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Effect of Cutout Orientation on Stress Concentration of Perforated Plates with Various Cutouts and Bluntness†

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Abstract

Perforated plates with cutouts (or holes) are widely used in structural members. These cutouts provide stress concentration in plates. Extensive studies have been carried out on stress concentration in perforated plates, which consider cutout shapes, boundary conditions, bluntness of cutouts, and more. This study presents stress concentration analyses of perforated plates with not only various cutouts and bluntness but also different cutout orientations. Especially, the effect of cutout orientation on stress concentration is emphasized since structural members have become more complicated recently. To obtain stress concentration patterns, a finite element program, ANSYS, is used. For the designated goal, three parameters are considered as follows: the shapes of polygonal cutouts (circle, triangle, and square), bluntness (a counter measure of radius ratio, r/R), and rotation of cutouts (θ). From the analyses, it is shown that, in general, as bluntness increases, the stress concentration increases, regardless of the shape and rotation. A more important finding is that the stress concentration increases as the cutouts become more oriented from the baseline, which is the positive horizontal axis ($+x$). This fact demonstrates that the orientation is also a relatively significant design factor to reduce stress concentration. In detail, in the case of the triangle cutout, orienting one side of the triangle cutout to be perpendicular to the applied tensile forces is preferable. Similarly, in the case of the square cutout, it is more advantageous to orient two sides of square cutout to be perpendicular to the applied tensile force. Therefore, at the design stage, determining the direction of a major tensile force is required. Then, by aligning those polygon cutouts properly, we can reduce stress concentration.

Keywords: Stress concentration analysis, Perforated plates, Radius of curvature, Finite element analysis, Cutout orientation

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1. Introduction

In general, plates are easily manufactured and are widely used for fabrication of structural members and eventually for the construction of marine and civil structures. Especially, a bolt-connection is a way of fastening plates by inserting bolts into the cutouts (or holes), which are made through designated areas of target plates that are to be connected. Making cutouts is not merely for connecting but also for reducing the weight of structural members. This type of connection causes stress concentration near the cutouts, causing high stresses, and sometimes results in the failure of structural members. To quantify the degree of stress concentration, the concept of stress concentration factor (SCF) is used, which is defined by the ratio of the maximum stress to nominal stress.

Most research on stress concentration focuses on the structural members that are mostly subject to (or weakened by) stress concentration [1-3]. Among such research, the work on stress concentration on cutouts include: analytical works for flat plates with circular holes and notches [4], optimum design of holes and notches by considering fatigue life [5], stress concentration analysis of differently materia-

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lized circular holes [6], etc. These works mainly focus on stress concentration on circularly-perforated plates.

In reality, other cutout shapes, rather than circles, are widely used for plate-like structures. In the polygonal cases, it is known that significant design variables or factors are edge bluntness and rotation. Bluntness can be indirectly defined by the ratio (*r/R*) of edge radius (*r*) to inscribing circle radius (*R*). However, it should be noted here that bluntness is a counter measure to the radius ratio (*r/R*) because bluntness decreases as the radius ratio increases. For an extreme example, a circular cutout has a unit radius ratio but it has zero bluntness.

A number of previous works have been carried out on bluntness, including a study done by Rezaeepazhand and Jafari (2005) who focuses on composite plates [7]. They consider bluntness effect on stress concentration in perforated composite plates. However, it seems to be difficult to locate a work that quantifies the rotation effect of polygonal cutouts on stress concentration. Therefore, this study mainly focuses on stress concentration analyses of steel plates according to cutout orientation. For the analyses, firstly, we select three different cutouts: circle, triangle, and square; secondly, we identify a number of degrees of bluntness to describe the radius ratio; and finally, we consider the rotation of cutouts. In the paper, stress concentration analyses are performed by ANSYS, a general purpose finite element program. From the analysis results, we estimate stress concentration pattern and factor.

2. Finite element model

Finite element analyses are conducted for the stress concentration analyses of perforated steel plates. The structural steel plates have dimensions 200 mm (*x*-direction), 200 mm (*y*-direction), and 5 mm (*z*-direction) as shown in Fig. 1. Material properties are shown in Table 1 and the location of cutout is the center of the plates. To clearly observe the concentration effect, the plate size is modeled as rather large for the cutout size.

