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Semi-resolution Practicability of Three-Dimensional Statics of Cables from Computer Programs

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ABSTRACT: *he purpose of this paper is to present a rational method for analyzing, designing, or evaluating the spread mooring systems used with floating drilling units. This paper presents a validated model to calculate the catenary static configuration. A semi-resolution approach is presented in this paper that is capable of predicting the static performance of a caisson mooring system. The solution is derived as a function of only three parameters, which can be solved numerically by implementing different kinds of boundary conditions. The efficiency and accuracy of the method permit quick parametric studies for the optimal selection of the system particle, which is undoubtedly useful for a preliminary design. A number of numerical examples demonstrate the validity of the adopted approach. The paper contains a complete description of the test cases and reports the results in such a way that it can provide a "benchmark" test for users and programmers of computer codes for flexible riser analysis.*

1. Introduction

The main idea to create a program which could calculate cable configurations was brought about by the lack of numerical simulators in naval and offshore engineering. In recent years, several computer programs for analysis of flexible risers have been developed. The programs apply various methods for static calculations. The methods and computer codes are also different with respect to user friendliness, generality, efficiency and accuracy. Cables are widely used in offshore engineering, for example in mooring floating structures or in deploying tethered subsea units from floating vessels. Neglecting the dynamic excitation due to the wave action on the moored structure or the support vessel and the variation of the underwater current in the time domain, the cable system can be treated as a static one under the action of steady forces due to current or the thrusters of the subsea unit. In addition to its strength characteristics, marine cables often contain power conductors, instrumentation lines, fibre optics, etc. As a result, the diameter of the cable increases such that the hydrodynamic drag effect of the underwater current. There are three different types of marine cable system which can be tackled by a static theory. 1. Submersible-cable system. The cable is fixed at both ends to stationary structures. It is necessary to evaluate the variation in cable tension along its length and the configuration of the cable. 2. Towing cable system (Fig. 1) The upper end is adjoined to an advancing ship on the sea surface. The lower end is connected to a subsea unit. The unit itself may have its own propulsion system, in which case it is necessary to know the location of the subsea unit. This is of practical importance for subsea operations as it can show whether certain locations can be reached or not. 3. Mooring cable system (Fig. 2), The upper end of the cable is connected to a floating structure on the free surface, while the lower end is fixed at the sea floor by an anchor. The tension in the cable and the horizontal trail of the moored structure are the primary concerns in this case.

The importance of the static analysis of marine cables is threefold: 1. Many questions of practical importance can be

Fig. 1 Towing cable

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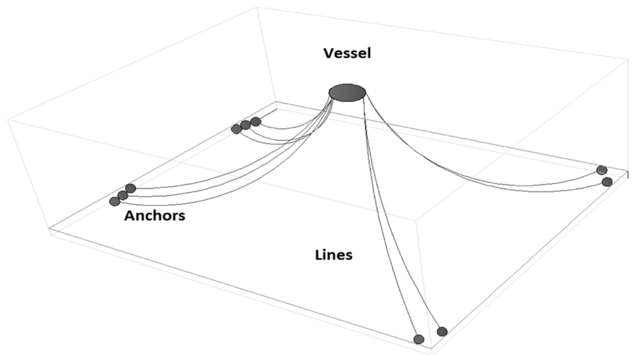

Fig. 2 Mooring system

answered without the full dynamic analysis which is undoubtedly more expensive. 2. When dynamic analysis is needed, it is often linearized as a perturbation problem around the static solution. In this case, the static analysis becomes a prerequisite to the dynamic analysis. 3. Static modelling and theoretical prediction provides a cheap and quick technique through which the interaction between the various parameters as well as the role of each parameter can be understood and a better design achieved. It is especially useful for the preliminary design. A number of different approaches have been adopted to solve the static problem under various assumptions and simplifications accordingly (Eames, 1968; Triantafyllou, 1982; Every, 1982; Macgregor, 1990). The frequently used methods for the three-dimensional quasi-static analysis of mooring cable systems include the lumped-parameter (mass) method (Leonard and Nath, 1981), the standard finite-element technique (Webster, 1975), the cable element method (Peyrot and Goulios, 1979; Peyrot, 1980) .Most assumptions are made on the following aspects:

