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($\tilde{g}$, s)-Continuous Functions between Topological Spaces

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Abstract. In this paper, we introduce ($\tilde{g}$, s)-continuous functions between topological spaces, study some of its basic properties and discuss its relationships with other topological functions.

1. Introduction

It is well known that the concept of closedness is fundamental with respect to the investigation of general topological spaces. Levine [28] initiated the study of generalized closed sets. The concept of $\tilde{g}$-closed sets was introduced by Jafari et al [23]. Initiation of contra-continuity was due to Dontchev [10]. Many different forms of contra-continuous functions have been introduced over the years by various authors [5, 11, 14, 15, 17, 19, 20, 22, 39].

In this paper, new generalizations of contra-continuity by using $\tilde{g}$-closed sets called ($\tilde{g}$, s)-continuity are presented. Characterizations and properties of ($\tilde{g}$,s)-continuous functions are discussed in detail. Finally, we obtain many important results in topological spaces.

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2. Preliminaries

In this paper, spaces X and Y mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, cl(A) and int(A) represent the closure of A and the interior of A respectively.

A subset A of a space X is said to be regular open (resp. regular closed) if A = int(cl(A)) (resp. A = cl(int A))) [46]. The δ-interior [51] of a subset A of X is the union of all regular open sets of X contained in A and it is denoted by δ-int(A). A subset A is called δ-open [51] if A = δ-int(A). The complement of δ-open set is called δ-closed. The δ-closure of a set A in a space (X,τ) is defined by δ-cl(A)= {x $\in$ X: A $\cap$ int(cl(U)) $\neq \phi$, U $\in \tau$ and x $\in$ U} and it is denoted by δ-cl(A).

The finite union of regular open set is said to be π-open [53]. The complement of π-open set is said to be π-closed. A subset A is said to be semi-open [27] (resp. α-open [33], preopen [32], β-open [1] or semi-preopen [2]) if A $\subset$ cl(int(A)) (resp. A $\subset$ int(cl(int(A))), A $\subset$ int(cl(A)), A $\subset$ cl(int(cl(A)))). The complement of semi-open (resp. α-open, preopen, β-open) is said to be semi-closed (resp. α-closed, preclosed, β-closed). The union (resp. intersection) of all α-open (resp. α-closed) sets, each contained in (resp. containing) a set S in a topological space X is called α-interior (resp. α-closure) of S and it is denoted by αint(S) (resp. αcl(S)). The union (resp. intersection) of all semi-open (resp. semi-closed) sets, each contained in (resp. containing) a set S in a topological space X is called semi-interior (resp. semi-closure) of S and it is denoted by sint(S) (resp. scl(S)). The union (resp. intersection) of all preopen (resp. preclosed) sets, each contained in (resp. containing) a set S in a topological space X is called preinterior (resp. preclosure) of S and it is denoted by pint(S) (resp. pcl(S)).

A subset A of a space X is said to be generalized closed (briefly, g-closed) [28] (resp. πg-closed [13], $\hat{g}$-closed [48], *g-closed [49]) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open (resp. π-open, semi-open, $\hat{g}$-open, *g-open) in X. The complement of g-closed (resp. πg-closed, $\hat{g}$-closed, *g-closed) is said to be g-open (resp. πg-open, $\hat{g}$-open, *g-open). A subset A of a space X is said to be $^{\#}$gs-closed [50] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is *g-open in X. The complement of $^{\#}$gs-closed is called $^{\#}$gs-open. A subset A of a space X is said to be $\tilde{g}$-closed [23] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $^{\#}$gs-open in X. The complement of $\tilde{g}$-closed is said to be $\tilde{g}$-open. The union (resp. intersection) of all $\tilde{g}$-open (resp. $\tilde{g}$-closed) sets, each contained in (resp. containing) a set S in a topological space X is called $\tilde{g}$-interior (resp. $\tilde{g}$-closure) of S and it is denoted by $\tilde{g}$-int(S) (resp. $\tilde{g}$-cl(S)).

A point x $\in$ X is said to be a θ-semi-cluster point [25] of a subset A of X if cl(U) $\cap$ A $\neq \phi$ for every semi-open set U containing x. The set of all θ-semi-cluster points of A is called the θ-semi-closure of A and is denoted by θ-s-cl(A). A subset A is called θ-semi-closed [25] if A = θ-s-cl(A). The complement of a θ-semi-closed set is called θ-semi-open.

The family of all δ-open (resp. $\tilde{g}$-open, $\tilde{g}$-closed, πg-open, πg-closed, regular open, regular closed, semi-open, closed) sets of X containing a point x $\in$ X is denoted by δO(X, x) (resp. $\tilde{G}$O(X, x), $\tilde{G}$C(X, x), πGO (X, x), πGC(X, x), RO(X, x), RC(X, x), SO(X, x), C(X, x)). The family of all δ-open (resp. $\tilde{g}$-open, $\tilde{g}$-closed, πg-open, πg-closed, semi-open, β-open, preopen, regular open, regular closed) sets of X is denoted by δO(X) (resp $\tilde{G}$O(X), $\tilde{G}$C(X), πGO(X), πGC(X), SO(X), βO(X), PO(X), RO(X), RC(X)).

Definition 2.1. A space X is said to be

1. *s-Urysohn* [3] if for each pair of distinct points x and y in X, there exist U $\in$ SO(X, x) and V $\in$ SO(X, y) such that cl(U) $\cap$ cl(V) = ϕ;
2. *weakly Hausdorff* [44] if each element of X is an intersection of regular closed sets.