ANSYS, a general purpose finite element program, is used. An eight-node solid element is used for modeling. To investigate stress concentration in an elastic range, the plates are modeled as a linear elastic material. The loading condition is a uni-axial tensile force at the left and right sides as shown in Fig. 1. Based on Rezaeepazhand and Jafari (2005), stress concentration reaches up to eleven times, depending on cutout shapes; hence, in the study, to limit the maximum stress to the elastic range, 20 MPa is loaded as the tensile loading condition. Since element size is critical for precise analysis, in the study, the size is 2 mm in most parts and 0.5 mm near the cutout areas.

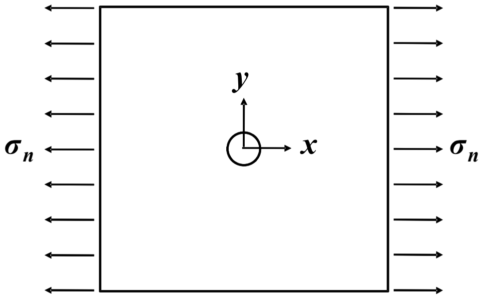

Fig. 1. Loading condition: uni-axial tensile force

Table 1. Material properties of structural steel

Young's modulus (GPa)	200
Poisson ratio	0.3
Tensile yield strength (MPa)	250
Tensile ultimate strength (MPa)	460

3. Cutout shapes, bluntness, and rotation

We consider three cutout shapes – circle, square, and regular triangle. For the square and triangle cutouts the concept of inscribing circle is used, as shown in Figs. 2 and 3, to compare with the corresponding circular cutout. In the figures, the solid-lined circles are the inscribing circles in the polygons. The radius size of the circular cutout is 10 mm.

In general, to reduce the stress concentration at the edges of cutouts, the edges are fabricated to be rounded. In the study, rather than 'roundness', we use 'bluntness' as a physical terminology to effectively describe stress concentration. As shown in Fig. 4, a term 'radius ratio' is defined as the ratio of the edge radius (*r*) to the inscribing circle radius (*R*). Accordingly, bluntness is a counter measure to the radius ratio (*r/R*) because bluntness decreases as the radius ratio increases. For an extreme example, a circular cutout has a unit radius ratio but it has zero bluntness. In other words, the degree of bluntness decreases as *r/R* increases. Here, again, we emphasize that the term ‘bluntness’ is used to describe that the edges of polygons are blunt. We consider a total of six different degrees of bluntness, including 0.1, 0.3, 0.5, 0.7, 0.9, and 1.0 for the polygon cutouts. Figs. 2 and 3 only show three of the six cases for the square and triangle cutouts.

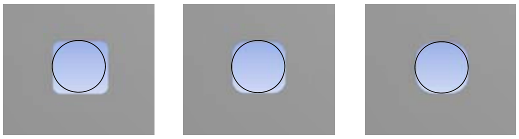

Fig. 2. Square cutout with *r/R* = 0.3 (left), 0.5 (center), 0.7 (right)

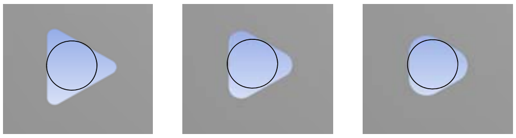

Fig. 3. Triangle cutout with *r/R* = 0.3 (left), 0.5 (center), 0.7 (right)

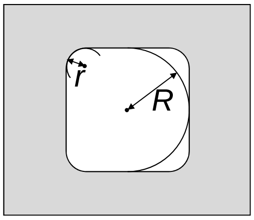

Fig. 4. Radius ratio (*r/R*) defined by edge radius (*r*) and inscribing circle radius (*R*)

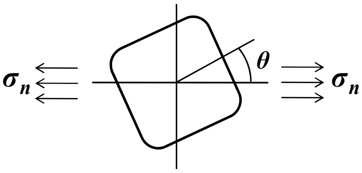

Fig. 5. Rotation of cutout

In addition to the shapes and bluntness, the last design consideration for cutout patterns is orientation. Fig. 5 shows the definition of orientation. The rotation angle θ represents how the cutouts are oriented from the baseline (+*x* axis). As shown in the figure, the loading directions are fixed as they are. Figs. 6 and 7 show a number of parts of the rotated cutouts for each case. By considering the symmetry of the polygonal cutouts, the angle increment 15° is applied; hence, a total of four cases are considered (0°, 15°, 30°, and 45°) for the square cutouts and three cases (0°, 15°, 30°) for the square cutouts.