l. Dimensionality. Quite a few practical problems can be modelled as two-dimensional problems which, needless to say, are easier than three-dimensional ones. 2. Elasticity. A lot of static problems involve little elastic deformation which provides a sound base to treat the cable as an inelastic one. The governing equations for an inelastic cable are simpler than those for an elastic cable. 3. Nature of the drag. The drag caused by the current flowing past the cable is still an unsolved problem; hence, it is an area open to different manipulations.

The cable analysis is basically a two point boundary value problem where some or all of the boundary values are known at either end of the cable. Dependent upon the types of practical problem, boundary conditions of different kinds are imposed. A common flaw present in most available methods is the inability to handle the different types of boundary conditions in a uniform manner. Another feature of the existing research is that most authors have experimented with different numerical techniques in solving the governing equations, but failed to explore and hence exploit the analytic properties, rendering the numerical solutions less efficient and less accurate. Whilst in comparison with the vast application of cables in marine operation, the theoretical research work is hardly proportional, experimental research work is even less so. Little work has been done specifically on the equilibrium configuration of marine cables, especially in the three-dimensional case. As a result, it is difficult to verify and confirm theoretical results against experimental ones. In this paper a rather general semi-analytical method has been developed. The basic idea is to consider a three-dimensional cable under a given distribution of many point loads. A compact exact solution is derived as a function of three parameters only, which can be solved numerically by implementing different kinds of boundary conditions. The real marine cable, where the drag load cannot be given beforehand, is solved by using an iterative procedure.

2. MATHEMATICAL MODELLING

2.1 Fundamental assumptions

The basic assumptions in the present modelling of mooring cables are as follows:

1. Zero torsional stiffness
2. Zero bending stiffness.
3. Non-negative tension.
4. The cable is uniform.
5. Hydrodynamic loading acting on an element of cable depends only upon the dimensions of that element, the angle of that element to the current and the current speed, and is not affected by neighbouring elements. The loading can be resolved into two components of normal and tangential forces which are dependent upon the normal component and the tangential component of the current velocity, respectively.

2.2 Coordinate system and discretization

A Cartesian coordinate system (x,y,z) is adopted, as shown in Fig. 3. Let s and p be the unstrained and the strained arch lengths along the cable, respectively. Whilst maintaining the generality, it is possible to let one end of the cable stay at the origin of the coordinate system. Conceptually a continuous marine cable can be discretised into many small segments, each under one point load which as a whole represents the distributed drag force along the cable. The end points of the segments and the point loads are numbered by the index i which runs from 0 at one end to N at the other.

2.3 Equilibrium equation

The statement of equilibrium at a point P between the coordinates P_n and P_{n+1} on the strained cable profile gives Equations (1)-(3):

$$T\frac{dx}{dp}=-V_x-\sum_{i=0}^{N}F_x^i \quad (1)$$

$$T\frac{dy}{dp}=-V_y-\sum_{i=0}^{N}F_y^i \quad (2)$$

$$T\frac{dz}{dp}=-V_z-\sum_{i=0}^{N}F_z^i-\frac{W}{L}s \quad (3)$$

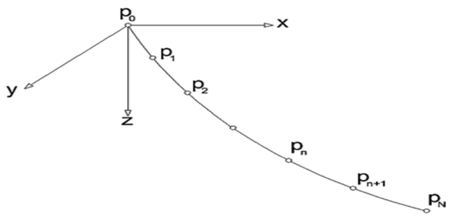

Fig. 3 Coordinate system and discretisation

where T is the cable tension, V_x, V_y and V_z are the three components of the force acting at the end s=0, F_x^i, F_y^i, and F_z^i are the external force components acting on the cable segment, L is the unstrained length of the whole cable and W is its weight in the fluid. In addition to the force equilibrium, the cable must satisfy the compatibility and constitutive relations. These are:

- *Compatibility relation,* Equations (4)*:*

$$(\frac{dx}{dp})^2+(\frac{dy}{dp})^2+(\frac{dz}{dp})^2=1 \quad (4)$$

- Constitutive relation: this is a mathematical expression of Hooke's law Equation (5) in the form:

$$T=EA(\frac{dp}{ds}-1) \quad (5)$$

where E is Young's modulus and A is the cross-sectional area of the cable in the unstrained profile.