Definition 2.2([20]). Let B be a subset of a space X. The set $\cap$ {A $\in$ RO(X) : B $\subset$ A } is called the r-kernel of B and is denoted by r-ker (B).

Proposition 2.3([20]). *The following properties hold for subsets A, B of a space X:*

1. x $\in$ r-ker(A) if and only if A $\cap$ K $\neq \phi$ for any regular closed set K containing x.
2. A $\subset$ r-ker(A) and A = r-ker(A) if A is regular open in X.
3. If A $\subset$ B, then r-ker(A) $\subset$ r-ker(B).

Lemma 2.4([30]). *If V is an open set, then* $scl(V) = int(cl(V))$.

Definition 2.5. A space X is said to be

1. *S-closed* [47] if every regular closed cover of X has a finite subcover,
2. *Countably S-closed* [1] if every countable cover of X by regular closed sets has a finite subcover,
3. *S-Lindelof* [29] if every cover of X by regular closed sets has a countable subcover.

Theorem 2.6([23]). *Union (intersection) of any two $\tilde{g}$-closed sets is again $\tilde{g}$-closed.*

Remark 2.7([13, 23, 48])**.** We have the following relations: closed $\Rightarrow$ $\tilde{g}$-closed $\Rightarrow$ $\hat{g}$-closed $\Rightarrow$ g-closed $\Rightarrow$ πg-closed.

None of these implications are reversible.

The subset {(x, f(x)) : x $\in$ X} $\subset$ X x Y is called the graph of a function f: X $\rightarrow$ Y and is denoted by G(f).

3. Characterizations of $\tilde{g}$-open sets

Lemma 3.1. *For any subset K of a topological space X,* $X\backslash\tilde{g}\text{-}cl(K) = \tilde{g}\text{-}int\,(X\backslash K)$.

Lemma 3.2. *If a subset A is $\tilde{g}$-closed in a space X, then* $A = \tilde{g}\text{-}cl(A)$.

Lemma 3.3. *If A is $\tilde{g}$-closed and $^{\#}$gs-open set, then A is closed.*

Theorem 3.4. *A set A is $\tilde{g}$-open in (X, τ) if and only if $F \subseteq int(A)$ whenever F is $^{\#}$gs-closed in X and $F \subseteq A$.*

Proof. Assume that A is $\tilde{g}$-open, F ⊆ A and F is $^{\#}$gs-closed. Then X\F is $^{\#}$gs-open and X\A ⊆ X\F. Since X\A is $\tilde{g}$-closed, cl(X\A) ⊆ X\F. It implies that X\ int(A) ⊆ X\F and hence F ⊆ int(A).

Conversely, put X\A = B. Suppose B ⊆ U where U is $^{\#}$gs-open. Now if X\A ⊆ U, then F = X\U ⊆ A and F is $^{\#}$gs-closed. It implies that F ⊆ int(A) and hence X\int(A) ⊆ X\F = U. Therefore X\int(X\B) ⊆ U and consequently cl(B) ⊆ U. Hence B is $\tilde{g}$-closed and therefore A is $\tilde{g}$-open. □

Theorem 3.5. *Suppose that A is $\tilde{g}$-open in X and that B is $\tilde{g}$-open in Y. Then A × B is $\tilde{g}$-open in X × Y.*

Proof. Suppose that F is closed and hence $^{\#}$gs-closed in X × Y and that F ⊆ A × B. By the previous theorem, it suffices to show that F ⊆ int(A × B).

Let (x,y) ∈ F. Then, for each (x,y) ∈ F, cl({x}) x cl({y}) = cl({x} x {y}) = cl({x,y}) ⊂ cl(F) = F ⊂ A x B. Two closed sets cl({x}) and cl({y}) are contained in A and B respectively. It follows from the assumption that cl({x}) ⊆ int(A) and that cl({y}) ⊆ int(B). Thus (x,y) ∈ cl({x}) x cl({y}) ⊆ int(A) x int(B) ⊆ int(A x B). It means that, for each (x,y) ∈ F, (x,y) ∈ int(A x B) and hence F ⊆ int(A x B). Therefore A x B is $\tilde{g}$-open in X x Y. □

Definition 3.6. A function f: X → Y is called $\tilde{g}$*-closed [24] if f (V) is $\tilde{g}$-closed set in Y for each $\tilde{g}$-closed set V in X.

Theorem 3.7([24]). *If a function f: X → Y is $\tilde{g}$*-closed, then for each subset B of Y and each $\tilde{g}$-open set U of X containing $f^{-1}(B)$, there exists a $\tilde{g}$-open set V in Y containing B such that $f^{-1}(V) \subset U$.*

4. Properties of ($\tilde{g}$, s)-continuous functions

Definition 4.1. A function f: X → Y is called ($\tilde{g}$, s)-continuous if the inverse image of each regular open set of Y is $\tilde{g}$-closed in X.

