수리가능한 품목의 예방교체를 위한 주문정책

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A Spare Ordering Policy for Preventive Replacement with Repair

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Key Words : Ordering Policy, Preventive Replacement, Minimal Repair.

Abstract

This paper presents a spare ordering policy for preventive replacement with minimal repair. To analyze the ordering policy, the failure process is modeled by a non-homogeneous Poisson process. Introducing the ordering, repair, downtime, replacement costs and salvage value, we derive the expected cost effectiveness as a criterion of optimality when the lifetime and lead times for the regular and expedited orders are generally distributed random variables. It is shown that, under certain conditions, there exists a finite and unique optimum ordering time which maximizes the expected cost effectiveness. A numerical example is also included to explain the proposed model.

1. Introduction

Most of maintenance policies treated in the literature have assumed implicitly that whenever a unit is to be replaced, a new unit is immediately available (Wang 2002). If, however as is often the case, the procurement lead time is not negligible, we should consider an ordering policy to determine the ordering time for a spare. Previous works on ordering policies have assumed that the costs of preventive and corrective replacements are equal (Chien 2005, Dohi et. al. 1998, Kaio and Osaki 1978a, Kaio and Osaki 1978b, Kalpakam and Hameed 1981, Park and Park 1986, Thomas and Osaki 1978), which implies in essential that there is no particular need for preventive replacement.

This paper presents a spare ordering policy for preventive replacement. Specifically, we consider the following problem. A 1-unit system begins operating at time 0 and there may exist two kinds of failures, major and minor ones. If a major failure occurs before a scheduled ordering time t, we place an expedited order immediately at the failure time instant and replace the failed unit with the new one as soon as it is delivered. On the other hand, if no major failure occurs before t, we place a regular order for a spare at t, and replace the unit on spare's arrival. Minimal repairs are performed for minor failures until a replacement or a major failure whichever occurs earlier. The lifetime of the

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unit and lead times for the regular and expedited orders are generally distributed random variables. The time between successive replacements is a cycle and the behavior in each cycle repeats. The problem is to determine the scheduled ordering time t so as to maximize the expected cost effectiveness.

The cost effectiveness is defined as "steady-stated availability/expected cost rate" which reflects the efficiency per dollar outlay. This criterion is useful in the case that the benefits obtained from the system operation are not reducible to monetary terms as in weapon systems (Hadly and Whitin 1963). Let us first define some terminology and symbols for lucid explanation.

Definition

Minor Failure: A failure which is repairable and the repair cost dose not exceed a certain amount (; usually known as 'repair cost limit' in maintenance theory).

Major Failure: A failure which is irreparable, or repairable but the repair is economically not justified since the cost exceeds the repair cost limit.

Minimal Repair: A repair which restores a failed system to its operating condition immediately before the failure. Examples of minimal repairs include complex systems where the repair of one component does not materially affect the condition of the whole system.

Symbols

$f(x), F(x), \overline{F}(x)$: pdf, cdf, survivor function of time to failure

$h(x)$: $f(x)/\overline{F}(x)$, failure rate at age x

$H(x)$: $\int_0^x h(u)du$, cumulative hazard of time to failure

$g(x), G(x), \overline{G}(x)$: pdf, cdf, survivor function of time to major failure

$r(x)$: $g(x)/\overline{G}(x)$, major failure rate at age x

$r_l(x)$: $g(x)/\overline{G}(x+l)$

$R_l(x)$: $[G(x+l)-G(x)]/\overline{G}(x+l)$

$w(l), W(l), L$: pdf, cdf, and mean value of regular lead time

L_e: mean value of expedited lead time($;L_e \le L$)

c_e, c_r: costs of expedited and regular orders respectively($;c_e \ge c_r$)

c_c, c_p: costs of corrective and preventive replacements respectively($;c_c \ge c_p$)

c_f: average cost of a minimal repair

c_d: downtime cost per unit time due to spare shortage

ν_s: salvage value per unit time for residual lifetime

p: probability that a failure is a minor one

t: scheduled time for spare ordering

$E(t)$: expected cost effectiveness for an infinite time span

$poim(k;m)$: $m^k \exp(-m)/k!$, Poisson pmf with mean value m

Other symbols are defined when needed.

