한국지능시스템학회 논문지 2011, Vol. 21, No. 3, pp. 389-394
DOI:10.5391/JKIIS.2011.21.3.389

# Generalized Intuitionistic Fuzzy Soft Sets

**Jin Han Park[1], Young Chel Kwun[2], Jinsoo Hwang[3]**

[1] **Department of Applied Mathematics, Pukyong National University, Busan 608-737, Korea**
[2] **Department of Mathematics, Dong-A University, Busan 604-714, Korea**
[3] **Department of Mathematics Education, Daegu University, Gyeongsan, Gyeongbuk 712-714, Korea**

**Abstract**

The notion of generalized intuitionistic fuzzy soft set theory is proposed. Our generalized intuitionistic fuzzy soft set theory is a combination of the generalized intuitionistic fuzzy set theory and the soft set theory. In other words, our generalized intuitionistic fuzzy soft set theory is an extension of the intuitionistic fuzzy soft set theory. The complement, "and" and "or" operations are defined on the generalized intuitionistic fuzzy soft sets. Their basic properties for the generalized intuitionistic fuzzy soft sets are also presented and discussed.

**Key words** : Soft sets, Generalized intuitionistic fuzzy soft sets, Operations.

## 1. Introduction

The theories such as probability theory [1], fuzzy set theory [2, 3], intuitionistic fuzzy set theory [4, 5], vague set theory [6] and rough set theory [7], which can be considered as mathematical tools for dealing with uncertainties, have their inherent difficulties (see [8]). The reason for these difficulties is possibly the inadequacy of parameterization tool of the theories. Molodtsov [8] introduced soft sets as a mathematical tool for dealing with uncertainties which is free from the above-mentioned difficulties. Since the soft set theory offers mathematical tool for dealing with uncertain, fuzzy and not clearly defined objects, it has a rich potential for applications to problems in real life situation. The concept and basic properties of soft set theory are presented in [8, 9]. He also showed how soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory and game theory. However, several assertions presented by Maji et al. [9] are not true in general [10].

Maji et al. [12] presented the concept of fuzzy soft sets which is based on a combination of the fuzzy sets and soft set models. Roy and Maji [13] provided its properties and an application in decision making under imprecise environment. Kong et al. [14] argued that the Roy-Maji method [13] was incorrect and presented a revised algorithm. Zou and Xiao [15] used soft sets and fuzzy soft sets to develop the data analysis approaches under incomplete environment, respectively. Xu et al. [16] introduced the notion of vague soft sets which is an extension to soft sets and is based on a combination of vague sets and soft set models. Majumdar and Samanta [17] further generalized the concept of fuzzy soft sets, in the other words, a degree is attached with the parameterization of fuzzy sets while defining a fuzzy soft set. Maji and his coworker [18, 19, 20] introduced the notion of intuitionistic fuzzy soft set theory which is based on a combination of the intuitionistic fuzzy sets and soft set models and studied the properties of intuitionistic fuzzy soft sets.

The purpose of this paper is to combine the generalized intuitionistic fuzzy sets [21] and soft sets [8], from which we can obtain a new soft set model: generalized intuitionistic fuzzy soft set theory. Intuitively, generalized intuitionistic fuzzy soft set theory presented in this paper is an extension of the intuitionistic fuzzy soft set theory. The rest of this paper is organized as follows. The following section briefly reviews some background on soft sets, fuzzy soft sets and intuitionistic fuzzy soft sets. In Section 3, we propose the concepts and operations of generalized intuitionistic fuzzy soft sets and

접수일자 : 2011년 3월 19일
완료일자 : 2011년 5월 21일
Corresponding Author : Young Chel Kwun
본 논문은 본 학회 2011년도 춘계학술대회에서 선정된 우수논문입니다.

discuss their properties in detail.

## 2. Preliminaries

Before introduce the notion of intuitionistic fuzzy soft sets, let us give the concept of intuitionistic fuzzy sets [4, 5]. Let $E$ be a fixed set. An intuitionistic fuzzy set (IFS) in $E$ is an object having the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in E\}$, where the functions $\mu_A : E \to [0,1]$ and $\gamma_A : E \to [0,1]$ define the degree of membership and the degree of non-membership respectively of the element $x \in E$ to the set $A$ and satisfy the condition $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for any $x \in E$.

**Definition 2.1.** [18] Consider $U$ and $E$ as a universe set and a set of parameters, respectively. $\mathcal{IF}(U)$ denotes the set of all intuitionistic fuzzy sets of $U$. Let $A \subseteq E$. A pair $\langle F, A\rangle$ is called an intuitionistic fuzzy soft set over $U$, where $F$ is a mapping given by $F : A \to \mathcal{IF}(U)$.