Theorem 4.2. *The following are equivalent for a function f: X → Y:*

1. f is ($\tilde{g}$, s)-continuous,
2. The inverse image of a regular closed set of Y is $\tilde{g}$-open in X,

3. f^{-1}(int(cl(V))) is $\tilde{g}$-closed in X for every open subset V of Y,
4. f^{-1}(cl(int(F))) is $\tilde{g}$-open in X for every closed subset F of Y,
5. f^{-1}(cl(U)) is $\tilde{g}$-open in X for every U $\in \beta$O(Y),
6. f^{-1}(cl(U)) is $\tilde{g}$-open in X for every U $\in$ SO(Y),
7. f^{-1}(int(cl(U))) is $\tilde{g}$-closed in X for every U $\in$ PO(Y).

Proof. (1) $\Leftrightarrow$ (2) : Obvious

(1) $\Leftrightarrow$ (3) : Let V be an open subset of Y. Since int(cl(V)) is regular open, f^{-1}(int cl(V))) is $\tilde{g}$-closed. The converse is similar.

(2) $\Leftrightarrow$ (4) : Similar to (1) $\Leftrightarrow$ (3)

(2) $\Rightarrow$ (5) : Let U be any β-open set of Y. By Theorem 2.4 of [2] that cl(U) is regular closed. Then by (2) f^{-1}(cl(U)) is $\tilde{g}$-open in X.

(5) $\Rightarrow$ (6) : Obvious from the fact that SO(Y) $\subset \beta$O(Y).

(6) $\Rightarrow$ (7): Let U $\in$ PO(Y). Then Y$\backslash$int(cl(U)) is regular closed and hence it is semi-open. Then, we have X$\backslash f^{-1}$(int(cl(U))) = f^{-1}(Y$\backslash$int(cl(U))) = f^{-1}(cl(Y$\backslash$int(cl(U)))) is $\tilde{g}$-open in X. Hence f^{-1}int(cl(U))) is $\tilde{g}$-closed in X.

(7) $\Rightarrow$ (1) : Let U be any regular open set of Y. Then U $\in$ PO(Y) and hence f^{-1}(U) = f^{-1}(int(cl(U))) is $\tilde{g}$-closed in X. □

Lemma 4.3([37]). *For a subset A of a topological space* (Y, σ) *the following properties hold:*

1. αcl(A) = cl(A) for every A $\in \beta$O(Y),
2. pcl(A) = cl(A) for every A $\in$ SO(Y),
3. scl(A) = int(cl(A)) for every A $\in$ PO(Y).

Corollary 4.4. *The following are equivalent for a function f: X $\to$ Y:*

1. f is ($\tilde{g}$,s)-continuous,
2. f^{-1}(αcl(A)) is $\tilde{g}$-open in X for every A $\in \beta$O(Y),
3. f^{-1}(pcl(A)) is $\tilde{g}$-open in X for every A $\in$ SO(Y),
4. f^{-1}(scl(A)) is $\tilde{g}$-closed in X for every A $\in$ PO(Y).

Proof. It follows from Lemma 4.3. □

Theorem 4.5. *Suppose that $\tilde{G}C(X)$ is closed under arbitrary intersections. The following are equivalent for a function f: X $\to$ Y:*

1. f is ($\tilde{g}$, s)-continuous,
2. the inverse image of a θ-semi-open set of Y is $\tilde{g}$-open,
3. the inverse image of a θ-semi-closed set of Y is $\tilde{g}$-closed,
4. f($\tilde{g}$-cl(U)) $\subset$ r-ker(f(U)) for every subset U of X,

5. $\tilde{g}$-cl(f^{-1}(V)) ⊂ f^{-1}(r-ker(V)) for every subset V of Y,
6. for each x ∈ X and each V ∈ SO(Y, f(x)), there exists a $\tilde{g}$-open set U in X containing x such that f(U) ⊂ cl(V),
7. f^{-1}(V) ⊂ $\tilde{g}$-int(f^{-1}(cl(V))) for every V ∈ SO(Y),
8. f($\tilde{g}$-cl(A)) ⊂ θ-s-cl(f(A)) for every subset A of X,
9. $\tilde{g}$-cl(f^{-1}(B)) ⊂ f^{-1}(θ-s-cl(B)) for every subset B of Y,
10. $\tilde{g}$-cl(f^{-1}(V)) ⊆ f^{-1}(θ-s-cl(V)) for every open subset V of Y,
11. $\tilde{g}$-cl(f^{-1}(V)) ⊆ f^{-1}(scl(V)) for every open subset V of Y,
12. $\tilde{g}$-cl(f^{-1}(V)) ⊆ f^{-1}(int(cl(V)) for every open subset V of Y.

Proof. (1) ⇒ (2): Since any θ-semi-open set is a union of regular closed sets, by using Theorem 4.2, (2) holds.

(2) ⇒ (6): Let x ∈ X and V ∈ SO(Y) containing f(x). Since cl(V) is θ-semi-open in Y, there exists a $\tilde{g}$-open set U in X containing x such that x ∈ U ⊂ f^{-1}(cl(V)). Hence f(U) ⊂ cl(V).

(6) ⇒ (7): Let V ∈ SO(Y) and x ∈ f^{-1}(V). Then f(x) ∈ V. By (6), there exists a $\tilde{g}$-open set U in X containing x such that f(U) ⊂ cl(V). It follows that x ∈ U ⊂ f^{-1}(cl(V)). Hence x ∈ $\tilde{g}$-int(f^{-1}(cl(V))). Thus, f^{-1}(V)⊂ $\tilde{g}$-int(f^{-1}(cl(V))).

