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Tightened-Normal-Tightened Group Acceptance Sampling Plan for Assuring Percentile Life

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ABSTRACT

The present paper extends the idea of tightened-normal-tightened sampling scheme to group acceptance sampling plans under the time truncated life tests. We consider three famous distributions that are widely used in the area of reliability such as the generalized exponential distribution, the Weibull distribution, and the Birnbaum-Saunders distribution in the proposed sampling plan. The plan parameters are determined such that the producer's risk and the consumer's risk are satisfied at the specified median life. Extensive tables showing plan parameters are provided at various values of the experiment time and the consumer's risk for each of three distributions for the practical use. Some examples are given to illustrate the procedure of the proposed plan.

Keywords: Tightened Inspection, Normal Inspection, Group Acceptance Sampling Plan, Life Test, Producer and Consumer Risks

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1. INTRODUCTION

Acceptance sampling plans are widely used for automotive products, pharmaceutical products and so on in the areas of compliance testing and quality assurance. The tightened-normal-tightened (TNT) sampling schemes are popularly used in compliance sampling (Schilling, 1982). The tightened inspection can be used when the quality of a product deteriorates and the normal inspection is used when the quality is found to be good.

Calvin (1977) proposed a TNT sampling plan utilizing zero acceptance number in two single sampling plans of different sample sizes, namely, n_1 and n_2 ($< n_1$) together with the switching rules, which is designated as TNT-(n_1, n_2; 0). Soundararajan and Vijayaraghavan (1992) developed tables and procedures for the selection of TNT-(n_1, n_2; c) scheme for various entry parameters. Vijayaraghavan and Soundararajan (1996) proposed another type of TNT sampling scheme using one sample size and two acceptance numbers, designated as TNT-(n; c_1, c_2) scheme. Muthuraj and Senthilkumar (2006) studied variables TNT sampling schemes. Recently, Senthilkumar and Muthuraj (2010) proposed variable TNT variables sampling schemes known as TNTVSS (n_1, n_2; k) and Aslam *et al*. (2010) designed a TNT scheme for Weibull distributions.

Group acceptance sampling plans (GASP) are considered more economical than the ordinary sampling plans in terms of cost savings due to the reduction in the number of testers needed. GASP based on truncated life tests are also useful to reduce the inspection time. The GASP based on truncated life tests for various lifetime distributions have been studied by many authors including Pascual and Meeker (1998), Vlcek *et al*. (2003), Jun *et al*. (2006), Aslam and Jun (2009), and Aslam *et al*. (2011).

To the best of our knowledge, there is no work on TNT sampling scheme applied to the GASP based on time truncated life tests. Therefore, the purpose of this paper is to propose TNT sampling scheme applied to the GASP based on time truncated life tests for three well-known life distributions: the generalized exponential distribution, the Weibull distribution, and the Birnbaum-Saunders distribution. The rest of the paper is organized as follows: the design of the proposed plan is given in Section 2. Plan parameters under various distributions are given in Section 3. Some examples and comparison of results are given in Section 4. Concluding remarks are given in the last section.

2. DESIGN OF THE PROPOSED PLAN

We propose the following TNT group acceptance sampling plan (TNTGASP) using testers having group size r based on the time truncated life test with duration t_0.

Step 1: Start with the tightened inspection. During the tightened inspection, select a random sample of size n from the lot and distribute r items to g groups. Accept the lot if the total number of failures from all groups by time t_0 is smaller than or equal to c_1. Reject the lot and truncate the test as soon as the number of failures reaches c_1. If t consecutive lots are accepted on the tightened inspection, then switch to the normal inspection as in Step 2 given below.

Step 2: During the normal inspection, select a random sample of size n from the lot and distribute r items to g groups. Accept the lot if the total number of failures by time t_0 is smaller than or equal to c_2 ($\geq c_1$). Reject the lot and truncate the test as soon as the number of failures reaches c_2. If an additional lot is rejected among the next s ($\geq$ t) lots after the rejection, then immediately revert to the tightened inspection as in Step 1 given above.

