cutting cannot decompose a domain into subdomains as indicated in Fig. 4.

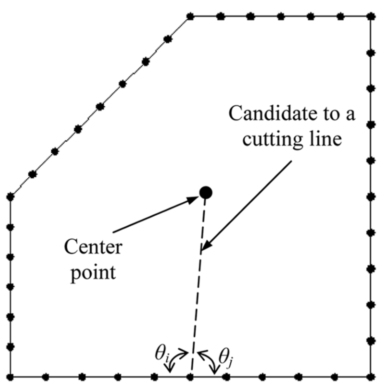


Fig. 4. Determine of a cutting line using the angles.

3.2 Decomposition at a convex boundary

A convex boundary of a domain may center a point outside the domain, for example in Fig. 2(a), since the point is located according to Eq. (1) using the boundary nodes. Therefore, it is necessary to remove the convex boundary at a domain. If the domain is divided at the convex boundary as shown Fig. 3(b), two subdomains may be generated without a convex boundary. The ideal splitting line [7] is used for the decomposition at a convex boundary. Splitting line index (*SLI*) in Eq. (3) is used to select the best splitting line at the convex boundary.

$$SLI = C_1\frac{\sum_{k=1}^{4}\left|\theta_k - \frac{\pi}{2}\right|}{2\pi} + C_2\frac{\sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2}}{L_{max}} \tag{3}$$

where C_1 and C_2 are constants, θ_k is the internal angle between a candidate to the best splitting line and the boundary of a domain as shown in Fig. 5, (X_i, Y_i) and (X_j, Y_j) are the coordinate of the two selected nodes, respectively, for the candidate, and L_{max} is the longest line of two boundary nodes. C_1 and C_2 are both 0.2 for this research. The first term of Eq. (3) is to select a splitting line that makes the internal angles close to 90° and the second one is to select a short splitting line because long splitting lines often make long narrow domains [7] at which the centering method in this research may generate bad-quality elements.

3.3 Node distribution

Nodes should be generated on a cutting line and a boundary line according to the element size at each end of the line. The input data for the boundary definition includes element size (*ES*). Eqs. (4)~(10) are used to place nodes on a cutting line or a boundary line. They use the average element size (F_1) of the two element sizes (ES_1 and ES_2) for the line, the number of elements (*No*) on the line, the element size (F_2) from the number of elements, the distance increment ($D1_{inc}$) from one side of the line, the distance increase ($D1$) from the side, node division number ($i_{Division}$) for the side, the distance increment ($D2_{inc}$) from the other side, the distance increase ($D2$) from the other side, and node division number ($j_{Division}$) for the other side, respectively.

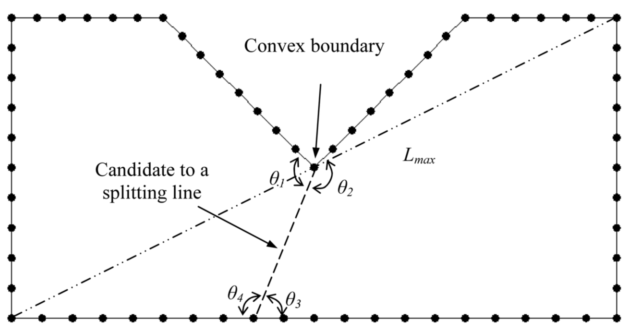


Fig. 5. Selection of the ideal splitting line at a convex boundary.

If a cutting line or a boundary line has the length of 10 and the element size of 1.0 and 2.0 at each side as shown in Fig. 6, nodes are placed with the different distances ($D1$ and $D2$) defined with $F1 = 1$, $No = 6$, $F2 = 1.667$, $D1_{inc} = 0.185$, and $D2_{inc} = -0.185$.