(7) ⇒ (1): Let F be any regular closed set of Y. Since F ∈ SO(Y), then by (7), f^{-1}(F) ⊂ $\tilde{g}$-int(f^{-1}(F)). This shows that f^{-1}(F) is $\tilde{g}$-open in X. Hence, by Theorem 4.2, (1) holds.

(2) ⇔ (3) : Obvious.

(1) ⇒ (4): Let U be any subset of X. Let y ∉ r-ker(f(U)). Then there exists a regular closed set F containing y such that f(U) ∩ F = ϕ. Hence, we have U ∩ f^{-1}(F) = ϕ and $\tilde{g}$-cl(U) ∩ f^{-1}(F) = ϕ. Therefore, we obtain f($\tilde{g}$-cl(U)) ∩ F = ϕ and y ∉ f($\tilde{g}$-cl(U)). Thus, f($\tilde{g}$-cl(U)) ⊂ r-ker(f(U)).

(4) ⇒ (5): Let V be any subset of Y. By (4), f($\tilde{g}$-cl(f^{-1}(V))) ⊂ r-ker(V) and $\tilde{g}$-cl(f^{-1}(V)) ⊂ f^{-1}(r-ker(V)).

(5) ⇒ (1): Let V be any regular open set of Y. By (5), $\tilde{g}$-cl(f^{-1}(V)) ⊂ f^{-1}(r-ker(V)) = f^{-1}(V) and $\tilde{g}$-cl(f^{-1}(V)) = f^{-1}(V). We obtain that f^{-1}(V) is $\tilde{g}$-closed in X.

(6) ⇒ (8): Let A be any subset of X. Suppose that x ∈ $\tilde{g}$-cl(A) and G is any semi-open set of Y containing f(x). By (6), there exists U ∈ $\tilde{G}$O(X, x) such that f(U) ⊂ cl(G). Since x ∈ $\tilde{g}$-cl(A), U ∩ A ≠ ϕ and hence ϕ ≠ f(U) ∩ f(A) ⊂ cl(G) ∩ f(A). Therefore, we obtain f(x) ∈ θ-s-cl(f(A)) and hence f($\tilde{g}$-cl(A)) ⊂ θ-s-cl(f(A)).

(8) ⇒ (9): Let B be any subset of Y. Then f($\tilde{g}$-cl(f^{-1}(B))) ⊂ θ-s-cl(f(f^{-1}(B)) ⊂ θ-s-cl(B) and $\tilde{g}$-cl(f^{-1}(B)) ⊂ f^{-1}(θ-s-cl(B)).

(9) ⇒ (6): Let V be any semi-open set of Y containing f(x). Since cl(V) ∩ (Y\ cl(V)) = ϕ, we have f(x) ∉ θ-s-cl(Y\cl(V)) and x ∉ f^{-1}(θ-s-cl(Y\cl(V))). By (9), x ∉ $\tilde{g}$-cl(f^{-1}(Y\cl(V))). Hence, there exists U ∈ $\tilde{G}$O(X, x) such that U ∩ f^{-1}(Y\cl(V)) = ϕ and f(U) ∩ (Y\cl(V)) = ϕ. It follows that f(U) ⊂ cl(V). Thus,(6) holds.

(9) ⇒ (10): Obvious.

(10) ⇒ (11): Obvious from the fact that θ-s-cl(V) = scl(V) for an open set V.

(11) ⇒ (12): Obvious from Lemma 2.4.

(12) ⇒ (1): Let V ∈ RO(Y). Then by (12) $\tilde{g}$-cl(f^{-1}(V)) ⊂ f^{-1}(int(cl(V))) = f^{-1}(V). Hence, f^{-1}(V) is $\tilde{g}$-closed which proves that f is ($\tilde{g}$, s)-continuous. □

Corollary 4.6. *Assume that $\tilde{G}C(X)$ is closed under arbitrary intersections. The following are equivalent for a function f: X → Y:*

1. f is ($\tilde{g}$,s)-continuous,
2. $\tilde{g}$-cl(f^{-1}(B)) ⊂ f^{-1}(θ-s-cl(B)) for every B ∈ SO(Y),
3. $\tilde{g}$-cl(f^{-1}(B)) ⊂ f^{-1}(θ-s-cl(B)) for every B ∈ PO(Y),
4. $\tilde{g}$-cl(f^{-1}(B)) ⊂ f^{-1}(θ-s-cl(B)) for every B ∈ βO(Y).

Proof. In Theorem 4.5, we have proved that the following are equivalent.

1. f is ($\tilde{g}$,s)-continuous.
2. $\tilde{g}$-cl(f^{-1}(B)) ⊂ f^{-1}(θ-s-cl(B)) for every subset B of Y.