The proposed plan is characterized by five parameters g, c_1, c_2, s, and t. It should be noted that the group size r and the test duration t_0 in the proposed plan are not plan parameters but they are pre-specified. The proposed plan is reduced to the TNT plan by Vijayaraghavan and Soundararajan (1996) when g = 1. Vijayaraghavan and Soundararajan (1996) developed the TNT plan by fixing the values of s but in the proposed plan we will consider it as a plan parameter.

According to Calvin (1977), the operating characteristics (OC) function of a TNT scheme is given by

$$P_a(p)=\frac{P_1(1-P_2^s)(1-P_1^t)(1-P_2)+P_2P_1^t(1-P_1)(1-P_2^s)}{(1-P_2^s)(1-P_1^t)(1-P_2)+P_1^t(1-P_1)(1-P_2^s)} \quad (1)$$

where p is the failure probability of an item before the termination time t_0, P_1 is the lot acceptance probability under the tightened inspection, and P_2 is the lot acceptance probability under the normal inspection.

Under the GASP with g and the acceptance number of c, the lot acceptance probability is given by

$$P=\sum_{i=0}^{c}\binom{rg}{i}p^i(1-p)^{rg-i}$$

So, P_1 and P_2 for the proposed plan are reduced to

$$P_1=\sum_{i=0}^{c_1}\binom{rg}{i}p^i(1-p)^{rg-i} \quad (2)$$

$$P_2=\sum_{i=0}^{c_2}\binom{rg}{i}p^i(1-p)^{rg-i} \quad (3)$$

The plan parameters of an acceptance sampling plan are usually determined by the two-point approach by considering the producer's risk and the consumer's risk at the same time. Let α be the producer's risk and β be the consumer's risk. Sometimes, $1-\alpha$ is called as the producer's confidence level, whereas $1-\beta$ is called as the consumer's confidence level. A producer prefers that the lot acceptance probability should be larger than or equal to the producer's confidence level, and a consumer wants it to be smaller than or equal to the consumer's risk. So, to find the plan parameters we need to solve two inequalities at the same time by considering both risks. These inequalities for the proposed plan are given in Eqs. (4) and (5):

$$P_a(p)=\frac{P_{11}(1-P_{21}^s)(1-P_{11}^t)(1-P_{21})+P_{21}P_{11}^t(1-P_{11})(1-P_{21}^s)}{(1-P_{21}^s)(1-P_{11}^t)(1-P_{21})+P_{11}^t(1-P_{11})(1-P_{21}^s)}\leq\beta \quad (4)$$

where

$$P_{11}=\sum_{i=0}^{c_1}\binom{rg}{i}p_1^i(1-p_1^i)^{rg-i}$$

$$P_{21}=\sum_{i=0}^{c_2}\binom{rg}{i}p_1^i(1-p_1^i)^{rg-i}$$

and p_1 is the failure probability of an item at the limiting quality level (LQL).

$$P_a(p_2)=\frac{P_{12}(1-P_{22}^s)(1-P_{12}^t)(1-P_{22})+P_{22}P_{12}^t(1-P_{12})(1-P_{22}^s)}{(1-P_{22}^s)(1-P_{12}^t)(1-P_{22})+P_{12}^t(1-P_{12})(1-P_{22}^s)}\geq 1-\alpha \quad (5)$$

where

$$P_{12}=\sum_{i=0}^{c_1}\binom{rg}{i}p_2^i(1-p_2^i)^{rg-i}$$

$$P_{22}=\sum_{i=0}^{c_2}\binom{rg}{i}p_2^i(1-p_2^i)^{rg-i}$$

and p_2 is the failure probability of an item at the acceptable quality level (AQL).