2. Preliminaries: failure process

When minimal repairs are performed to all failures, the failure process follows an NHPP (Non-Homogeneous Poisson Process). In the NHPP the number of failures in $(0,x]$,$N(x)$,follows a Poisson distribution with mean value $H(x)$ (Barlow and Hunter 1960, Nakagawa and Kowada 1983).

$$\Pr[N(x)=k] = poim(k;H(x)) \qquad (1)$$

Thus the cumulative density function of the time to nth failure, $F_n(x)$, is

$$F_n(x) = \Pr[N(x) \ge n] = \sum_{k=n}^{\infty} poim(k;H(x)) \qquad (2)$$

and the probability density function of the time to nth failure, $f(x)$, is

$$\begin{aligned} f_n(x) &= \frac{d}{dx} F_n(x) \\ &= \sum_{k=n}^{\infty} [poim((j-1);H(x)) \cdot h(x) \end{aligned}$$

$$-poim(j;H(x)) \cdot h(x)]$$
$$= poim((n-1);H(x)) \cdot h(x) \qquad (3)$$

The probability density function of time to major failure is

$$\begin{aligned} g(x) &= \sum_{k=1}^{\infty} [k\text{th failure is the first major failure}] \\ &\quad \cdot f_k(x) \\ &= \sum_{k=1}^{\infty} p^{k-1}(1-p) f_k(x) \\ &= (1-p)\sum_{k=1}^{\infty} p^{k-1} poim((k-1);H(x)) \cdot h(x) \\ &= (1-p)h(x)\exp[-(1-p)H(x)] \\ &= (1-p)f(x)\overline{F}(x)^{-p} \qquad (4) \end{aligned}$$

The cumulative density function and survivor function are

$$G(x) = \int_0^x g(u)du = 1-\overline{F}(x)^{1-p} \qquad (5)$$

and

$$\overline{G}(x) = \overline{F}(x)^{1-p} \qquad (6)$$

Hence major failure rate at age x is

$$r(x) = g(x)/\overline{G}(x) = (1-p)h(x) \qquad (7)$$

In fact, (7) is a corollary of Savits (1988) which shows a kind of decomposition property of NHPP. Let $M(x)$ be the number of minimal repairs in $(0,x]$. Then the probability mass function is

Pr[M(x)=k]=Pr[Only minor failures occur k times] +Pr[(k+1)th failure is the first major failure]

$$\begin{aligned} &= p^k \cdot \Pr[N(x)=k] + p^k(1-p) \cdot \Pr[N(x) \ge k+1] \\ &= p^k \cdot poim(k;H(x)) + p^k(1-p)\sum_{j=k+1}^{\infty} poim(j;H(x)) \\ &= p^k \cdot \sum_{j=k}^{\infty} poim(j;H(x)) - p^{k+1}\sum_{j=k+1}^{\infty} poim(j;H(x)) \end{aligned} \qquad (8)$$

Thus the expected number of minimal repairs, $E[M(x)]$,is

$$E[M(x)] = \sum_{k=1}^{\infty} k \cdot \Pr[M(x)=k] = \frac{p}{1-p}G(x) \qquad (9)$$

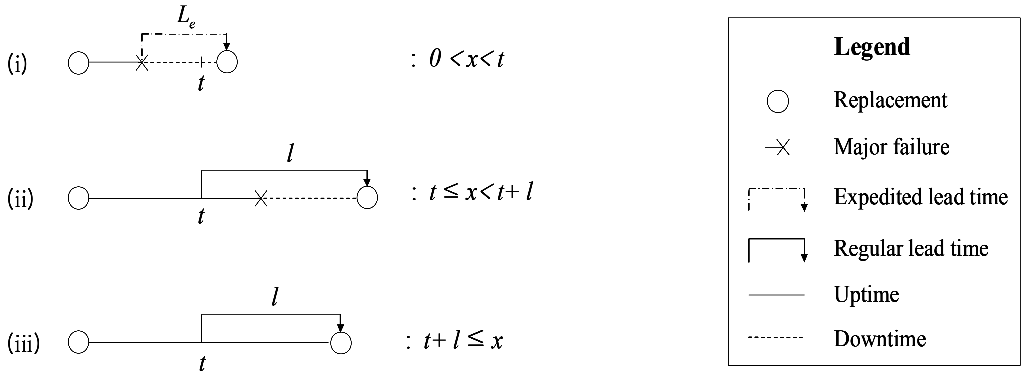
Notice that $E[M(x)]$ is the same as Cleroux, Dubuc and Tilquin (1979, p.1161).

3. Model and analysis

Since each replacement is a regeneration point, the following three mutually exclusive and exhaustive possibilities exist in every cycle (see Figure 1):

(i) A major failure occurs before the scheduled ordering time t.
(ii) A major failure occurs between t and the arrival of the ordered spare.
(iii) No major failure occurs before the arrival of the ordered spare.