Hence the corollary is proved. □

5. The related functions with ($\tilde{g}$, s)-continuous functions

Definition 5.1. A function f: X → Y is said to be

1. *perfectly continuous* [35] if f^{-1}(V) is clopen in X for every open set V of Y,
2. *regular set-connected* [12, 16] if f^{-1}(V) is clopen in X for every V ∈ RO(Y),
3. *almost s-continuous* [6, 38] if for each x ∈ X and each V ∈ SO(Y, f(x)), there exists an open set U in X containing x such that f(U) ⊂ scl(V),
4. *strongly continuous* [26] if the inverse image of every set in Y is clopen in X,
5. *RC-continuous* [11] if f^{-1}(V) is regular closed in X for each open set V of Y,
6. *contra R-map* [17] if f^{-1}(V) is regular closed in X for each regular open set V of Y,
7. *contra-super-continuous* [22] if for each x ∈ X and for each F ∈ C(Y, f(x)), there exists a regular open set U in X containing x such that f(U) ⊂ F,
8. *almost contra-super-continuous* [15] if f^{-1}(V) is δ-closed in X for every regular open set V of Y,
9. *contra continuous* [10] if f^{-1}(V) is closed in X for every open set V of Y,
10. *contra g-continuous* [5] if f^{-1}(V) is g-closed in X for every open set V of Y,

11. *(θ, s)-continuous* [25, 39] if for each x ∈ X and each V ∈ SO(Y, f(x)), there exists an open set U in X containing x such that f(U) ⊂ cl(V),
12. *contra πg-continuous* [19] if $f^{-1}(V)$ is πg-closed in X for each open set V of Y,
13. *$\hat{g}$-continuous* [48] if $f^{-1}(V)$ is $\hat{g}$-closed in X for each closed set V of Y,
14. *$\tilde{g}$-continuous* [24] if $f^{-1}(V)$ is $\tilde{g}$-closed in X for each closed set V of Y,
15. *(g, s)-continuous* [14] if $f^{-1}(V)$ is g-closed in X for each regular open set V of Y,
16. *($\hat{g}$, s)-continuous* [43] if $f^{-1}(V)$ is $\hat{g}$-closed in X for each regular open set V of Y,
17. *(πg, s)-continuous* [14] if f^{-1} (V) is πg-closed in X for each regular open set V of Y.

Definition 5.2. A function f: X → Y is said to be contra $\hat{g}$-continuous [43] (resp. contra $\tilde{g}$-continuous) if $f^{-1}(V)$ is $\hat{g}$-closed (resp. $\tilde{g}$-closed) in X for each open set V of Y.

Remark 5.3. The following diagram holds for a function f: X → Y:

strongly continuous	→	almost s-continuous
↓		↓
perfectly continuous	→	regular set-connected
↓		↓
RC continuous	→	contra R-map
↓		↓
contra-super-continuous	→	almost contra-super-continuous
↓		↓
contra-continuous	→	(θ, s)-continuous
↓		↓
contra $\tilde{g}$-continuous	→	($\tilde{g}$, s)-continuous
↓		↓
contra $\hat{g}$-continuous	→	($\hat{g}$, s)-continuous
↓		↓
contra g-continuous	→	(g, s)-continuous
↓		↓
contra πg-continuous	→	(πg, s)-continuous

None of these implications is reversible as shown in the following examples and in the related paper [43].

Example 5.4. Let X = Y = {a, b, c}, τ = {ϕ, X, {b}, {a, c}} and σ = {ϕ, Y, {a, b}}. Then the identity function f: X $\rightarrow$ Y is ($\tilde{g}$, s)-continuous but it is not contra $\tilde{g}$-continuous.

Example 5.5. Let X = Y = {a, b, c}, τ = {ϕ, X, {a}, {b, c}} and σ = {ϕ, Y, {a, b}}. Then the identity function f: X $\rightarrow$ Y is contra $\hat{g}$-continuous but it is not contra $\tilde{g}$-continuous.

Example 5.6. Let X = Y = {a, b, c, d}, $\tau = \sigma$ = {ϕ, X = Y, {b}, {a, c, d}}. Then the function f: X $\rightarrow$ Y which is defined as f(a) = b, f(b) = c, f(c) = d, f(d) = a is ($\hat{g}$, s)-continuous but it is not ($\tilde{g}$, s)-continuous.

Example 5.7. Let X = Y = {a, b, c, d}, $\tau = \sigma$ = {ϕ, X = Y, {c}, {a, d}, {a, c, d}}. Then the function f: X $\rightarrow$ Y which is defined as f(a) = c, f(b)= c, f(c) = b, f(d) = b is ($\tilde{g}$, s)-continuous but it is not (θ,s)-continuous.

Let X = {a, b, c, d} = Y, τ = {ϕ, X, {b}, {b, c}, {b, c, d}} and σ = {ϕ, Y, {a}, {a, b}. Then the identity function f: X $\rightarrow$ Y is contra $\tilde{g}$-continuous but it is not contra continuous.

A topological space (X, τ) is said to be extremely disconnected [4] if the closure of every open set of X is open in X.

Definition 5.8. A function f: X $\rightarrow$ Y is said to be almost $\tilde{g}$-continuous if f^{-1}(V) is $\tilde{g}$-open in X for every regular open set V of Y.

Theorem 5.9. *Let* (Y, σ) *be extremely disconnected. Then, the following are equivalent for a function f:* $(X, \tau) \rightarrow (Y, \sigma)$*:*

1. f is ($\tilde{g}$, s)-continuous,
2. f is almost $\tilde{g}$-continuous.

Proof. (1) $\Rightarrow$ (2): Let x $\in$ X and U be any regular open set of Y containing f(x). Since Y is extremely disconnected, by lemma 5.6 of [41] U is clopen and hence U is regular closed. Then f^{-1}(U) is $\tilde{g}$-open in X. Thus, f is almost $\tilde{g}$-continuous.

(2) $\Rightarrow$ (1): Let K be any regular closed set of Y. Since Y is extremely disconnected, K is regular open and f^{-1}(K) is $\tilde{g}$-open in X. Thus, f is ($\tilde{g}$, s)-continuous. □

Definition 5.10. A space is said to be P_Σ [52] or strongly s-regular [21] if for any open set V of X and each x $\in$ V, there exists K $\in$ RC(X, x) such that x $\in$ K $\subset$ V.