3. PROPOSED PLAN UNDER LIFETIME DISTRIBUTION

In this section, we will present the TNTGASP plan for the generalized exponential distribution, the Weibull distribution, and the Birnbaum-Saunders distribution. The brief introduction of each distribution is also given. Here, we are particularly designing the proposed plan for assuring the median life although any percentile life can be considered similarly.

3.1 Under Generalized Exponential Distribution

The generalized exponential distribution was originally derived by Gupta and Kundu (1999), and these authors observed that the two-parameter generalized exponential distribution can be used quite effectively in the area of acceptance sampling and reliability for the analysis of lifetime data in place of two-parameter Weibull and gamma distributions. According to Gupta and Kundu (1999), the generalized exponential distribution has an increasing or a decreasing failure rate depending on the value of the shape parameter. The two-parameter generalized exponential distribution has the following cumulative distribution function (cdf):

$$F(t;\delta,\lambda)=\left(1-exp\left(-\tfrac{t}{\lambda}\right)\right)^{\delta} \text{ for } \quad t>0 \tag{6}$$

Here, $\delta>0$ and $\lambda>0$ are the shape and the scale parameters, respectively. The qth percentile of the distribution is given by

$$\theta_q=-\lambda ln\left(1-q^{1/\delta}\right) \tag{7}$$

The median of the distribution is given by

$$\theta_m=-\lambda ln\left(1-(\tfrac{1}{2})^{1/\delta}\right) \tag{8}$$

It is convenient to write the experiment time as the multiple of the target median θ_m^0 such that $t_0=\theta_m^0$ for a constant a. Then, the failure probability of an item by time t_0 in terms of the median life is given as

$$p=\left[1-exp\left(aln(1-(1/2)^{1/\delta}\right)/(\theta_m/\theta_m^0)\right]^{\delta} \tag{9}$$

Under the generalized exponential distribution, the failure probability p_1 at the consumer's risk is determined for $\theta_m/\theta_m^0=1$ and the failure probability p_2 at the producer's risk is determined for various values of median ratio.

The plan parameters of the proposed plan by satisfying Eqs. (4) and (5) are determined for various values of the shape parameters, the consumer's risk, the median ratio, and the group size. The producer's risk is specified as 0.05 for all cases. The plan parameters are placed in Tables 1 and 2 for the shape parameters of 2 and 3, respectively.

From these tables, we note that the number of groups required increases as the consumer's risk decreases, but it remains unchanged within the same consumer's risk as the median ratio increases from 2 to 6. But, for the same group size, we note the decreasing trend in g as termination time a increases from 0.5 to 1.0. We also note that the value of g remains the same generally when the shape parameter changes from 2 to 3.

3.2 Under the Weibull Distribution

The Weibull distribution is widely used in the area of acceptance sampling and reliability analysis. The failure time or the lifetime of electronic components is usually modeled by this distribution. Several authors including Jun *et al*. (2006) used the Weibull distribution to develop variable sampling plans under various testing

Table 1. Proposed TNTGASP for generalized exponential distribution with $\delta=2$

β	θ_m/θ_m^0	r = 5				r = 10			
		a = 0.5		a = 1.0		a = 0.5		a = 1.0	
		g, c_1, c_2	s, t	g, c_1, c_2	s, t	g, c_1, c_2	s, t	g, c_1, c_2	s, t
0.25	2	2, 0, 7	10, 13	1, 0, 9	3, 17	1, 1, 4	17, 19	1, 1, 8	5, 17
	4	2, 0, 2	11, 18	1, 1, 2	10, 20	1, 0, 2	14, 14	1, 0, 6	8, 9
	6	2, 0, 6	12, 17	1, 0, 6	12, 16	1, 0, 3	12, 16	1, 1, 3	5, 20
0.10	2	2, 0, 8	14, 19	1, 0, 5	7, 10	1, 0, 4	5, 10	1, 2, 9	10, 11
	4	2, 0, 6	11, 15	1, 0, 5	15, 15	1, 0, 3	7, 15	1, 1, 3	6, 10
	6	2, 0, 3	13, 14	1, 0, 6	7, 16	1, 0, 4	7, 13	1, 1, 3	1, 17
0.05	2	3, 0, 3	4, 13	2, 0, 14	4, 13	1, 0, 4	2, 9	1, 1, 8	11, 11
	4	3, 0, 4	5, 15	2, 0, 8	6, 15	1, 0, 4	5, 7	1, 1, 7	5, 16
	6	3, 0, 3	16, 20	2, 0, 10	2, 5	1, 0, 2	7, 12	1, 1, 9	11, 14