2.4 Boundary conditions

The mathematical formulation of the problem is completed by the addition of the boundary conditions. Corresponding to the three types of marine cable system, there are three sets of mathematical boundary conditions: 1. Both ends of the cable are fixed at known points, that is Equations (6)-(7):

$$x(0)=y(0)=z(0)=0 \quad (6)$$

$$x(L)=x_L,y(L)=y_L,z(L)=z_L \quad (7)$$

where coordinates x_L, y_L and z_L are given. 2. One end is fixed, the other is subjected to known force components, that is Equations (8)-(11):

$$x(0)=y(0)=z(0)=0 \quad (8) \quad T\frac{dx}{dp}|_{s=L}=T_x \quad (9)$$

$$T\frac{dy}{dp}|_{s=L}=T_y \quad (10)$$

$$T\frac{dz}{dp}|_{s=L}=T_z \quad (11)$$

where coordinates T_x, T_y and T_z are given. 3. A combination of the previous two cases, that is Equations (12)-(15):

$$x(0)=y(0)=z(0)=0 \quad (12)$$

$$T\frac{dx}{dp}|_{s=L}=T_x \quad (13)$$

$$T\frac{dy}{dp}|_{s=L}=T_y \quad (14)$$

$$Z(L)=Z_L \quad (15)$$

3. PARAMETRIC ANALYTIC SOLUTION

By invoking the following relations:

$$\frac{dx}{ds}=\frac{dx}{dp}\frac{dp}{ds}\ ,\ \frac{dy}{ds}=\frac{dy}{dp}\frac{dp}{ds}\ ,\ \frac{dz}{ds}=\frac{dz}{dp}\frac{dp}{ds}$$

and noting that $\frac{dp}{ds}$ is given as a function of the tension T through Equation(5), we have Equations (16)-(18):

$$\frac{dx}{ds}=-\frac{1}{T}(V_x+\sum_{i=0}^{N}F_x^i)(\frac{T}{EA}+1) \quad (16)$$

$$\frac{dy}{ds}=-\frac{1}{T}(V_y+\sum_{i=0}^{N}F_y^i)(\frac{T}{EA}+1) \quad (17)$$

$$\frac{dz}{ds}=-\frac{1}{T}(V_z+\sum_{i=0}^{N}F_z^i+\frac{W}{L}s)(\frac{T}{EA}+1) \quad (18)$$

where by squaring Equations (1)-(3) and substituting them into the compatibility relation the following expression for T results Equation (19):

$$T=\sqrt{(V_x+\sum_{i=0}^{N}F_x^i)^2+(V_y+\sum_{i=0}^{N}F_y^i)^2+(V_z+\sum_{i=0}^{N}F_z^i+\frac{W}{L}s)^2} \quad (19)$$

Integrating these equations over the interval $[s_n,s_{n+1}]$ gives. Equations (20)-(22):

$$x(s_{n+1})=x(s_n)+\int_{s_n}^{s_{n+1}}\frac{dx}{ds}ds=x(s_n)+\Delta x(s_n) \quad (20)$$

$$y(s_{n+1})=y(s_n)+\int_{s_n}^{s_{n+1}}\frac{dy}{ds}ds=y(s_n)+\Delta y(s_n) \quad (21)$$

$$z(s_{n+1})=z(s_n)+\int_{s_n}^{s_{n+1}}\frac{dz}{ds}ds=z(s_n)+\Delta z(s_n) \quad (22)$$