In the following, we will introduce the notion of interval-valued fuzzy soft sets [22]. Let us briefly introduce the concept of the interval-valued fuzzy sets [23]. An interval-valued fuzzy set $X$ on a universe $U$ is a mapping such that $X : U \to \mathrm{Int}([0,1])$, where $\mathrm{Int}([0,1])$ stands for the set of all closed subintervals of $[0,1]$. The set of all interval-valued fuzzy sets on $U$ is denoted by $\mathcal{IVF}(U)$.

For $X \in \mathcal{IVF}(U)$, $\mu_X(x) = [\mu_X^-(x), \mu_X^+(x)]$ is called the degree of membership of an element $x$ to $X$. Here, $\mu_X^-(x)$ and $\mu_X^+(x)$ are referred to as the lower and upper degrees of membership of $x$ to $X$ and $0 \le \mu_X^-(x) \le \mu_X^+(x) \le 1$.

**Definition 2.2.** Let $U$ be an initial universe and $E$ be a set of parameters. $\mathcal{IVF}(U)$ denotes the set of all interval-valued fuzzy sets of $U$. Let $A \subseteq E$. A pair $\langle F, A\rangle$ is called an interval-valued fuzzy soft set over $U$, where $F$ is a mapping given by $F : A \to \mathcal{IVF}(U)$.

**Remark 2.3.** The equivalence between the structures of interval-valued fuzzy sets and intuitionistic fuzzy sets was proved by Deschrijver and Kerre [24]. That is, there exists an isomorphism from $\mathcal{IF}(U)$ to $\mathcal{IVF}(U)$. Then, for any parameter $\varepsilon \in A$, there exist maps $f$ and $g$ such that

(a) $f$ assigns to every interval-valued fuzzy set $F(\varepsilon) = \{\langle x, [\mu_{F(\varepsilon)}^-(x), \mu_{F(\varepsilon)}^+(x)]\rangle : x \in U\}$ an intuitionistic fuzzy set $H(\varepsilon) = f(F(\varepsilon))$ given by

$$\mu_{H(\varepsilon)}(x) = \mu_{F(\varepsilon)}^-(x) \text{ and } \gamma_{H(\varepsilon)}(x) = 1 - \mu_{F(\varepsilon)}^+(x).$$

(b) $g$ assigns to every intuitionistic fuzzy set $H(\varepsilon) = \{\langle x, \mu_{H(\varepsilon)}(x), \gamma_{H(\varepsilon)}(x)\rangle : x \in U\}$ an interval-valued fuzzy set $F(\varepsilon) = g(H(\varepsilon))$ given by

$$\mu_{F(\varepsilon)}^-(x) = \mu_{H(\varepsilon)}(x) \text{ and } \mu_{H(\varepsilon)}^+(x) = 1 - \gamma_{F(\varepsilon)}(x).$$

Thus intuitionistic fuzzy soft set and interval-valued fuzzy soft set are equipollent extensions of fuzzy soft set.

## 3. Generalized intuitionistic fuzzy soft sets

In this section, we present the generalized intuitionistic fuzzy soft set theory which is an extension of the intuitionistic fuzzy soft set theory.

### 3.1 Definitions

Obviously, by combining the generalized intuitionistic fuzzy sets and soft sets, it is natural to define the generalized intuitionistic fuzzy soft set model. First, let us briefly introduce the concept of the generalized intuitionistic fuzzy sets.

**Definition 3.1.** [21] A generalized intuitionistic fuzzy set (GIFS) $A$ on $X$ is an object having the form $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$, where $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote membership function and non-membership function, respectively, of $A$ and satisfy $\min\{\mu_A(x), \gamma_A(x)\} \le 0.5$ for all $x \in X$.

**Definition 3.2.** Let $U$ be an initial universe and $E$ be a set of parameters. $\mathcal{GIF}(U)$ denotes the set of all GIFSs of $U$. Let $A \subseteq E$. A pair $\langle F, A\rangle$ is a generalized intuitionistic fuzzy soft set over $U$, where $F$ is a mapping given by $F : A \to \mathcal{GIF}(U)$.

To illustrate this idea, let us consider the following example. Some of it is quoted from [8, 10, 11, 16, 22, 25].