TNTGASP: tightened-normal-tightened group acceptance sampling plan.

Table 2. Proposed TNTGASP for generalized exponential distribution with $\delta = 3$

β	θ_m / θ_m^0	r = 5				r = 10			
		$a = 0.5$		$a = 1.0$		$a = 0.5$		$a = 1.0$	
		g, c_1, c_2	s, t	g, c_1, c_2	s, t	g, c_1, c_2	s, t	g, c_1, c_2	s, t
	2	2, 0, 6	1, 15	1, 0, 5	6, 9	1, 0, 3	3, 8	1, 0, 8	5, 7
0.25	4	2, 0, 2	16, 17	1, 0, 5	5, 20	1, 0, 2	4, 16	1, 1, 8	11, 19
	6	2, 0, 8	14, 17	1, 0, 8	12, 19	1, 0, 4	12, 17	1, 2, 7	13, 16
	2	3, 0, 7	10, 18	2, 0, 10	11, 19	1, 0, 3	5, 5	1, 1, 7	12, 17
0.10	4	3, 0, 3	7, 19	2, 0, 4	1, 10	1, 0, 4	3, 19	1, 2, 5	15, 17
	6	3, 0, 9	4, 16	2, 0, 14	17, 17	1, 0, 3	2, 13	1, 2, 9	9, 12
	2	4, 0, 5	11, 18	2, 0, 14	17, 18	1, 0, 4	7, 8	1, 0, 8	4, 13
0.05	4	4, 0, 9	2, 20	2, 0, 7	7, 16	1, 0, 4	6, 6	1, 1, 8	3, 3
	6	4, 0, 8	11, 13	2, 0, 4	2, 5	1, 0, 1	11, 18	1, 1, 6	4, 4

TNTGASP: tightened-normal-tightened group acceptance sampling plan.

scheme. Aslam and Jun (2009) and Aslam *et al*. (2011) developed group sampling plans for the Weibull distribution. The cdf of the distribution is given by

$$F(t;\ \gamma,\ \lambda) = 1 - \exp(-(t/\lambda)^{\gamma}),\ \ t > 0 \tag{10}$$

where $\gamma > 0$ and $\lambda > 0$ are the shape and the scale parameters, respectively.

The q^th^ percentile of the Weibull distribution is given by

$$\theta_q = \lambda\left(\ln\left(\frac{1}{1-q}\right)\right)^{1/\gamma} \tag{11}$$

The median life under the Weibull distribution is given by

$$\theta_m = \lambda\left(\ln(2)\right)^{1/\gamma} \tag{12}$$

The failure probability under the Weibull distribution is given by

$$p = 1 - exp\left[-a^{\gamma}(\theta_m/\theta_m^0)^{-\gamma}\, ln(2)\right] \tag{13}$$

Under the Weibull distribution, the failure probability p_1 is determined at $\theta_m/\theta_m^0 = 1$ and the failure probability p_2 is determined at various values of the median ratio. The plan parameters for the proposed plan under the Weibull distribution are presented in Tables 3 and 4 for the shape parameter $\gamma = 2$ and 3, respectively.

From Tables 3 and 4, we see that the values of g increses as the consumer's risk β changes from 0.25 to 0.05. When all other things remain unchanged, we note the increasing trend in g as the shape parameter of Weibull distribution changes from 2 to 3.