After some mathematic manipulations, the integrations result in the following Equation (23):

$$\Delta x(s_n) = -\frac{V_x + \sum_{i=0}^{N} F_x^i}{EA}(s_{n+1} - s_n) + \frac{L(V_x + \sum_{i=0}^{N} F_x^i)}{W} \times \left[\sinh^{-1} \frac{-(V_Z + \sum_{i=0}^{N} F_z^i + \frac{W}{L} s_{n+1})}{\sqrt{(V_x + \sum_{i=0}^{N} F_x^i)^2 + (V_y + \sum_{i=0}^{N} F_y^i)^2}} - \sinh^{-1} \frac{-(V_Z + \sum_{i=0}^{N} F_z^i + \frac{W}{L} s_n)}{\sqrt{(V_x + \sum_{i=0}^{N} F_x^i)^2 + (V_y + \sum_{i=0}^{N} F_y^i)^2}} \right] \tag{23}$$

These analytical solutions are given as functions of three unknown

parameters, that is, V_x , V_y and V_z. The remaining part of this section is concerned with their solution Equations (24)-(25):

$$\Delta y(s_n) = -\frac{V_y + \sum_{i=0}^{N} F_y^i}{EA}(s_{n+1} - s_n) + \frac{L(V_y + \sum_{i=0}^{N} F_y^i)}{W} \times \left[\sinh^{-1} \frac{-(V_Z + \sum_{i=0}^{N} F_z^i + \frac{W}{L} s_{n+1})}{\sqrt{(V_x + \sum_{i=0}^{N} F_x^i)^2 + (V_y + \sum_{i=0}^{N} F_y^i)^2}} - \sinh^{-1} \frac{-(V_Z + \sum_{i=0}^{N} F_z^i + \frac{W}{L} s_n)}{\sqrt{(V_x + \sum_{i=0}^{N} F_x^i)^2 + (V_y + \sum_{i=0}^{N} F_y^i)^2}} \right] \tag{24}$$

$$\Delta z(s_n) = \frac{L}{2WEA} \left[\begin{array}{l} (V_z + \sum_{i=0}^{N} F_z^i + \frac{W}{L} s_n)^2 - \\ (V_z + \sum_{i=0}^{N} F_z^i + \frac{W}{L} s_{n+1})^2 \end{array} \right] + \frac{L}{W} \left[\begin{array}{l} \sqrt{(V_x + \sum_{i=0}^{N} F_x^i)^2 + (V_y + \sum_{i=0}^{N} F_y^i)^2 + (V_z + \sum_{i=0}^{N} F_z^i + \frac{W}{L} s_n)^2} - \\ \sqrt{(V_x + \sum_{i=0}^{N} F_x^i)^2 + (V_y + \sum_{i=0}^{N} F_y^i)^2 + (V_z + \sum_{i=0}^{N} F_z^i + \frac{W}{L} s_{n+1})^2} \end{array} \right] \tag{25}$$

3.1 Submersible cable

By using Equations (20)-(22) repeatedly, we have Equations (26)-(28):

$$\sum_{i=0}^{N-1} \Delta x(s_i) = x(L) - x(0) \tag{26}$$

$$\sum_{i=0}^{N-1} \Delta y(s_i) = y(L) - y(0) \tag{27}$$

$$\sum_{i=0}^{N-1} \Delta z(s_i) = z(L) - z(0) \tag{28}$$

The solution of this nonlinear algebraic equation system gives the answer for the three unknowns. Once, V_x , V_y and V_z are known, the coordinates of any points between s_n and s_{n+1} on the strained cable profile are given by Equations (29)-(31):

$$x(s) = x(0) + \sum_{i=0}^{N-1} \Delta x(s_i) + \int_{s_n}^{s} \frac{dx}{ds} ds \tag{29}$$

$$y(s) = y(0) + \sum_{i=0}^{N-1} \Delta y(s_i) + \int_{s_n}^{s} \frac{dy}{ds} ds \tag{30}$$

$$z(s) = z(0) + \sum_{i=0}^{N-1} \Delta z(s_i) + \int_{s_n}^{s} \frac{dz}{ds} ds \tag{31}$$

and the tension is given by Equation (19).