**Example 3.3.** Consider a generalized intuitionistic fuzzy soft set $\langle F, A\rangle$, where $U$ is a set of six houses under the consideration of decision maker to purchase, which is denoted by $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$, and $A$ is parameter set, where $A = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5\} = \{\text{expensive, beautiful, wooden, in good repair, in the green surroundings}\}$. The generalized intuitionistic fuzzy soft set $\langle F, A\rangle$ describe the "attractiveness of the houses" to the decision maker. Suppose that

$$\begin{aligned}
F(\varepsilon_1) = \{&\langle h_1, 0.9, 0.2\rangle, \langle h_2, 0.7, 0.3\rangle, \langle h_3, 0.5, 0.4\rangle, \langle h_4, 0.6, 0.5\rangle,\\
&\langle h_5, 0.6, 0.4\rangle, \langle h_6, 0.7, 0.4\rangle\};\\
F(\varepsilon_2) = \{&\langle h_1, 0.8, 0.2\rangle, \langle h_2, 0.7, 0.2\rangle, \langle h_3, 0.7, 0.4\rangle, \langle h_4, 0.8, 0.3\rangle,\\
&\langle h_5, 0.5, 0.4\rangle, \langle h_6, 0.5, 0.3\rangle\};\\
F(\varepsilon_3) = \{&\langle h_1, 0.6, 0.2\rangle, \langle h_2, 0.5, 0.3\rangle, \langle h_3, 0.4, 0.5\rangle, \langle h_4, 0.7, 0.5\rangle,\\
&\langle h_5, 0.6, 0.4\rangle, \langle h_6, 0.8, 0.3\rangle\};\\
F(\varepsilon_4) = \{&\langle h_1, 0.4, 0.6\rangle, \langle h_2, 0.9, 0.3\rangle, \langle h_3, 0.7, 0.4\rangle, \langle h_4, 0.8, 0.3\rangle,\\
&\langle h_5, 0.8, 0.4\rangle, \langle h_6, 0.8, 0.4\rangle\};\\
F(\varepsilon_5) = \{&\langle h_1, 0.9, 0.2\rangle, \langle h_2, 0.8, 0.3\rangle, \langle h_3, 0.7, 0.4\rangle, \langle h_4, 0.6, 0.5\rangle,\\
&\langle h_5, 0.7, 0.4\rangle, \langle h_6, 0.7, 0.2\rangle\}.
\end{aligned}$$

The generalized intuitionistic fuzzy soft set $\langle F, A\rangle$ is a parameterized family $\{F(\varepsilon_i) : i = 1, 2, 3, 4, 5\}$ of generalized intuitionistic fuzzy sets on $U$, and $\langle F, A\rangle = \{$expensive houses $= \{\langle h_1, 0.9, 0.2\rangle, \langle h_2, 0.7, 0.3\rangle, \langle h_3, 0.5, 0.4\rangle, \langle h_4, 0.6, 0.5\rangle, \langle h_5, 0.6, 0.4\rangle, \langle h_6, 0.7, 0.4\rangle\}$, beautiful houses $= \{\langle h_1, 0.8, 0.2\rangle, \langle h_2, 0.7, 0.2\rangle, \langle h_3, 0.7, 0.4\rangle, \langle h_4, 0.8, 0.3\rangle, \langle h_5, 0.5, 0.4\rangle, \langle h_6, 0.5, 0.3\rangle\}$, wooden houses $= \{\langle h_1, 0.6, 0.2\rangle, \langle h_2, 0.5, 0.3\rangle, \langle h_3, 0.4, 0.5\rangle, \langle h_4, 0.7, 0.5\rangle, \langle h_5, 0.6, 0.4\rangle, \langle h_6, 0.8, 0.3\rangle\}$, in good repair houses $= \{\langle h_1, 0.4, 0.6\rangle, \langle h_2, 0.9, 0.3\rangle, \langle h_3, 0.7, 0.4\rangle, \langle h_4, 0.8, 0.3\rangle, \langle h_5, 0.8, 0.4\rangle, \langle h_6, 0.8, 0.4\rangle\}$, in the green surroundings houses $= \{\langle h_1, 0.9, 0.2\rangle, \langle h_2, 0.8, 0.3\rangle, \langle h_3, 0.7, 0.4\rangle, \langle h_4, 0.6, 0.5\rangle, \langle h_5, 0.7, 0.4\rangle, \langle h_6, 0.7, 0.2\rangle\}\}$.

Table 1 gives the tabular representation of the generalized intuitionistic fuzzy soft set $\langle F, A\rangle$.

Table 1. Tabular representation of the generalized intuitionistic fuzzy soft set $\langle F, A\rangle$