3.3 Under Birnbaum-Saunders Distribution

The Birnbaum-Saunders distribution is used to model the failure time of the fatigue process, originally derived by Birnbaum and Saunders (1969). Desmond

Table 3. Proposed TNTGASP for Weibull distribution with $\gamma = 2$

β	θ_m / θ_m^0	r = 5				r = 10			
		$a = 0.5$		$a = 1.0$		$a = 0.5$		$a = 1.0$	
		g, c_1, c_2	s, t	g, c_1, c_2	s, t	g, c_1, c_2	s, t	g, c_1, c_2	s, t
	2	2, 0, 4	8, 11	1, 0, 4	5, 9	1, 0, 4	10, 13	1, 3, 9	9, 15
0.25	4	2, 0, 1	5, 19	1, 1, 3	2, 20	1, 0, 3	10, 15	1, 0, 5	4, 17
	6	2, 0, 5	8, 12	1, 1, 2	7, 17	1, 0, 5	19, 19	1, 0, 9	4, 7
	2	3, 0, 7	11, 16	1, 0, 4	7, 12	2, 0, 8	7, 20	1, 2, 6	2, 12
0.10	4	3, 0, 5	1, 20	1, 0, 1	8, 9	2, 0, 8	14, 15	1, 0, 8	15, 18
	6	3, 0, 5	2, 11	1, 0, 3	2, 15	2, 0, 13	3, 16	1, 1, 6	4, 19
	2	4, 0, 8	10, 17	1, 0, 3	2, 4	2, 0, 7	13, 13	1, 1, 4	2, 5
0.05	4	4, 0, 2	9, 13	1, 0, 3	3, 17	2, 0, 18	12, 19	1, 0, 4	12, 17
	6	4, 0, 5	2, 19	1, 0, 2	9, 20	2, 0, 15	9, 20	1, 0, 5	1, 11

TNTGASP: tightened-normal-tightened group acceptance sampling plan.

Table 4. Proposed TNTGASP for Weibull distribution with $\gamma = 3$

β	θ_m / θ_m^0	r = 5				r = 10			
		a = 0.5		a = 1.0		a = 0.5		a = 1.0	
		g, c_1, c_2	s, t	g, c_1, c_2	s, t	g, c_1, c_2	s, t	g, c_1, c_2	s, t
	2	4, 0, 2	4, 5	1, 0, 4	17, 20	2, 0, 8	14, 15	1, 2, 9	1, 8
0.25	4	4, 0, 4	2, 18	1, 0, 1	4, 20	2, 0, 1	13, 15	1, 0, 3	3, 12
	6	4, 0, 2	13, 18	1, 0, 1	18, 19	2, 0, 6	13, 20	1, 3, 5	9, 10
	2	6, 0, 5	5, 8	1, 0, 4	7, 14	3, 0, 14	10, 17	1, 2, 9	10, 14
0.10	4	6, 0, 5	16, 17	1, 0, 3	17, 18	3, 0, 10	2, 10	1, 2, 4	10, 11
	6	6, 0, 11	8, 15	1, 0, 1	4, 5	3, 0, 2	5, 13	1, 0, 7	9, 19
	2	7, 0, 13	11, 16	1, 0, 2	8, 10	4, 0, 15	3, 14	1, 1, 6	10, 19
0.05	4	7, 0, 6	10, 18	1, 0, 4	13, 20	4, 0, 5	4, 12	1, 1, 8	7, 12
	6	7, 0, 12	1, 20	1, 0, 3	2, 8	4, 0, 10	8, 19	1, 0, 7	11, 18

TNTGASP: tightened-normal-tightened group acceptance sampling plan.