$$F_1 = \frac{ES_1 + ES_2}{2} \tag{4}$$

$$No = \frac{L}{F_1} \tag{5}$$

$$F_2 = \frac{L}{No} \tag{6}$$

$$D1_{inc} = \left(\frac{ES_2 - ES_1}{No}\right)\left(\frac{F_2}{F_1}\right) \tag{7}$$

$$D1 = D1_{inc} \times i_{Division} + ES_1 \times \frac{F_2}{F_1} \tag{8}$$

$$D2_{inc} = \left(\frac{ES_1 - ES_2}{No}\right)\left(\frac{F_2}{F_1}\right) \tag{9}$$

$$D2 = D2_{inc} \times j_{Division} + ES_2 \times \frac{F_2}{F_1} \tag{10}$$

The element size of each node on the boundary of a domain or a subdomian is determined with the average of its two neighboring elements. For example, the element size for the node between the second element ($i = 2$) and the third one ($i = 3$) is 1.575 (= 1.48 + 1.67 / 2). When a node is centered at a domain, its element size is determined as the average of the element size of all the nodes on the boundary of the domain and is, therefore, able to produce a smooth mesh transition at the domain.

3.4 Narrow domain with eight nodes

A narrow domain with 8 nodes needs to be directly divided without a point centered at the domain and its cutting lines. If a point is centered at a narrow domain and cutting lines are generated from the point as shown in Fig. 7(a), the domain would be divided into quite small subdomains which may produce bad quality elements. Therefore, it is necessary to divide the domain with the shortest cutting line within the domain as shown in Fig. 7(b). This direct division produces fewer subdomains than the division using the center point and the cutting lines and therefore, generates fewer elements at the domain.

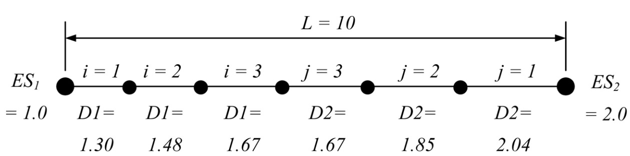


Fig. 6. Placement of nodes on a line with the element sizes.

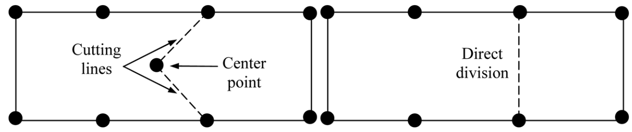


Fig. 7. Narrow domain with 8 nodes and its decomposition.

3.5 Closure of six node domains

Six node domains should be divided into quadrilateral elements. The centering method decomposes an initial 2D domain into 4 or 6 node subdomains at the end. The 4 node domains are directly accepted as quadrilateral elements but the 6 node domains should be divided into quadrilateral elements. Splitters have been used for 6 node domains depending on their configuration [7,10] because the direct division of a 6-node domain into only two quadrilateral elements may produce a great internal angle close to 180°.

In this research, 6 node domains are divided, as shown in Table 1, depending on their configuration and presence on the boundary of the initial domain in order to reduce the number of elements and nodes generated. The number of nodes and the number of elements affect the computation time of finite element analysis (FEA). If a triangular domain is present on the boundary of the initial domain as Case 3 in Table 1, it generates five elements and 3 extra nodes but it generates 2 elements and zero extra node if it is not on the boundary. The smooth method employed for the centering method improves bad quadrilateral elements within the initial domain by moving their nodes. However, it cannot improve bad elements composed of boundary nodes of the initial domain since the boundary nodes cannot be moved to define the domain.

3.6 Smoothing

Laplacian smoothing technique [1,7] is used to improve the mesh generated by the centering method. It moves a node to the centroid of the polygon constructed by its neighboring nodes. However, it does not move any boundary nodes of the initial domain. Eq. (11) is used to reposition a node. Eq. (11) is similar to Eq. (1) in that the location of a node is determined by its surrounding nodes.