| $U$ | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ | $\varepsilon_4$ | $\varepsilon_5$ |
|---|---|---|---|---|---|
| $h_1$ | $\langle 0.9, 0.2\rangle$ | $\langle 0.8, 0.2\rangle$ | $\langle 0.6, 0.2\rangle$ | $\langle 0.4, 0.6\rangle$ | $\langle 0.9, 0.2\rangle$ |
| $h_2$ | $\langle 0.7, 0.3\rangle$ | $\langle 0.7, 0.2\rangle$ | $\langle 0.5, 0.3\rangle$ | $\langle 0.9, 0.3\rangle$ | $\langle 0.8, 0.3\rangle$ |
| $h_3$ | $\langle 0.5, 0.4\rangle$ | $\langle 0.7, 0.4\rangle$ | $\langle 0.4, 0.5\rangle$ | $\langle 0.7, 0.4\rangle$ | $\langle 0.7, 0.4\rangle$ |
| $h_4$ | $\langle 0.6, 0.5\rangle$ | $\langle 0.8, 0.3\rangle$ | $\langle 0.7, 0.5\rangle$ | $\langle 0.8, 0.3\rangle$ | $\langle 0.6, 0.5\rangle$ |
| $h_5$ | $\langle 0.6, 0.4\rangle$ | $\langle 0.5, 0.4\rangle$ | $\langle 0.6, 0.4\rangle$ | $\langle 0.8, 0.4\rangle$ | $\langle 0.7, 0.4\rangle$ |
| $h_6$ | $\langle 0.7, 0.4\rangle$ | $\langle 0.5, 0.3\rangle$ | $\langle 0.8, 0.3\rangle$ | $\langle 0.8, 0.4\rangle$ | $\langle 0.7, 0.2\rangle$ |

**Definition 3.4.** Let $U$ be an initial universe and $E$ be a set of parameters. Suppose that $A, B \subseteq E$, $\langle F, A\rangle$ and $\langle G, B\rangle$ are two generalized intuitionistic fuzzy soft sets over $U$. Then $\langle F, A\rangle$ is said to be a generalized intuitionistic fuzzy soft subset of $\langle G, B\rangle$ if

(1) $A \subseteq B$;

(2) for any $\varepsilon \in A$, $F(\varepsilon)$ is a generalized fuzzy subset of $G(\varepsilon)$, that is, for all $x \in U$ and $\varepsilon \in A$, $\mu_{F(\varepsilon)}(x) \leq \mu_{G(\varepsilon)}(x)$ and $\gamma_{F(\varepsilon)}(x) \geq \gamma_{G(\varepsilon)}(x)$.

This relationship is denoted by $\langle F, A\rangle \sqsubseteq \langle G, B\rangle$. Similarly, $\langle F, A\rangle$ is said to be a generalized intuitionistic fuzzy soft superset of $\langle G, B\rangle$, if $\langle G, B\rangle$ is called a generalized intuitionistic fuzzy soft subset of $\langle F, A\rangle$. We denote it by $\langle F, A\rangle \sqsupseteq \langle G, B\rangle$.

**Definition 3.5.** Let $A, B \subseteq E$, $\langle F, A\rangle$ and $\langle G, B\rangle$ be two generalized intuitionistic fuzzy soft sets over a universe $U$. Then $\langle F, A\rangle$ and $\langle G, B\rangle$ are said to be generalized intuitionistic fuzzy soft equal, denoted by $\langle F, A\rangle = \langle G, B\rangle$, if

(1) $\langle F, A\rangle$ is a generalized intuitionistic fuzzy soft subset of $\langle G, B\rangle$;

(2) $\langle G, B\rangle$ is a generalized intuitionistic fuzzy soft subset of $\langle F, A\rangle$.

**Example 3.6.** Given two generalized intuitionistic fuzzy soft sets $\langle F, A\rangle$ and $\langle G, B\rangle$ over $U$, where $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ is the set of six houses, $A = \{\varepsilon_1, \varepsilon_2\} = \{\text{expensive, beautiful}\}$, $B = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\} = \{\text{expensive, beautiful, wooden}\}$, and

$$\begin{aligned}
F(\varepsilon_1) = \{&\langle h_1, 0.9, 0.2\rangle, \langle h_2, 0.7, 0.3\rangle, \langle h_3, 0.5, 0.4\rangle,\\
&\langle h_4, 0.6, 0.5\rangle, \langle h_5, 0.6, 0.4\rangle, \langle h_6, 0.7, 0.4\rangle\};\\
F(\varepsilon_2) = \{&\langle h_1, 0.8, 0.2\rangle, \langle h_2, 0.7, 0.2\rangle, \langle h_3, 0.7, 0.4\rangle,\\
&\langle h_4, 0.8, 0.3\rangle, \langle h_5, 0.5, 0.4\rangle, \langle h_6, 0.5, 0.3\rangle\};\\
G(\varepsilon_1) = \{&\langle h_1, 0.9, 0.1\rangle, \langle h_2, 0.8, 0.3\rangle, \langle h_3, 0.6, 0.5\rangle,\\
&\langle h_4, 0.7, 0.4\rangle, \langle h_5, 0.6, 0.4\rangle, \langle h_6, 0.8, 0.3\rangle\};\\
G(\varepsilon_2) = \{&\langle h_1, 0.8, 0.1\rangle, \langle h_2, 0.9, 0.2\rangle, \langle h_3, 0.7, 0.3\rangle,\\
&\langle h_4, 0.8, 0.2\rangle, \langle h_5, 0.8, 0.4\rangle, \langle h_6, 0.8, 0.3\rangle\};\\
G(\varepsilon_3) = \{&\langle h_1, 0.9, 0.2\rangle, \langle h_2, 0.8, 0.3\rangle, \langle h_3, 0.7, 0.4\rangle,\\
&\langle h_4, 0.6, 0.5\rangle, \langle h_5, 0.7, 0.4\rangle, \langle h_6, 0.7, 0.2\rangle\}.
\end{aligned}$$