3.2 Towing cable

This is an easier situation. The unknown , V_x , V_y and V_z can be calculated directly by the following relations Equations (32)-(34):

$$V_x = -T_x - \sum_{i=0}^{N} F_x^i \tag{32}$$

$$V_y = -T_y - \sum_{i=0}^{N} F_y^i \tag{33}$$

$$V_z = -T_z - \sum_{i=0}^{N} F_z^i - W \tag{34}$$

3.3 Mooring cable

In this case, two force components can be found readily from Equations (35)-(36):

$$V_x = -T_x - \sum_{i=0}^{N} F_x^i \tag{35}$$

$$V_y = -T_y - \sum_{i=0}^{N} F_y^i \tag{36}$$

and the third one T_z is solved from Equation (37):

$$\sum_{i=0}^{N-1} \Delta z(s_i) = z(L) - z(0) \tag{37}$$

4. DRAG FORCE

Let $U = \{U_x, U_y, U_z\}$ represent the averaged current velocity vector at an ith cable segment which has two end points with coordinates (x_i, y_i, z_i) and $(x_{i-1}, y_{i-1}, z_{i-1})$. The normal drag component F_N and the tangential drag component F_τ are given by Equations (38)-(39):

$$F_N = \frac{1}{2} \rho C_N dl \, |U_N| U_N \tag{38}$$

$$F_\tau = \frac{\pi}{2} \rho C_\tau dl \, |U_\tau| \, U_\tau \tag{39}$$

Where C_N and C_τ are the normal and the tangential drag coefficients, respectively, d is the diameter of the cable and ρ is the density of the fluid. l, U_τ , U_N are given by:

$$l=\sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2+(z_i-z_{i-1})^2}$$

$$U_\tau=\{U_{\tau x},U_{\tau y},U_{\tau z}\},$$

$$U_{\tau x}=\frac{1}{l^2}\left[U_x(x_i-x_{i-1})^2+U_y(x_i-x_{i-1})(y_i-y_{i-1})+U_z(x_i-x_{i-1})(z_i-z_{i-1})\right]$$

$$U_{\tau y}=\frac{1}{l^2}\left[U_x(x_i-x_{i-1})(y_i-y_{i-1})+U_y(y_i-y_{i-1})^2+U_z(y_i-y_{i-1})(z_i-z_{i-1})\right]$$

$$U_{\tau z}=\frac{1}{l^2}\left[U_x(x_i-x_{i-1})(z_i-z_{i-1})+U_y(y_i-y_{i-1})(z_i-z_{i-1})+U_z(z_i-z_{i-1})^2\right]$$

$$U_N=\{U_x-U_{\tau x},U_y-U_{\tau y},U_z-U_{\tau z}\}.$$

5. NUMERICAL ITERATION

To calculate the cable profile by using Equations(29)-(31), F_x^i, F_y^i, and F_z^i (i=0,1,2,...N) must be known. These drag forces, however, depend upon the cable profile, and they cannot be known beforehand. In fact they form a part of the solution themselves. As a result, an iterative scheme must be used to solve the problem. Inside this global iteration, there exists another smaller iterative loop to solve the nonlinear algebraic equation system of Equations(26)-(28) or Equation (37), if the submersible cable problem or mooring cable problem is to be solved. The *Newton- Raphson* method is used here to seek the solution by solving a succession of linear equation systems. As is always true for a nonlinear problem involving an iterative solution procedure, the initial estimation plays an important role. A bad starting estimate can either deteriorate the overall efficiency or make the procedure totally unworkable. In the present study the initial approximation is based upon the zero hydrodynamic load situation. This is good for the cases where the cable experiences light drag force in comparison with its own weight in the fluid. When the drag force becomes dominant, however, more iterations are needed to increase the current speed step-by-step until the prescribed value. The results of the previous iteration serve as the starting estimate of the current iteration.