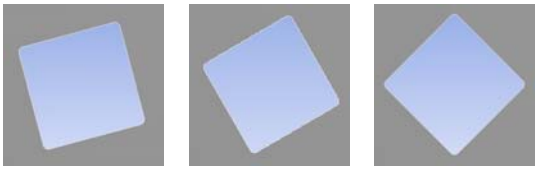

Fig. 6. Square cutout with θ = 15° (left), 30° (center), and θ = 45° (right)

Fig. 7. Triangle cutout with θ = 0° (left), 15° (center), and θ = 30° (right)

4. Results

By considering the design variables or factors – cutout shape, the degree of bluntness, and cutout rotation – we obtained the stress concentration pattern, the maximum von-Mises stress, and the stress concentration factor. These results are as shown in the following sections.

4.1 Cutout shapes and bluntness

As mentioned previously, there are three different cutout shapes – circle, square, and triangle. In addition, for considering bluntness (a counter measure of *r/R*), a total of six radius ratios are considered: *r/R* = 0.1, 0.3, 0.5, 0.7, 0.9, and 1.0, respectively. This section discusses the variation of stress concentration with respect to the cutout shapes and bluntness. All of the other factors remain the same, for example the uni-axial tensile forces are fixed at 20 MPa.

Table 2 shows the maximum von-Mises stress and stress concentration factor. It should be noted here that the zero bluntness (*r/R* = 1) actually means that the cutout shape is a circle; hence, from the table, we can see how the shapes and the degrees of bluntness vary the Maximum von-Mises stress and stress concentration factor. Fig. 8 shows how the stress concentration factor (SCF) varies with respect to cutout shapes and the radius ratio (a counter measure of degree of bluntness).

Table 2. Maximum von-Mises stress and stress concentration factor with respect to bluntness

r/R	Square cutout		Triangle cutout	
	(MPa)	(SCF)	(MPa)	(SCF)
0.1	63.33	3.17	131.56	6.58
0.3	68.21	3.41	96.77	4.84
0.5	57.16	2.86	82.42	4.12
0.7	56.66	2.83	75.35	3.77
0.9	56.50	2.83	65.17	3.26
1	60.26	3.01	60.26	3.02

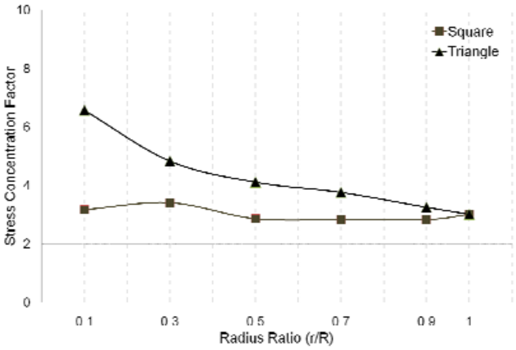

Fig. 8. SCF with respect to radius ratio

In the case of the circular cutout, the maximum stress is 60.26 MPa and the stress concentration factor is 3.01. According to previous studies, the maximum stress is about three times the tensile force [8]. Since our tensile force is 20 MPa, the magnitude of 60.26 MPa exactly concurs with the previous observation. As shown in Table 2, the maximum von-Mises stresses and accordingly stress concentration factors change, depending on the cutout shapes and bluntness.

In the case of the square cutouts, although the quantities range between 56.50 and 63.33 MPa, they do not significantly differ from 60.26 MPa, which is the maximum von-Mises stress that occurred in the circularly-perforated steel plate. It is interesting to note that: (1) the stresses for *r/R* = 0.5, 0.7, and 0.9 are smaller than that of *r/R* = 1.0 which is the circular cutout case, and (2) the maximum stress (68.21 MPa) occurs in the case of *r/R* = 0.3.

In the case of the triangle cutouts, the results are quite consistent because: (1) all the stresses exceed that of the circular cutout case, and (2) unlike the square cases, starting from the maximum stress (131.55 MPa) the stresses decrease as the degrees of radius ratio increases. In other words, the stresses increase as the degree of bluntness increases.