From Definition 3.4, we can obtain $\langle F, A\rangle \sqsubseteq \langle G, B\rangle$.

## 3.2 Operators and properties

Now we define some operations on generalized intuitionistic fuzzy soft sets.

**Definition 3.7.** Let $E = \{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n\}$ be a set of parameters. The *not* of $E$, denoted by $\rceil E$, is defined by $\rceil E = \{\neg\varepsilon_1, \neg\varepsilon_2, \ldots, \neg\varepsilon_n\}$, where $\neg\varepsilon_i = \textit{not}\ \varepsilon_i$.

**Definition 3.8.** The complement of a generalized intuitionistic fuzzy soft set $\langle F, A\rangle$, denoted by $\langle F, A\rangle^c$, is defined by $\langle F, A\rangle^c = \langle F^c, \rceil A\rangle$, where $F^c : \rceil A \to \mathcal{GIF}(U)$ is a mapping given by $F^c(\varepsilon) = \langle x, \gamma_{F(\neg\varepsilon)}(x), \mu_{F(\neg\varepsilon)}(x)\rangle$ for all $x \in U$ and $\varepsilon \in \rceil A$.

**Example 3.9.** For Example 3.3, the complement of the generalized intuitionistic fuzzy soft set $\langle F, A\rangle$ is given as follows:

$$\begin{aligned}
\langle F, A\rangle^c = \{&\text{not expensive houses} = \{\langle h_1, 0.2, 0.9\rangle, \langle h_2, 0.3, 0.7\rangle,\\
&\langle h_3, 0.4, 0.5\rangle, \langle h_4, 0.5, 0.6\rangle, \langle h_5, 0.4, 0.6\rangle, \langle h_6, 0.4, 0.7\rangle\},\\
&\text{not beautiful houses} = \{\langle h_1, 0.2, 0.8\rangle, \langle h_2, 0.2, 0.7\rangle,\\
&\langle h_3, 0.4, 0.7\rangle, \langle h_4, 0.3, 0.8\rangle, \langle h_5, 0.4, 0.5\rangle, \langle h_6, 0.3, 0.5\rangle\},\\
&\text{not wooden houses} = \{\langle h_1, 0.2, 0.6\rangle, \langle h_2, 0.3, 0.5\rangle,\\
&\langle h_3, 0.5, 0.4\rangle, \langle h_4, 0.5, 0.7\rangle, \langle h_5, 0.4, 0.6\rangle, \langle h_6, 0.3, 0.8\rangle\},\\
&\text{not in good repair houses} = \{\langle h_1, 0.6, 0.4\rangle, \langle h_2, 0.3, 0.9\rangle,\\
&\langle h_3, 0.4, 0.7\rangle, \langle h_4, 0.3, 0.8\rangle, \langle h_5, 0.4, 0.8\rangle, \langle h_6, 0.4, 0.8\rangle\},\\
&\text{not in the green surroundings houses} = \{\langle h_1, 0.9, 0.2\rangle,\\
&\langle h_2, 0.3, 0.8\rangle, \langle h_3, 0.4, 0.7\rangle, \langle h_4, 0.5, 0.6\rangle, \langle h_5, 0.4, 0.7\rangle,\\
&\langle h_6, 0.2, 0.7\rangle\}\}.
\end{aligned}$$

**Definition 3.10.** A generalized intuitionistic fuzzy soft set $\langle F, A\rangle$ is said to be a null generalized intuitionistic fuzzy soft set, denoted by $\Phi$, if for any $\varepsilon \in A$, $\mu_{F(\varepsilon)}(x) = 0$ and $\gamma_{F(\varepsilon)}(x) = 1$, for all $x \in U$.

**Definition 3.11.** A generalized intuitionistic fuzzy soft set $\langle F, A\rangle$ is said to be an absolute generalized intuitionistic fuzzy soft set, denoted by $\Sigma$, if for any $\varepsilon \in A$, $\mu_{F(\varepsilon)}(x) = 1$ and $\gamma_{F(\varepsilon)}(x) = 0$, for all $x \in U$.