6. NUMERICAL EXAMPLES

The above analysis forms a suite of programs which predicts the equilibrium profile of a marine cable. In this section some results are presented to demonstrate the validity of the adopted method. Fig. 4 shows the configuration of an elastic steel catenary in air predicted by the present method. It is suspended between two rigid supports which are not at the same level. Shown on the same figure is the exact analytic solution (Irvine, 1981). They agree with each other very well. Fig. 5 shows the change in configuration of a hanging cable in air under the action of a point load. Fig. 6 and Fig. 7 show the theoretical predictions of marine cables in the presence of ocean current against the results of experimental measurement (Kojima et al., 1986). In Fig. 4 the cable is 340 m long and in Fig. 5 it is 310 m long. Both have a diameter of 0.02 m, Young's modulus of 2×10^{11} N/m^2, and the same weight distribution in water at 2.25 N/m. The experimental results correspond to a current speed of 0.514 m/sec for both cases. The agreement between the theoretical and the experimental results is good. Also shown on the two figures is the effect of the ocean current through illustration of the different cable configurations at different current speeds. It is clear that the current plays a significant role. Fig. 8 shows the configuration of a three-dimensional submersible cable. A verification of the results is not possible since no report on three-dimensional experimental work is available.

Table 1 shows the configuration of an elastic steel catenar

Table 1 Configuration of steel catenary in air

X	N	E (N/m^2)	d (m)
0	0	2×10^{11}	0.02
5.25	-2.5	2×10^{11}	0.02
25.67	-12.5	2×10^{11}	0.02
58.64	-17.5	2×10^{11}	0.02
65.85	-15.5	2×10^{11}	0.02
79.64	-12.65	2×10^{11}	0.02
82.91	-9.56	2×10^{11}	0.02
92.5	-8.6	2×10^{11}	0.02

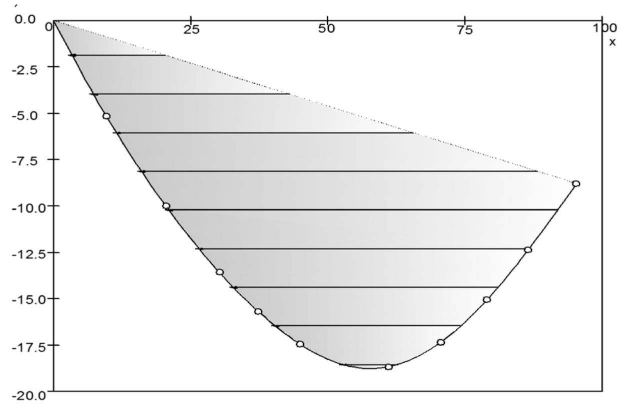

Fig. 4 Configuration of steel catenary in air. L=100m, d=0.02 m, E=2×10^{11} N/m^2 (-) presentmethod; (O) analytic solution (Irvine,1981)

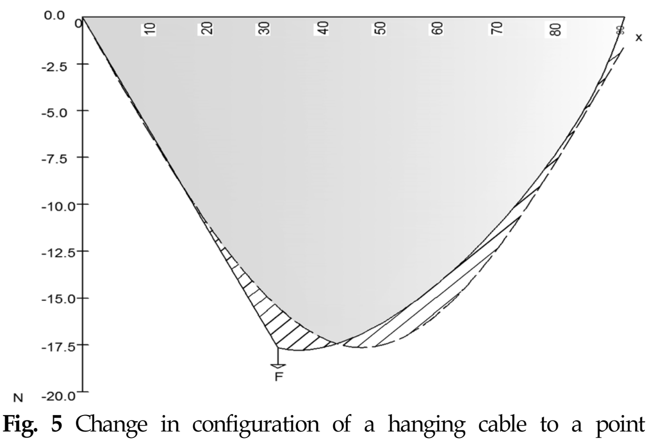

Fig. 5 Change in configuration of a hanging cable to a point load. L=100m, d=0.02 m, E=2×10^{11} N/m^2

yin air predicted by the present method. It is suspended between two rigid supports which are not at the same level. Shownon the Fig. 4 is the exact analytic solution (Irvine, 1981).