The expected cycle length is



<Figure 1> Possible realizations of one cycle.

$$\int_0^t (x+L_e)g(x)dx + \int_0^\infty \int_t^\infty (t+l)g(x)w(l)dxdl$$
$$= L-(L-L_e)G(t) + \int_0^t \overline{G}(x)dx \quad (10)$$

Downtime occurs in the cases (i) and (ii), and the expected downtime per cycle is

$$L_e\int_0^t g(x)dx + \int_0^\infty \int_t^{t+l}(t+l-x)g(x)w(l)dxdl$$
$$= \int_0^\infty \int_t^{t+l} G(x)w(l)dxdl - (L-L_e)G(t) \quad (11)$$

Since uptime per cycle is cycle length minus downtime, the expected uptime per cycle is

$$U(t) = L + \int_0^t G(x)dx - \int_0^\infty \int_t^{t+l} G(x)w(l)dxdl$$
$$= \int_0^\infty \int_0^{t+l} \overline{G}(x)w(l)dxdl \quad (12)$$

The expected cost per cycle is the sum of the ordering, repair, downtime, replacement costs and salvage value. The expected ordering cost per cycle is.

$$c_e\int_0^t g(x)dx + c_r\int_t^\infty g(x)dx = c_r + (c_e - c_r)G(t) \quad (13)$$

From (9), the expected repair cost per cycle is

$$c_f\{p/(1-p)\}\int_0^\infty G(t+l)w(l)dl \quad (14)$$

From (11), the expected downtime cost per cycle is

$$c_d\left[\int_0^\infty \int_t^{t+l} G(x)w(l)dxdl - (L-L_e)G(t)\right] \quad (15)$$

Corrective replacement occurs in the cases (i), (ii), and preventive replacement occurs in the case (iii). Thus the expected replacement cost per cycle is

$$c_c[\int_0^t g(x)dx + \int_0^\infty \int_t^{t+1} g(x)w(l)dxdl]$$
$$+ c_p\int_0^\infty \overline{G}(t+l)w(l)dl$$
$$= c_p + (c_c - c_p)\int_0^\infty G(t+l)w(l)dl \quad (16)$$

It seems reasonable that salvage value of a used unit, which is still operable, is proportional to the expected residual lifetime (Kaio and Osaki 1978b). Salvage value occurs in the case (iii), and the expected salvage value per cycle is

$$v_s\int_0^\infty \int_{t+l}^\infty (x-t-l)g(x)w(l)dxdl$$
$$= v_s\int_0^\infty \int_{t+l}^\infty \overline{G}(x)w(l)dxdl \quad (17)$$

Thus the expected cost per cycle is

$$C(t) = (c_r + c_p) + \{(c_e - c_r) - c_d(L-L_e)\}G(t)$$
$$+ \{(c_c - c_p) + c_f p/(1-p)\}\int_0^\infty G(t+l)w(l)dl$$
$$+ c_d\int_0^\infty \int_t^{t+l} G(x)w(l)dxdl$$
$$- \nu_s\int_0^\infty \int_{t+l}^\infty \overline{G}(x)w(l)dxdl \quad (18)$$

Since the behavior in each cycle repeats, the cost effectiveness, “steady-stated availability/expected cost rate”, can be rewritten as “expected uptime in a cycle/expected cost per cycle”. Thus, the expected cost effectiveness is

$$E(t) = U(t)/C(t) \quad (19)$$

where
$U(t)$ and $C(t)$ are given by (12) and (18) respectively.

To examine the optimum ordering policy, the followings are assumed.

(a) $h(x)$ is strictly increasing (that is, the failure rate of a unit increases as it gets old).

(b) The expected expedited lead time is smaller than that of regular lead time, but it costs more (i.e., $L_e \le L$ and $c_e \ge c_r$).

Lemma. $r_l(x)$ and $R_l(x)$ are strictly increasing if $h(x)$ is strictly increasing.

Proof. Since $r(x)$ and $h(x)$ have the same monotone properties from (7), strictly IFR (increasing failure rate) means

$$r(x+l)-r(x)>0 \tag{20}$$

and

$$r'(x)=[g'(x)+g(x)r(x)]/\overline{G}(x)>0 \tag{21}$$

From (20) and (21),

$$r_l'(x)=\frac{g'(x)+g(x)r(x+l)}{\overline{G}(x+l)}>\frac{g'(x)+g(x)r(x)}{\overline{G}(x+l)}>0 \tag{22}$$

and

$$R_l'(x)=\frac{\overline{G}(x)}{\overline{G}(x+l)}[r(x+l)-r(x)]>0$$
$$\text{for all } x\geq 0 \text{ and } l\geq 0. \tag{23}$$

Thus, $r_l(x)$ and $R_l(x)$ are strictly increasing functions.