(1985) discussed the applications of the Birnbaum-Saunders distribution in biological sciences. Johnson *et al.* (1994) provided the many applications of Birnbaum-Saunders distribution in a wide variety of contexts. Baklizi and EI Masri (2004) used the Birnbaum-Saunders distribution to develop the ordinary acceptance sampling plan. Balakrishnan *et al.* (2007) developed acceptance sampling plans based on truncated life tests for the generalized Birnbaum-Saunders distribution. Recently, Lio *et al.* (2010) developed GASP using the percentile life of the Birnbaum-Saunders distribution, and Shoaib *et al.* (2011) developed acceptance sampling plans for the Birnbaum-Saunders distribution. The cdf of the Birnbaum-Saunders distribution is expressed in terms of the cdf of a standard normal (Φ) as follows:

$$F(t;\, m, \lambda) = \Phi\left[\frac{1}{m}\left\{\left(\frac{t}{\lambda}\right)^{1/2} - \left(\frac{\lambda}{t}\right)^{1/2}\right\}\right],\ t > 0 \qquad (14)$$

where m is the shape parameter and λ is the scale parameter. The scale parameter of Birnbaum-Saunders distribution is itself the median of the distribution. Therefore, we will use λ as the quality parameter to develop the proposed plan. It is convenient to write the experiment time as the multiple of the target median (λ_m^0) such that $t_0 = a\lambda_m^0$ for a constant a. The failure probability of an item before experiment time is given by

$$p = \Phi\left[\frac{1}{m}\left\{\left(\frac{a}{\lambda/\lambda_m^0}\right)^{1/2} - \left(\frac{\lambda/\lambda_m^0}{a}\right)^{1/2}\right\}\right] \qquad (15)$$

Tables 5 and 6 are constructed for the Birnbaum-Saunders distribution when the shape parameter is 1 and 1.5, respectively. From these tables, we see that the shape parameter does not quite affect the number of groups required.

3.4 Examples

Suppose that an experimenter wants to adopt the TNTGASP having group size of 10 to test a product

Table 5. Proposed TNTGASP for the Birnbaum-Saunders distribution when m = 1

β	θ_m / θ_m^0	r = 5				r = 10			
		a = 0.5		a = 1.0		a = 0.5		a = 1.0	
		g, c_1, c_2	s, t	g, c_1, c_2	s, t	g, c_1, c_2	s, t	g, c_1, c_2	s, t
	2	2, 0, 3	2, 10	1, 1, 4	15, 18	1, 0, 4	1, 5	1, 2, 8	16, 20
0.25	4	2, 0, 4	7, 15	1, 1, 2	11, 16	1, 0, 6	9, 11	1, 0, 5	10, 20
	6	2, 0, 6	10, 20	1, 0, 1	1, 12	1, 0, 3	20, 20	1, 3, 4	10, 17
	2	2, 0, 7	5, 7	1, 0, 4	6, 6	1, 0, 9	14, 17	1, 2, 9	6, 12
0.10	4	2, 0, 4	7, 13	1, 0, 4	16, 17	1, 0, 6	7, 14	1, 1, 2	5, 13
	6	2, 0, 5	4, 15	1, 0, 1	11, 14	1, 0, 7	8, 11	1, 0, 9	19, 20
	2	3, 0, 14	7, 16	1, 0, 4	2, 5	2, 1, 6	5, 10	1, 1, 9	8, 11
0.05	4	3, 0, 8	4, 9	1, 0, 2	3, 6	2, 0, 16	15, 17	1, 1, 5	16, 17
	6	3, 0, 13	8, 18	1, 0, 2	3, 15	2, 0, 12	18, 18	1, 1, 9	4, 13

TNTGASP: tightened-normal-tightened group acceptance sampling plan.