**Definition 3.12.** Let $\langle F, A\rangle$ and $\langle G, B\rangle$ be two generalized intuitionistic fuzzy soft sets over a universe $U$. Then "$\langle F, A\rangle$ and $\langle G, B\rangle$" is a generalized intuitionistic fuzzy soft set, denoted by $\langle F, A\rangle \wedge \langle G, B\rangle$, is defined by $\langle F, A\rangle \wedge \langle G, B\rangle = \langle H, A \times B\rangle$, where

$H(\alpha,\beta) = F(\alpha) \cap G(\beta)$ for any $(\alpha,\beta) \in A \times B$, that is, $H(\alpha,\beta)(x) = \langle \min\{\mu_{F(\alpha)}(x), \mu_{G(\beta)}(x)\}$, $\max\{\gamma_{F(\alpha)}(x), \gamma_{G(\beta)}(x)\}\rangle$, for all $(\alpha,\beta) \in A \times B$ and $x \in U$.

**Definition 3.13.** Let $\langle F, A\rangle$ and $\langle G, B\rangle$ be two generalized intuitionistic fuzzy soft sets over a universe $U$. Then "$\langle F, A\rangle$ or $\langle G, B\rangle$" is a generalized intuitionistic fuzzy soft set, denoted by $\langle F, A\rangle \vee \langle G, B\rangle$, is defined by $\langle F, A\rangle \vee \langle G, B\rangle = \langle O, A \times B\rangle$, where $O(\alpha,\beta) = F(\alpha) \cup G(\beta)$ for any $(\alpha,\beta) \in A \times B$, that is, $O(\alpha,\beta)(x) = \langle \max\{\mu_{F(\alpha)}(x), \mu_{G(\beta)}(x)\}$, $\min\{\gamma_{F(\alpha)}(x), \gamma_{G(\beta)}(x)\}\rangle$, for all $(\alpha,\beta) \in A \times B$ and $x \in U$.

**Theorem 3.14.** For two generalized intuitionistic fuzzy soft sets $\langle F, A\rangle$ and $\langle G, B\rangle$ over a universe $U$, the following properties hold:

(1) $(\langle F, A\rangle \wedge \langle G, B\rangle)^c = \langle F, A\rangle^c \vee \langle G, B\rangle^c$;

(2) $(\langle F, A\rangle \vee \langle G, B\rangle)^c = \langle F, A\rangle^c \wedge \langle G, B\rangle^c$.

*Proof.* (1) Suppose that $\langle F, A\rangle \wedge \langle G, B\rangle = \langle H, A \times B\rangle$. Then we have $(\langle F, A\rangle \wedge \langle G, B\rangle)^c = \langle H, A \times B\rangle^c = \langle H^c, \rceil(A \times B)\rangle$. Since $\langle F, A\rangle^c = \langle F^c, \rceil A\rangle$ and $\langle G, B\rangle^c = \langle G^c, \rceil B\rangle$, we have $\langle F, A\rangle^c \vee \langle G, B\rangle^c = \langle F^c, \rceil A\rangle \vee \langle G^c, \rceil B\rangle$. Assume that $\langle F^c, \rceil A\rangle \vee \langle G^c, \rceil B\rangle = \langle O, \rceil A \times \rceil B\rangle = \langle O, \rceil(A \times B)\rangle$. Then for all $(\neg\alpha, \neg\beta) \in \rceil A \times \rceil B$ and $x \in U$, we have

$$\begin{aligned}\mu_{O(\neg\alpha,\neg\beta)}(x) &= \max\{\mu_{F^c(\neg\alpha)}(x), \mu_{G^c(\neg\beta)}(x)\},\\ \gamma_{O(\neg\alpha,\neg\beta)}(x) &= \min\{\gamma_{F^c(\neg\alpha)}(x), \gamma_{G^c(\neg\beta)}(x)\},\end{aligned}$$

Since $\langle F, A\rangle^c = \langle F^c, \rceil A\rangle$ and $\langle G, B\rangle^c = \langle G^c, \rceil B\rangle$, $F^c(\neg\alpha) = \langle x, \gamma_{F(\alpha)}(x), \mu_{F(\alpha)}(x)\rangle$ for all $\neg\alpha \in \rceil A$ and $x \in U$, and $G^c(\neg\beta) = \langle x, \gamma_{G(\beta)}(x), \mu_{G(\beta)}(x)\rangle$ for all $\neg\beta \in \rceil B$ and $x \in U$, i.e., $\mu_{F^c(\neg\alpha)}(x) = \gamma_{F(\alpha)}(x)$, $\gamma_{F^c(\neg\alpha)}(x) = \mu_{F(\alpha)}(x)$, $\mu_{G^c(\neg\alpha)}(x) = \gamma_{G(\alpha)}(x)$ and $\gamma_{G^c(\neg\alpha)}(x) = \mu_{G(\alpha)}(x)$. Therefore, for all $(\neg\alpha, \neg\beta) \in \rceil(A \times B)$ and $x \in U$, we have

$$\begin{aligned}\mu_{O(\neg\alpha,\neg\beta)}(x) &= \max\{\gamma_{F(\alpha)}(x), \gamma_{G(\beta)}(x)\},\\ \gamma_{O(\neg\alpha,\neg\beta)}(x) &= \min\{\mu_{F(\alpha)}(x), \mu_{G(\beta)}(x)\}.\end{aligned}$$