Definition 5.11. A space (X, τ) is called $\tilde{g}$-$T_{1/2}$ if every $\tilde{g}$-closed set is closed.

Theorem 5.12. *Let f: X* $\rightarrow$ *Y be a function from a* $\tilde{g}$-$T_{1/2}$*- space X to a topological space Y. The following are equivalent.*

1. f is (θ, s)-continuous,

2. f is ($\tilde{g}$, s)-continuous.

Theorem 5.13. *Let f: $X \to Y$ be a function. Then, if f is $(\tilde{g}, s)$-continuous, X is $\tilde{g}$-$T_{1/2}$ and Y is P_Σ, then f is continuous.*

Proof. Let G be any open set of Y. Since Y is P_Σ, there exists a subfamily Φ of RC(Y) such that G = $\cup$ {A: A $\in \Phi$}. Since X is $\tilde{g}$-$T_{l/2}$ and f is ($\tilde{g}$, s)-continuous, f^{-1}(A) is open in X for each A $\in \Phi$ and f^{-1}(G) is open in X. Thus, f is continuous.

Theorem 5.14. *Let f: $X \to Y$ be a function from a $\tilde{g}$-$T_{1/2}$-space (X, τ) to an extremely disconnected space (Y, σ). Then the following are equivalent.*

1. f is ($\tilde{g}$,s)-continuous.

2. f is (θ, s)-continuous.

3. f is almost contra-super-continuous.

4. f is contra R-map.

5. f is regular set-connected.

6. f is almost s-continuous.

Proof. (6) $\Rightarrow$ (5) $\Rightarrow$ (4) $\Rightarrow$ (3) $\Rightarrow$ (2) $\Rightarrow$ (1): Obvious.
(1) $\Rightarrow$ (6) Let V be any semi-open and semi-closed set of Y. Since V is semi-open, cl(V) = cl(int (V)) and hence cl(V) is open in Y. Since V is semi-closed, int(cl(V)) $\subset$ V $\subset$cl(V) and hence int(cl(V)) = V = cl(V). Therefore, V is clopen in Y and V $\in$ RO(Y) $\cap$ RC(Y). Since f is ($\tilde{g}$, s)-continuous, f^{-1}(V) is $\tilde{g}$-open and $\tilde{g}$-closed in X. Since X is $\tilde{g}$-$T_{1/2}$-space , $\tau = \tilde{G}$O(X). Thus, f^{-1}(V) is clopen in X and hence f is almost s-continuous [38, Theorem 3.1]. □

Definition 5.15. A space is said to be weakly P_Σ [36] if for any V $\in$ RO(X) and each x $\in$ V, there exists F $\in$ RC(X,x) such that x $\in$ F $\subset$ V.

Theorem 5.16. *Let f: $(X, \tau) \to (Y, \sigma)$ be a $(\tilde{g}, s)$-continuous function and $\tilde{G}C(X)$ be closed under arbitrary intersections. If Y is weakly P_Σ and X is $\tilde{g}$-$T_{1/2}$, then f is regular set-connected.*

Proof. Let V be any regular open set of Y. Since Y is weakly P_Σ, there exists a subfamily Φ of RC(Y) such that V = $\cup$ {A: A $\in \Phi$}. Since f is ($\tilde{g}$, s)-continuous, f^{-1}(A) is $\tilde{g}$-open in X for each A $\in \Phi$ and f^{-1}(V) is $\tilde{g}$-open in X. Also f^{-1}(V) is $\tilde{g}$-closed in X since f is ($\tilde{g}$, s)-continuous. Since X is $\tilde{g}$-$T_{1/2}$ space, then $\tau = \tilde{G}$O(X). Hence f^{-1}(V) is clopen in X and then f is regular set-connected. □

Definition 5.17. A function f: X $\to$ Y is said to be $\tilde{g}$-irresolute [24] if f^{-1}(V) is $\tilde{g}$-open in X for every V $\in \tilde{G}$O(X).

Theorem 5.18. *Let f: $X \to Y$ and g: $Y \to Z$ be functions. Then, the following properties hold:*

1. If f is $\tilde{g}$-irresolute and g is ($\tilde{g}$, s)-continuous, then g o f: X $\to$ Z is ($\tilde{g}$, s)-continuous.
2. If f is ($\tilde{g}$, s)-continuous and g is contra R-map, then g o f: X $\to$ Z is almost $\tilde{g}$-continuous.
3. If f is $\tilde{g}$-continuous and g is (θ, s)-continuous, then g o f: X $\to$ Z is ($\tilde{g}$, s)-continuous.
4. If f is ($\tilde{g}$, s)-continuous and g is RC continuous, then g o f: X $\to$ Z is $\tilde{g}$-continuous.
5. If f is almost $\tilde{g}$-continuous and g is contra R-map, then g o f: X $\to$ Z is ($\tilde{g}$, s)-continuous.