From the lemma, we have the following theorem regarding the optimum ordering time t^* which ma-imizes $E(t)$.

Theorem 1. Suppose that $(c_e-c_r)\geq c_d(L-L_e)$.

(i) If $p(0)\leq 0$, then the optimum ordering time $t^*=0$ i.e., place a regular order at the same instant when a unit is put in service and never place an expedited order.

(ii) If $p(0)>0$ and $P(\infty)<0$then there exists a finite and unique optimum ordering time $t^*(0<t^*<\infty)$ satisfying $p(t^*)=0$.

(iii) If $p(\infty)\geq 0$,then the optimum ordering time $t^*=\infty$, i.e., place an expedited order at the instant of failure and never place a regular order.

Proof. Define the numerator of the derivative of $E(t)$ in (19) divided by $\overline{G}(t+l)$ as

$$p(t)=C(t)-U(t)[\{(c_e-c_r)-c_d(L-L_e)\}r_l(t)$$
$$+\{(c_c-c_p)+c_fp/(1-p)\}\int_0^\infty r(t+l)w(l)dl$$
$$+c_d\int_0^\infty R_l(t)w(l)dl+\nu_s] \tag{24}$$

Differentiating $E(t)$ with respect to t and setting it equal to zero implies $p(t)=0$. Further,

$$p'(t)=-U(t)[\{(c_e-c_r)-c_d(L-L_e)\}r_l'(t)$$
$$+\{(c_c-c_p)+c_fp/(1-p)\}\int_0^\infty r'(t+l)w(l)dl$$
$$+c_d\int_0^\infty R_l'(t)w(l)dl] \tag{25}$$

Since $r_l(t)$ and $R_l(t)$ are strictly increasing, $p(t)$ is strictly decreasing. Thus, the existence of t^* in the theorem follows trivially.

A sufficient condition for the optimality in the theorem, $(c_e-c_r)>c_d(L-L_e)$, has been widely used in spare ordering policies (Chien 2005, Dohi et. al. 1998, Kaio and Osaki 1978a, Kaio and Osaki 1978b, Kalpakam and Hameed 1981). However it should be noted that the assumption does not economically justify placing an expedited order since the additional cost for the expedition(c_e-c_r) is larger than the savings obtained from the expedition $c_d(L-L_e)$. Hence it is meaningful only when there exist such intangibles as loss of goodwill, reputation and credit which are difficult to be quantified and included in downtime cost.

4. Numerical example

For the purpose of illustration, let us consider the following case: Both the lifetime and regular lead time are gamma distributed with integer modulus.

Lifetime cdf

$$F(t)=1-[1+0.01t+(0.01t)^2/2]\exp(-0.01t)$$

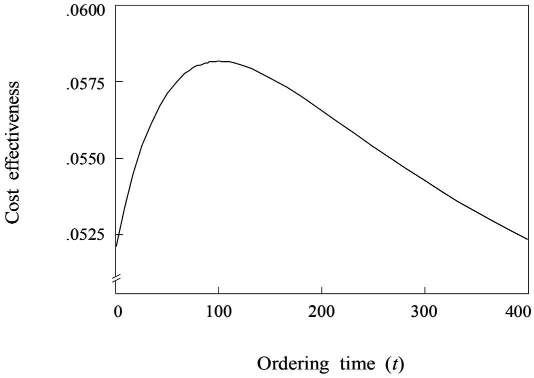
Regular lead time cdf

$$W(l)=1-(1+0.02l)\exp(-0.02l),$$
where mean $L=100$.

Expedited lead time follows any general distribution with mean $L_e=50$.

The probability of minor failure $p=0.7$, and the cost parameters are $c_e=\$8000$, $c_r=\$6000$, $c_c=\$3000$, $c_p=\$1000$, $c_f=\$100$, $c_d=\$50$, $\nu_s=\$10$.

Figure 2 shows how the expected cost effectiveness $E(t)$ changes with respect to the scheduled ordering time t .The optimal policy is placing a regular order at $t^* = 100$ and the corresponding cost effectiveness is $E^*(t) = .0576$.



<Figure 2> Cost effectiveness as function of ordering time t_0.

5. Concluding remarks

A spare ordering policy for preventive replacement with minimal repair is proposed in this paper. In general, there are three interrelated policies (; ordering, stocking, and maintenance policies) to be considered in the joint optimization of maintenance and spares inventory models.

The joint optimization of the three policies is an important future research area.

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