Since $\langle H, A \times B\rangle^c = \langle H^c, \rceil(A \times B)\rangle$, we have $H^c(\neg\alpha, \neg\beta) = \langle x, \gamma_{H(\alpha,\beta)}(x), \mu_{H((\alpha,\beta)}(x)\rangle$, i.e., $\mu_{H^c(\neg\alpha,\neg\beta)}(x) = \gamma_{H(\alpha,\beta)}(x)$ and $\gamma_{H^c(\neg\alpha,\neg\beta)}(x) = \mu_{H(\alpha,\beta)}(x)$. Since $(\neg\alpha, \neg\beta) \in \rceil(A \times B)$, $(\alpha, \beta) \in A \times B$. Since $\langle F, A\rangle \wedge \langle G, B\rangle = \langle H, A \times B\rangle$, we have $H(\alpha,\beta)(x) = \langle \min\{\mu_{F(\alpha)}(x), \mu_{G(\beta)}(x)\}, \max\{\gamma_{F(\alpha)}(x), \gamma_{G(\beta)}(x)\}\rangle$ for all $x \in U$. Therefore, we have

$$\begin{aligned}\mu_{H(\alpha,\beta)}(x) &= \min\{\mu_{F(\alpha)}(x), \mu_{G(\beta)}(x)\},\\ \gamma_{H(\alpha,\beta)}(x) &= \max\{\gamma_{F(\alpha)}(x), \gamma_{G(\beta)}(x)\}.\end{aligned}$$

Consequently, $H^c$ and $O$ are the same operators. Thus, we have $(\langle F, A\rangle \wedge \langle G, B\rangle)^c = \langle F, A\rangle^c \vee \langle G, B\rangle^c$.

(2) Similar to that of (1). □

**Theorem 3.15.** For three generalized intuitionistic fuzzy soft sets $\langle F, A\rangle$, $\langle G, B\rangle$ and $\langle H, C\rangle$ over a universe $U$, the following properties hold:

(1) $\langle F, A\rangle \wedge (\langle G, B\rangle \wedge \langle H, C\rangle) = (\langle F, A\rangle \wedge \langle G, B\rangle) \wedge \langle H, C\rangle$;

(2) $\langle F, A\rangle \vee (\langle G, B\rangle \vee \langle H, C\rangle) = (\langle F, A\rangle \vee \langle G, B\rangle) \vee \langle H, C\rangle$.

*Proof.* (1) Let $\langle G, B\rangle \wedge \langle H, C\rangle = \langle I, B \times C\rangle$, where $I(\alpha,\beta) = G(\alpha) \cap H(\beta)$ for all $(\alpha,\beta) \in B \times C$. That is, $I(\alpha,\beta)(x) = \langle \min\{\mu_{G(\alpha)}(x), \mu_{H(\beta)}(x)\}, \max\{\gamma_{G(\alpha)}(x), \gamma_{H(\beta)}(x)\}\rangle$ for all $(\alpha,\beta) \in B \times C$ and $x \in U$. Since $\langle F, A\rangle \wedge (\langle G, B\rangle \wedge \langle H, C\rangle) = \langle F, A\rangle \wedge \langle I, B \times C\rangle$, we suppose that $\langle F, A\rangle \wedge \langle I, B \times C\rangle = \langle K, A \times (B \times C)\rangle$, where $K(\delta,\alpha,\beta) = F(\delta) \cap I(\alpha,\beta)$, $(\delta,\alpha,\beta) \in A \times (B \times C) = A \times B \times C$. Hence, for all $x \in U$, we have

$$\begin{aligned}&K(\delta,\alpha,\beta)(x)\\ &\quad= \big\langle \min\{\mu_{F(\delta)}(x), \mu_{I(\alpha,\beta)}(x)\}, \max\{\gamma_{F(\delta)}(x),\\ &\qquad \gamma_{I(\alpha,\beta)}(x)\}\big\rangle\\ &\quad= \big\langle \min\{\mu_{F(\delta)}(x), \min\{\mu_{G(\alpha)}(x), \mu_{H(\beta)}(x)\}\},\\ &\qquad \max\{\gamma_{F(\delta)}(x), \max\{\gamma_{G(\alpha)}(x), \gamma_{H(\beta)}(x)\}\}\big\rangle.\end{aligned}$$