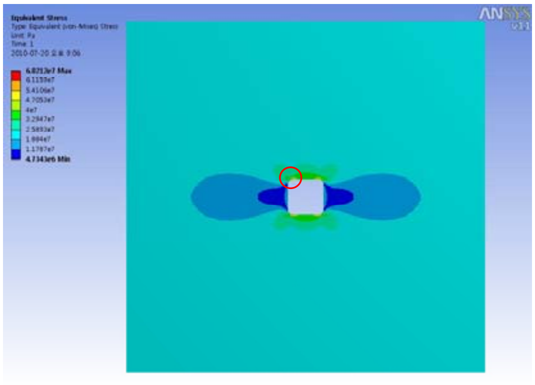

Fig. 9. Stress contour of plate with square cutout (*r/R* = 0.3)

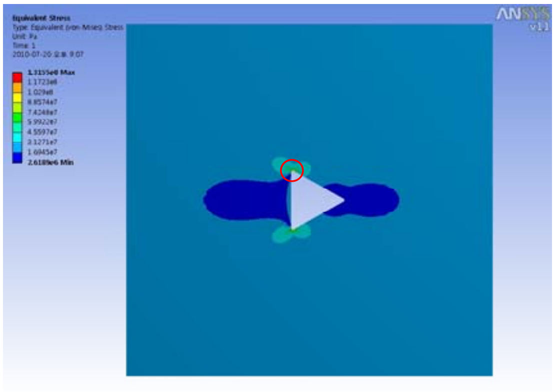

Fig. 10. Stress contour of plate with triangle cutout (*r/R* = 0.1)

To visualize the stress patterns, two stress contours are shown in Figs. 9 and 10. Fig. 9 shows the stress contour in the case of the square cutout with *r/R* = 0.3. The circle on the contour indicates the area having the maximum von-Mises stress. In addition, the left and right balloon shapes represent the areas under 11 MPa. Fig. 10 shows the stress contour in the case of the triangle cutout with *r/R* = 0.1. The circle on the contour shows the area having the maximum von-Mises stress. Similarly, the left and right balloon shapes represent the area under 16 MPa. It is interesting to note that stress concentration occurs in the broad range of the top and bottom sides in the case of the square cutout while stress concentration occurs in the narrow range of the top and bottom edges in the case of the triangle cutout.

From the observation, we can conclude that the bluntness effect on the stress concentration patterns is also dependent on cutout shapes. However, in general, as bluntness increases, stress concentration increases.

4.2 Rotation of cutouts

This section discusses the stress analysis results by considering the rotation of the cutouts. In the cases of the square cutout, four rotation angles are considered, 0°, 15°, 30°, and 45°, while three angles, 0°, 15°, and 30°, are considered in the case of the triangle cutout.

Table 3 shows the maximum von-Mises stresses and stress concentration factors for the steel plates with square cutouts, which have the four rotations. As a result, we can see that many differences occur in the maximum stresses, depending on the rotation angle. However, for all of the cases consistently, the stresses increase as the rotation angles increase. By combining the rotation effect with the bluntness effect, the maximum stress (132.48 MPa) occurs in the case of the *r/R* = 0.1 (maximum bluntness) and the rotation of 45° (maximum rotation). In addition, we can see that with the exception of the zero rotation case, all the cases show that the maximum stress increases as the bluntness increases, as shown in Fig. 11.

Table 4 shows the maximum von-Mises stresses and stress concentration factors for the steel plates with triangle cutouts, which have the three rotations. For all of the cases, the stresses increase as the rotation angles increase, as clearly shown in Fig. 12. With both effects of the rotation angle and bluntness, the maximum stress (163.53 MPa) occurs in the case of the bluntness of *r/R* = 0.1 (maximum bluntness) and the rotation of 30° (maximum rotation).

Fig. 13 shows the stress contour in the case of the square cutout with *r/R* = 0.1 and rotation 45°, which gives the maximum stress. This figure represents different patterns from that of Fig. 9 showing the case of 0°. The maximum stress concentration occurs in the top and bottom edges. Fig. 14 shows the stress contour in the case of the triangle cutout with *r/R* = 0.1 and rotation 30°, which also gives the maximum stress. The maximum stress concentration occurs in the top edge.