Theorem 5.19. *Let Y be a regular space and f: X $\to$ Y be a function. Suppose that the collection of $\tilde{g}$-closed sets of X is closed under arbitrary intersections. Then if f is $(\tilde{g}, s)$-continuous, f is $\tilde{g}$-continuous.*

Proof. Let x be an arbitrary point of X and V an open set of Y containing f(x). Since Y is regular, there exists an open set G in Y containing f(x) such that cl(G) $\subset$ V. Since f is ($\tilde{g}$, s)-continuous, there exists U $\in \tilde{G}$O(X, x) such that f(U) $\subset$ cl(G). Then f(U) $\subset$ cl(G) $\subset$ V. Hence, f is $\tilde{g}$-continuous. □

6. Fundamental properties

Definition 6.1([24]). A space X is said to be

1. $\tilde{g}$-T_2 if for each pair of distinct points x and y in X, there exist U $\in \tilde{G}$O(X, x) and V $\in \tilde{G}$O(X, y) such that U $\cap$ V = ϕ.
2. $\tilde{g}$-T_1 if for each pair of distinct points x and y in X, there exist $\tilde{g}$-open sets U and V containing x and y, respectively, such that y $\notin$ U and x $\notin$ V.

Remark 6.2. The following implications are hold for a topological space X.

1. $T_2 \Rightarrow \tilde{g}$-$T_2$
2. $T_1 \Rightarrow \tilde{g}$-$T_1$

These implications are not reversible.

Example 6.3. Let X = {a, b, c} with τ = {ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}}. Then X is both $\tilde{g}$-T_2 and $\tilde{g}$-T_1 but it is neither T_1 nor T_2.

Theorem 6.4. *The following properties hold for a function f: X $\to$ Y:*

1. If f is a ($\tilde{g}$, s)-continuous injection and Y is s-Urysohn, then X is $\tilde{g}$-T_2.
2. If f is a ($\tilde{g}$, s)-continuous injection and Y is weakly Hausdorff, then X is $\tilde{g}$-T_1.

Proof. (1) Let Y be s-Urysohn. By the injectivity of f, f(x) $\neq$ f(y) for any distinct points x and y in X. Since Y is s-Urysohn, there exist A $\in$ SO(Y, f(x)) and B $\in$ SO(Y, f(y)) such that cl(A) $\cap$ cl(B) = ϕ. Since f is a ($\tilde{g}$, s)-continuous, by theorem 4.5, there exist $\tilde{g}$-open sets C and D in X containing x and y, respectively, such that f(C) $\subset$ cl(A) and f(D) $\subset$ cl(B) such that C $\cap$ D = ϕ. Thus, X is $\tilde{g}$-T_2.

(2) Let Y be weakly Hausdorff. For x $\neq$ y in X, there exist A, B $\in$ RC(Y) such that f(x) $\in$ A, f(y) $\notin$ A, f(x) $\notin$ B and f(y) $\in$ B. Since f is ($\tilde{g}$, s)-continuous, f^{-1}(A) and f^{-1}(B) are $\tilde{g}$-open subsets of X such that x $\in$ f^{-1}(A), y $\notin$ f^{-1}(A), x $\notin$ f^{-1} (B) and y $\in$ f^{-1}(B). Hence, X is $\tilde{g}$-T_1. □

Theorem 6.5([24]). *A space X is $\tilde{g}$-T_2 if and only if for any pair of distinct points x, y of X there exist $\tilde{g}$-open sets U and V such that x $\in$ U and y $\in$ V and $\tilde{g}$-cl(U) $\cap$ $\tilde{g}$-cl(V) = ϕ.*

Definition 6.6. A graph G(f) of a function f: X $\to$ Y is said to be ($\tilde{g}$, s)-graph if there exist a $\tilde{g}$-open set A in X containing x and a semi-open set B in Y containing y such that (A $\times$ cl(B)) $\cap$ G(f) = ϕ for each (x, y) $\in$ (X $\times$ Y)\G(f).

Proposition 6.7. *The following properties are equivalent for a function f: X $\to$ Y:*

1. G(f) is ($\tilde{g}$, s)-graph,
2. For each (x, y) $\in$ (X x Y)\G(f), there exist a $\tilde{g}$-open set A in X containing x and a semi-open set B in Y containing y such that f(A) $\cap$ cl(B) = ϕ.
3. For each (x, y) $\in$ (X x Y)\G(f), there exist a $\tilde{g}$-open set A in X containing x and a regular closed set K in Y containing y such that f(A) $\cap$ K = ϕ.

Definition 6.8. A subset S of a space X is said to be S-closed relative to X [34] if for every cover {A_i: i $\in$ I} of S by semi-open sets of X, there exists a finite subset I_0 of I such that S $\subset$ $\cup${cl(A_i) : i $\in$ I_0}.

Theorem 6.9. *If a function f: X $\to$ Y has a $(\tilde{g}, s)$-graph and the collection of $\tilde{g}$-closed sets of X is closed under arbitrary intersections, then $f^{-1}(A)$ is $\tilde{g}$-closed in X for every subset A which is S-closed relative to Y.*

Proof. Suppose that A is S-closed relative to Y and x $\notin$ f^{-1}(A). We have (x, y) $\in$ (X x Y)\G(f) for each y $\in$ A and there exist a $\tilde{g}$-open set B_y containing x and a semi-open set C_y containing y such that f(B_y) $\cap$ cl(C_y) = ϕ. Since {C_y: y $\in$ A} is a cover by semi-open sets of Y, there exists a finite subset {y_1, y_2, ..., y_n} of A such that A $\subset$ $\cup${cl(C_{yi}): i = 1, 2, ..., n}. Take B = $\cup${B_{yi} : i = 1, 2, ..., n}. Then B is a $\tilde{g}$-open containing x and f(B) $\cap$ A = ϕ. Thus, B $\cap$ f^{-1}(A) = ϕ and hence f^{-1}(A) is $\tilde{g}$-closed in X. □

Theorem 6.10. *Let f: X $\to$ Y be a $(\tilde{g}, s)$-continuous function. Then the following properties hold:*

1. G(f) is a ($\tilde{g}$, s)-graph if Y is an s-Urysohn.

2. f is almost $\tilde{g}$-continuous if Y is s-Urysohn S-closed space and $\tilde{G}$C(X) is closed under arbitrary intersections.