Suppose that $\langle F, A\rangle \wedge \langle G, B\rangle = \langle J, A \times B\rangle$, where $J(\delta,\alpha) = F(\delta) \cap G(\alpha)$ for all $(\delta,\alpha) \in A \times B$. Hence we have $J(\delta,\alpha) = \langle \min\{\mu_{F(\delta)}(x), \mu_{G(\alpha)}(x)\}, \max\{\gamma_{F(\delta)}(x), \gamma_{G(\alpha)}(x)\}\rangle$ for all $(\delta,\alpha) \in A \times B$ and $x \in U$. Since $(\langle F, A\rangle \wedge \langle G, B\rangle) \wedge \langle H, C\rangle = \langle J, A \times B\rangle \wedge \langle H, C\rangle$, we suppose that $\langle J, A \times B\rangle \wedge \langle H, C\rangle = \langle O, (A \times B) \times C\rangle$, where $O(\delta,\alpha,\beta) = J(\delta,\alpha) \cap H(\beta)$, for all $(\delta,\alpha,\beta) \in (A \times B) \times C = A \times B \times C$. Hence, for all $x \in U$, we have

$$\begin{aligned}&O(\delta,\alpha,\beta)(x)\\ &\quad= \big\langle \min\{\mu_{J(\delta,\alpha)}(x), \mu_{H(\beta)}(x)\}, \max\{\gamma_{J(\delta,\alpha)}(x),\\ &\qquad \gamma_{H(\beta)}(x)\}\big\rangle\\ &\quad= \big\langle \min\{\min\{\mu_{F(\delta)}(x), \mu_{G(\alpha)}(x)\}, \mu_{H(\beta)}(x)\},\\ &\qquad \max\{\max\{\gamma_{F(\delta)}(x), \gamma_{G(\alpha)}(x)\}, \gamma_{H(\beta)}(x)\}\big\rangle\\ &\quad= \big\langle \min\{\mu_{F(\delta)}(x), \min\{\mu_{G(\alpha)}(x), \mu_{H(\beta)}(x)\}\},\\ &\qquad \max\{\gamma_{F(\delta)}(x), \max\{\gamma_{G(\alpha)}(x), \gamma_{H(\beta)}(x)\}\}\big\rangle\\ &\quad= K(\delta,\alpha,\beta)(x).\end{aligned}$$

Consequently, $K$ and $O$ are the same operators. Thus, $\langle F, A\rangle \wedge (\langle G, B\rangle \wedge \langle H, C\rangle) = (\langle F, A\rangle \wedge \langle G, B\rangle) \wedge \langle H, C\rangle$.

(2) Similar to that of (1). □

## 4. Conclusions

In this paper, the notion of the generalized intuitionistic fuzzy soft set theory is proposed. Our generalized intuitionistic fuzzy soft set is a combination of the generalized intuitionistic fuzzy set and the soft set. The complement, "and", "or" operations are then defined on the generalized intuitionistic fuzzy soft sets. This new extension not only provides a significant addition to existing theories for handling uncertainties, but also leads to potential areas of further field research and pertinent applications.

## Acknowledgments

This study was supported by research funds from Dong-A University.

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## 저 자 소 개

**Jin Han Park** received the B.S. degree, M.S. degree, and Ph.D. degree from the Department of Mathematics, Dong-A University, Korea, in the field of general topology. He is currently a professor of the Department of Applied Mathematics, Pukyong National University, Korea. His research interests include general topology, fuzzy mathematics, decision making,

**Young Chel Kwun** received the B.S. and M.S. degree in mathematics from Dong-A University, Busan, Korea in 1981 and 1983, respectively and the Ph.D. degrees in applied mathematics from Dong-A University, Busan, Korea in 1990. He was a Visiting Scholar in the Department of applied mathematics, Kobe University, Kobe, Japan, from 2004 to 2005. He is currently a Professor in the Department of Mathematics, Dong-A University, Korea. His current research interests include fuzzy differential equations, informatics, mathematical theory, patial differential equations, system theory and control.

**Jin-soo Hwang** received the B.S. degree in mathematics and M. S. degree in applied mathematics from Donga University, Busan, Korea in 1999 and 2001, respectively.
And he received Ph.D. degree in applied mathematics from Kobe university, Kobe, Japan in 2005. He was a postdoc researcher in the Department of Mathematics, Virginia Polytech University at Blacksburg, Virginia, U.S.A., from 2005 to 2006. He is currently a fulltime lecturer in the Department of Mathematics Education, Daegu University, Korea. His current research interests include Optimal control theory in partial differential equations, Identification problems in partial differential equations, Well-posedness of partial differential equations and Fuzzy differential equations.