Table 3. Maximum von-Mises stress and stress concentration factor (SCF) of square cutouts with rotation angle

r/R	0° (MPa)	15° (MPa)	30° (MPa)	45° (MPa)
0.1	63.33	91.13	126.17	132.48
0.3	68.21	89.45	100.15	97.41
0.5	57.16	71.67	83.68	84.47
0.7	56.66	64.15	70.07	70.11
0.9	56.50	60.83	63.48	63.98
1	60.26	60.26	60.26	60.26
r/R	0° (SCF)	15° (SCF)	30° (SCF)	45° (SCF)
0.1	3.17	4.56	6.31	6.62
0.3	3.41	4.47	5.01	4.87
0.5	2.86	3.58	4.18	4.22
0.7	2.83	3.21	3.50	3.51
0.9	2.83	3.04	3.17	3.20
1	3.01	3.01	3.01	3.01

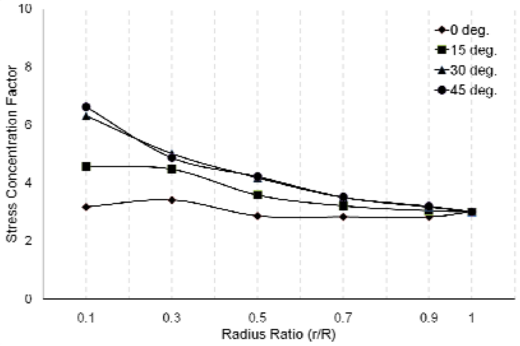

Fig. 11. SCF with respect to rotation for square cutouts

From the results (see Figs. 9, 10, 13, and 14), in the case of the square cutout, it is more advantageous to orient two sides of the square cutout to be perpendicular to the applied tensile force because this reduces the maximum stress. For example, in the case of square cutouts with *r/R* = 0.1, the maximum stress decreases from 132.48 (θ = 45°) to 63.33 MPa (θ = 0°), which is a 69.15 MPa or 209% decrease. Similarly, in the case of the triangle cutout, it is also preferable to orient one side of the triangle cutout to be perpendicular to the applied tensile forces because of stress reduction. For example, in the case of triangle cutouts with *r/R* = 0.1, the maximum stress decreases from 163.53 (*θ* = 45°) to 131.55 MPa (*θ* = 0°), which is a 31.98 MPa or 124% decrease. Accordingly, at the design stage, determining the direction of a major tensile force is required. By aligning these polygon cutouts as observed here, we can then reduce stress concentration.

Table 4. Maximum von-Mises stress and stress concentration factor (SCF) of triangle cutouts with rotation angle

r/R	0° (MPa)	15° (MPa)	30° (MPa)	45° (MPa)
0.1	131.55	150.66	163.53	N/A
0.3	96.77	99.89	113.14	
0.5	82.42	84.89	84.07	
0.7	75.35	73.93	74.46	
0.9	65.17	62.66	63.80	
1	60.26	60.26	60.26	

r/R	0° (SCF)	15° (SCF)	30° (SCF)	45° (SCF)
0.1	6.58	7.53	8.18	N/A
0.3	4.84	4.99	5.66	
0.5	4.12	4.24	4.20	
0.7	3.77	3.70	3.72	
0.9	3.26	3.13	3.19	
1	3.01	3.01	3.01	

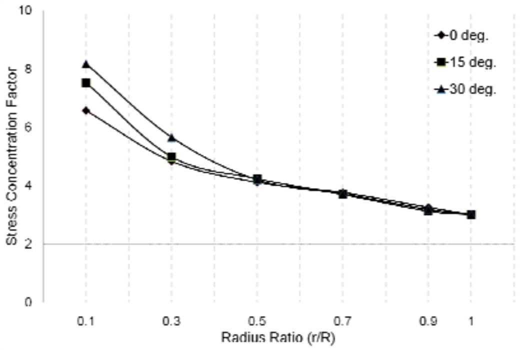

Fig. 12. SCF with respect to rotation for triangle cutouts

In addition to rotation, similarly to the previous section, for all the degrees of orientation, it is also observed that the stress concentration decreases as the bluntness of the cutouts decreases. For example, in the case of square cutouts with 45° rotation, the maximum stress decreases from 132.48 (*r/R* = 0.1) to 63.98 MPa (*r/R* = 0.9), which is a 68.50 MPa or 207% decrease. Similarly, in the case of triangle cutouts with 30° rotation, the maximum stress decreases from 163.53 (*r/R* = 0.1) to 63.80 MPa (*r/R* = 0.9) with a 99.73 MPa or 256% decrease.