Proof. (1) Let Y be s-Urysohn and (x, y) $\in$ (X x Y) $\setminus$ G(f). Then f(x) $\neq$ y. Since Y is s-Urysohn, there exist M $\in$ SO(Y, f(x)) and N $\in$ SO(Y, y) such that cl(M) $\cap$ cl(N) = ϕ. Since f is ($\tilde{g}$, s)-continuous, by theorem there exists a $\tilde{g}$-open set A in X containing x such that f(A) $\subset$ cl(M). Hence, f(A) $\cap$ cl(N) = ϕ and G(f) is ($\tilde{g}$, s)-graph in X x Y.
(2) Let F be a regular closed set in Y. By theorem 3.3 and 3.4 [34], F is S-closed relative to Y. Hence, by theorem 6.9 and (1), $f^{-1}(F)$ is $\tilde{g}$-closed in X and hence f is almost $\tilde{g}$-continuous. □

Theorem 6.11. *Let f, g: X → Y be functions and $\tilde{g}$-cl(S) be $\tilde{g}$-closed for each S ⊂ X. If*

1. f and g are ($\tilde{g}$, s)-continuous,

2. Y is s-Urysohn,

then A = {x ∈ X : f(x) = g(x)} is $\tilde{g}$-closed in X.

Proof. Let x $\in$ X$\setminus$A, then it follows that f(x) $\neq$ g(x). Since Y is s-Urysohn, there exist M $\in$ SO(Y, f(x)) and N $\in$ SO(Y, g(x)) such that cl(M) $\cap$ cl(N) = ϕ. Since f and g are ($\tilde{g}$, s)-continuous, there exist $\tilde{g}$-open sets U and V containing x such that f(U) $\subset$ cl(M) and g(V) $\subset$ cl(N). Hence, U $\cap$ V = P $\in$ $\tilde{G}$O(X), f(P) $\cap$ g(P) = ϕ and then x $\notin$ $\tilde{g}$-cl(A). Thus, A is $\tilde{g}$-closed in X. □

Definition 6.12. A subset A of a topological space X is said to be $\tilde{g}$-dense in X if $\tilde{g}$-cl(A) = X.

Theorem 6.13. *Let f, g: X → Y be functions and $\tilde{g}$-cl(S) be $\tilde{g}$-closed for each S ⊂ X. If*

1. Y is s-Urysohn,

2. f and g are ($\tilde{g}$, s)-continuous,

3. f = g on $\tilde{g}$-dense set A $\subset$ X, then f = g on X.

Proof. Since f and g are ($\tilde{g}$, s)-continuous and Y is s-Urysohn, by Theorem 6.11, B = {x $\in$ X : f(x) = g(x)} is $\tilde{g}$-closed in X. We have f = g on $\tilde{g}$-dense set A $\subset$ X. Since A $\subset$ B and A is $\tilde{g}$-dense set in X, then X = $\tilde{g}$-cl(A) $\subset$ $\tilde{g}$-cl(B) = B. Hence f = g on X. □

Definition 6.14([24]). A space X is said to be

1. countably $\tilde{g}$-compact if every countable cover of X by $\tilde{g}$-open sets has a finite subcover,
2. $\tilde{g}$-Lindelof if every $\tilde{g}$-open cover of X has a countable subcover.

Theorem 6.15. *Let f: $X \to Y$ be a $(\tilde{g}, s)$-continuous surjection. Then the following statements hold:*

1. if X is $\tilde{g}$-Lindelof, then Y is S-Lindelof.
2. if X is countably $\tilde{g}$-compact, then Y is countably S-closed.

Definition 6.16. A space X is called $\tilde{g}$-connected if X is not the union of two disjoint nonempty $\tilde{g}$-open sets.

Example 6.17. Let X = {a, b, c} with τ = {ϕ, X, {a}, {b, c}}. Then X is not $\tilde{g}$-connected.

Example 6.18. Let X = {a, b, c} with τ = {ϕ, X, {a}, {a, b}}. Then X is $\tilde{g}$-connected.

Theorem 6.19. *Let f: $X \to Y$ be a $(\tilde{g}, s)$-continuous surjection. If X is $\tilde{g}$-connected, then Y is connected.*

Proof. Assume that Y is not connected space. Then there exist nonempty disjoint open sets A and B such that Y = A ∪ B. Also A and B are clopen in Y. Since f is ($\tilde{g}$, s)-continuous, f^{-1}(A) and f^{-1}(B) are $\tilde{g}$-open in X. Moreover f^{-1}(A) and f^{-1}(B) are nonempty disjoint and X = f^{-1}(A) ∪ f^{-1}(B). This shows that X is not $\tilde{g}$-connected. This contradicts the assumption that Y is not connected. □

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