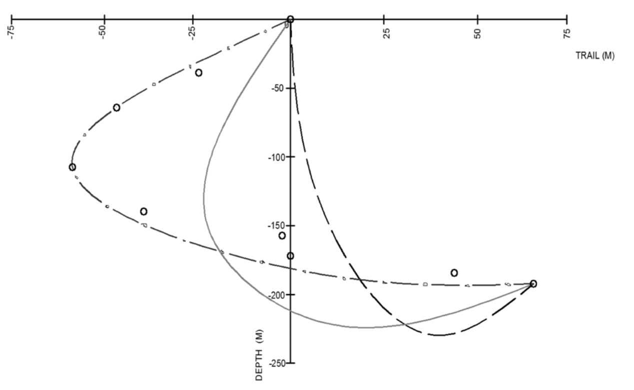

Fig. 6 Measured and predicted configurations of a marine cable. (O) measurements, current speed U=0.514 m/sec; (-) present method, U=0.514 m/sec; (-.-) present method, U=0.2 m/sec; (---) present method, no current

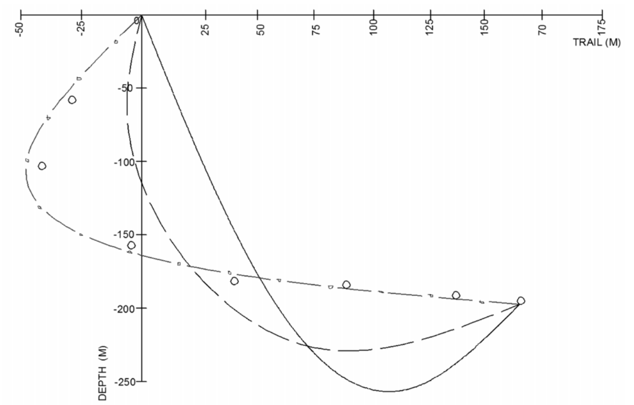

Fig. 7 Measured and predicted configurations of a marine cable. (O) measurements, current speed U=0.514 m/sec; (-) present method, U=0.514 m/sec; (-.-) present method, U= 0.2 m/sec; (---) present method, no current

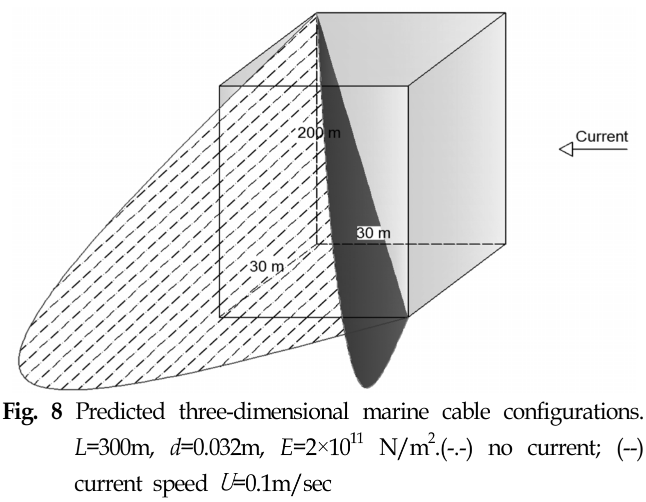

Fig. 8 Predicted three-dimensional marine cable configurations. L=300m, d=0.032m, E=2×10^{11} N/m^2.(-.-) no current; (--) current speed U=0.1m/sec

Conclusions

The approach developed in this paper is of practical importance, with the capacity of answering various questions arising from subsea interventions involving tethered subsea systems. In addition to that, the semi-analytic method, within the limitations of all assumptions made, is highly accurate and efficient. The semi-analytic method analysis, which shows whether a facility will pay for itself, is by far the most important tools to be used in reaching the right investment decision. However, they entail rather laborious and routine calculations which are less interesting to make than the calculation. The appeal of such performance calculation techniques (for example, that of simulation with the use of a computer) can lead to an excessive amount of the valuable project time being spent on them. If a team has a suitable simulation model available, and is experienced in its use, then the work can be done quickly.

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