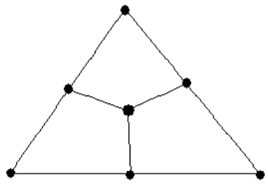
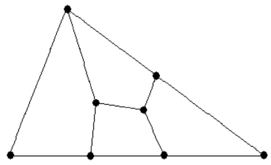
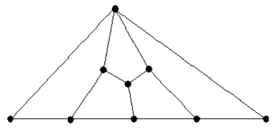
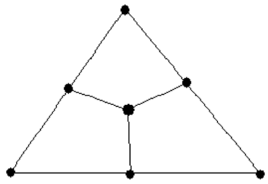
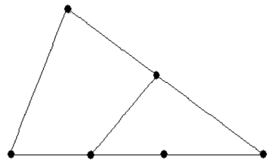
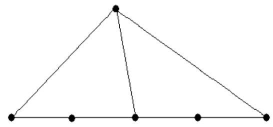
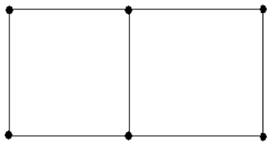
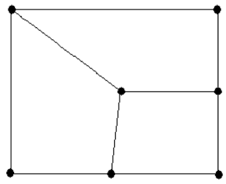
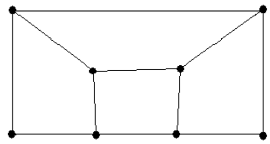
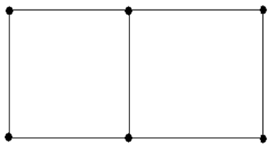
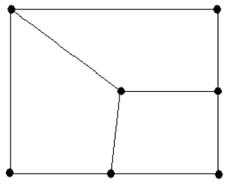
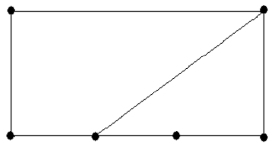
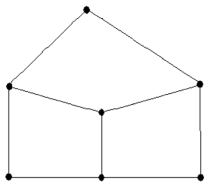
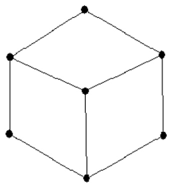
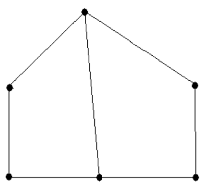
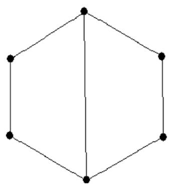
$$(X_{New}, Y_{New}) = \frac{1}{N}\sum_{k=1}^{N}(X_{BN}^k, Y_{BN}^k) \tag{11}$$

Here, (X_{New}, Y_{New}) is the new location of the node to be moved, N is the number of the nodes connected with the node, and (X_{BN}^k, Y_{BN}^k) is the coordinate of the connected nodes.

4. Examples

The centering method developed in this research was implemented in the computer language, FORTRAN, and applied to generate a quadrilateral mesh at arbitrary

Table 1. Configuration of six node domains and their decomposition

Configuration	Presence on the initial boundary	Case 1	Case 2	Case 3
Triangle	Presence			
	Absence			
Rectangle	Presence			
	Absence			
Pentagon or hexagon	Presence			
	Absence			

2D domains. They are a domain in a rugby ball shape, a square with a rhombus hole inside, and a circle with 4 circular holes inside. The mesh quality is evaluated with the internal angles of elements. If all the four angles of every element are greater than 30° and less than 150°, the mesh is acceptable for FEA [14].

4.1 Rugby ball shape

A domain in a rugby ball shape was used as a simple example to verify the quality of the mesh generated by the centering method. Fig. 8 shows the mesh for the domain. It has 64 boundary nodes, 318 nodes in total, and 351 elements. The maximum angle is 141° and the minimum angle is 42°. Therefore, the mesh is acceptable for FEA. It can be seen from Fig. 8 that the size of the internal elements is similar to that of the boundary

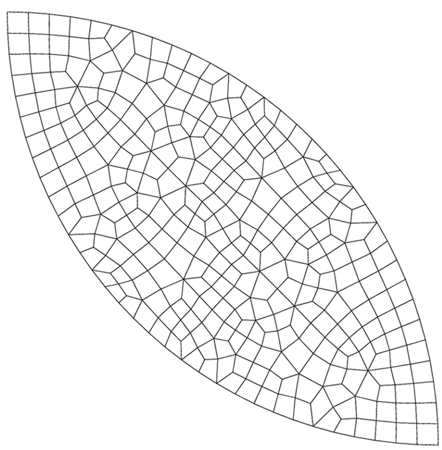


Fig. 8. Mesh generation for a domain in a rugby ball shape.

elements. It indicates that the centering method prevents elements from growing gradually smaller away from the curved concave boundary.