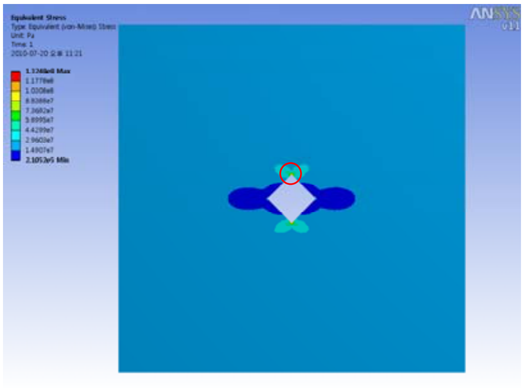

Fig. 13. Stress contour for square cutout (*r/R* = 0.1, *θ* = 45°)

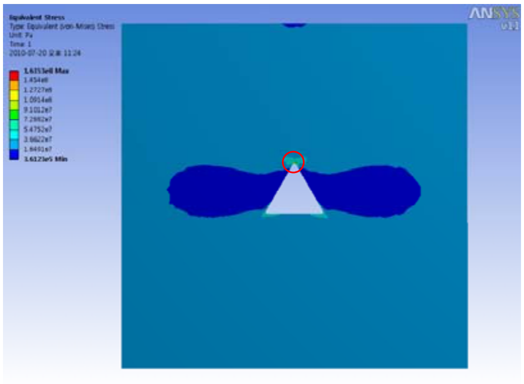

Fig. 14. Stress contour for triangle cutout (*r/R* = 0.1, *θ* = 30°)

Therefore, the next question among these two factors (rotation and bluntness) is which factor should preferably be controlled to minimize the stress concentration. Based on Tables 3 and 4, and Figs. 11 and 12, we can clearly see that bluntness is a more effective factor. For example, as the bluntness approaches zero (*r/R* approached to unit), the maximum stresses tend to converge to 60.26 MPa (the maximum stress in the case of circular cutout) and naturally the rotation effect vanishes. However, this does not reflect the manufacturing ease and cost. In a sense, it may be preferable to control orientation to reduce the manufacturing costs and cumbersome manufacturing. Therefore, at this analysis stage, the judgment should be handed over.

In summary, to minimize the stress concentration of the steel plates with polygon cutouts, the cutouts should have smooth edges and proper rotations. In other words, by controlling the smoothness (or bluntness) and rotation, we can minimize the stress concentration of the perforated steel plates. Among bluntness and rotation, controlling bluntness is analytically preferable to minimize the stress concentration.

5. Conclusions

This study presents stress concentration analyses of perforated steel plates with various shapes, bluntness, and rotation of polygonal cutouts. For the analysis we intentionally limit resulting stresses in an elastic range by controlling the applied uniaxial tensile forces. We observed that the maximum stress in the perforated steel plate with the circular cutout is about three times the applied force; hence, the previous observation performed by Pilkey et al. (2008) is verified. From the finite element analyses, the following findings are reported. Depending on cutout shapes, bluntness and rotation effects on stress concentration vary. However, in general, as bluntness increases, the stress concentration increases, regardless of the shape and rotation. A more important finding is that the stress concentration increases as the cutouts become more oriented from the baseline, which is the positive horizontal axis ($+x$) and one of the directions of the applied tensile forces. This fact demonstrates that the orientation is also a relatively significant design factor to reduce stress concentration. In general, in the case of the triangle cutout, it is preferable to orient one side of the triangle cutout to be perpendicular to the applied tensile forces. Similarly, in the case of the square cutout, it is more advantageous to orient two sides of square cutout to be perpendicular to the applied tensile force. Therefore, at the design stage, determining the direction of a major tensile force is required. By aligning these polygon cutouts properly, we can then reduce stress concentration. This finding is mainly for uni-axial tensile forces in an elastic range. Other cases such as uni-axial compressive forces and bi-axial tensile and/or compressive forces should be considered for the future work. In addition, stress concentration analyses in a non-elastic range could be an interesting topic for future work.

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