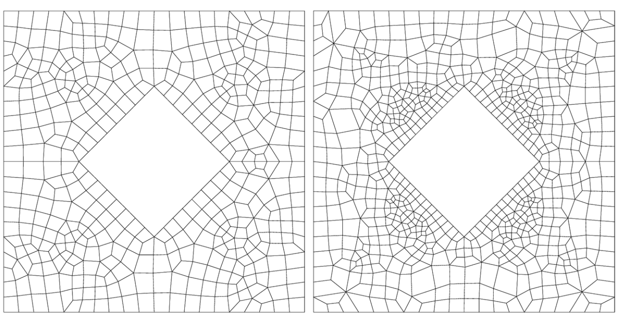
4.2 Squares with a rhombus hole inside

Fig. 9 shows two squires in the same configuration but with a different element size on the boundary. They were used as examples at an intermediate level. The hole in Fig. 9(b) has almost half of the element size of the hole in Fig. 9(a). The elements on the hole boundary are good since they have approximately 90° of the internal angles to the boundary. The centering method was intended to produce good-quality elements on the boundary of a domain.

The mesh of the domain in Fig. 9(a) has 498 nodes and 450 elements and shows that the size of the internal elements is relatively similar to that of the boundary elements. The maximum angle is 138° and the minimum angle is 47°. These angles are acceptable for FEA. The mesh in Fig. 9(b) has 819 nodes and 740 elements and shows that the centering method yields a relatively smooth transition from large elements to small ones. The maximum and the minimum angles are 147° and 36°, respectively, and therefore, the mesh is acceptable for FEA.

4.3 Circular domain with four holes inside

The circular domain in Fig. 10 was employed as a complex geometry for the application of the centering method. The centering method generated 224 boundary nodes, 1182 nodes, and 1069 elements at the domain. The maximum and the minimum angles are 160° and 22°, respectively, and therefore, the mesh is not considered acceptable for FEA. Bad elements were generated around the holes because the centering method first divided the domain at the convex boundary of the holes into narrow subdomains. Many cutting lines were first made at the nodes of the convex boundary. This decomposition produced many long narrow areas that produced bad-quality elements around the internal holes. It is necessary to make improvements, such as combination of elements and re-split, for the bad elements as a clean-up process [10,14]. However, the centering method produced a smooth transition from the external boundary and the internal boundary of the domain.



(a) (b)

Fig. 9. Square domains with a rectangular hole inside. (a) uniform element size, and (b) small element size on the hole.

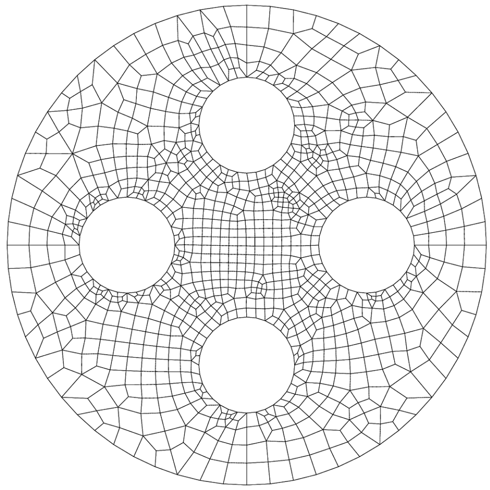


Fig. 10. Circle with four circular holes inside.

5. Conclusion

A new method, named centering method, has been developed to automatically generate a quadrilateral mesh for 2D domains in arbitrary configuration. It centers a point at a domain, selects the best cutting lines based on the criterion using the between-angles, and divides the domain into as many sub-domains as the cutting lines. This decomposition process is continued until all subdomains have 4 or 6 nodes. Each of four-node subdomain is directly converted into a quadrilateral element but 6-node subdomains are split into quadrilateral elements depending on their configuration and presence on the initial boundary of the domain. This method was designed to produce good-quality elements on the boundary of a domain and a smooth mesh transition from large elements to small ones.

The new method has been shown to be capable of quadrilateral mesh generation for the 2D domains used for the examples. It generated good boundary elements for the domain of the simple and the intermediate-level geometries. However, for the domain of the complex geometry, it generated the mesh that is considered unacceptable for FEA because some elements have a large or small internal angle due to the decomposition at convex boundaries producing long narrow subdomians and therefore, it is necessary to improve them, for example, by combination and re-split. The mesh transition was smooth for all the domains. This new method can be adapted as a mesh generator for FEA since it is capable of generate a quadrilateral mesh at 2D domains of a low, medium, or high complexity in geometry.

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