Table 6. Proposed TNTGASP for the Birnbaum-Saunders distribution when m = 1.5

β	θ_m / θ_m^0	r = 5				r = 10			
		a = 0.5		a = 1.0		a = 0.5		a = 1.0	
		g, c_1, c_2	s, t	g, c_1, c_2	s, t	g, c_1, c_2	s, t	g, c_1, c_2	s, t
0.25	2	2, 0, 6	1, 20	1, 0, 3	2, 7	1, 0, 4	1, 9	1, 1, 9	17, 18
	4	2, 0, 1	12, 12	1, 0, 2	2, 4	1, 0, 6	19, 20	1, 1, 6	13, 17
	6	2, 0, 6	10, 15	1, 0, 3	6, 10	1, 0, 2	8, 17	1, 2, 4	9, 11
0.10	2	3, 0, 9	4, 11	1, 0, 4	4, 9	2, 0, 14	6, 12	1, 1, 7	9, 13
	4	3, 0, 1	9, 20	1, 0, 4	6, 8	2, 0, 9	7, 14	1, 0, 4	9, 14
	6	3, 0, 1	8, 11	1, 0, 3	1, 15	2, 0, 5	2, 8	1, 0, 5	1, 6
0.05	2	4, 0, 12	7, 15	2, 0, 9	2, 10	2, 0, 15	2, 14	1, 0, 9	15, 15
	4	4, 0, 6	8, 9	2, 0, 8	6, 9	2, 0, 6	12, 12	1, 0, 5	9, 11
	6	4, 0, 7	17, 20	2, 0, 7	2, 4	2, 0, 1	6, 9	1, 0, 3	6, 19

TNTGASP: tightened-normal-tightened group acceptance sampling plan.

using the time truncated life test with the duration of t_0 = 500 hours. Let the target median life of the product is 1,000 hours, while the producer's risk is α = 0.05 at the median ratio of 2 and the consumer's risk is β = 0.10. Suppose that the lifetime of the product follows the generalized exponential distribution with the shape parameter δ = 2. Then the plan parameters from Table 1 are as follows: g = 1, c_1 = 0, c_2 = 4, s = 5, t = 10. The plan is implemented as follows:

Step 1: Start with the tightened inspection. Select a random sample of size 10 from a lot and distribute 10 items to the single group. Accept the lot if there are no failures by 500 hours. Otherwise, reject the lot. If 10 consecutive lots are accepted on the tightened inspection, then switch to the normal inspection given below.

Step 2: During the normal inspection, select a random sample of size 10 from a lot and assign them to the single group. Accept the lot if the number of failures by 500 hours is smaller than or equal to 4. Otherwise, reject the lot. If an additional lot is rejected among the next 5 lots after the rejection, then immediately revert to the tightened inspection given above.

If the life time of a product follows the Weibull distribution with shape parameter 2, then from Table 3 we have the plan parameters g = 2, c_1 = 0, c_2 = 8, s = 7, t = 20. The plan parameters when the lifetime of the product follows the Birnbaum-Saunders distribution with shape parameter m = 1 are given as g = 1, c_1 = 0, c_2 = 9, s = 14, t = 17.

We can compare the results of these distributions as we set their quality levels equally using the median lifetime. The number of groups obtained under the generalized exponential distribution and the Birnbaum-Saunders distribution are the same, but the number of groups obtained under the Weibull distribution is larger than the generalized exponential distribution or the Birnbaum-Saunders distribution. This means that the plans under these two distributions are more economical than the one under the Weibull distribution.

4. CONCLUDING REMARKS

We propose the TNTGASP plan based on time truncated life tests for three widely used distributions. We used the median as the quality parameter of the product, but the use of other percentile life will be straightforward. We provided the extensive tables for the practical use and explained the implementation procedure through examples.

We compared the results of three distributions in terms of the number of groups required. It was found that the plans under the generalized exponential distribution or the Birnbaum-Saunders distributions are more economical than under the Weibull distribution in that the former plans require smaller sample size than the latter. An economic design of the proposed sampling plan by considering various cost such as sampling and testing can be an interesting topic for the future research.

REFERENCES

Aslam, M. and Jun, C. H. (2009), A group acceptance sampling plan for truncated life test having Weibull distribution, *Journal of Applied Statistics*, **36**(9), 1021-1027.

Aslam, M., Balamurali, S., Jun, C. H., and Ahmad, M. (2010), A two-plan sampling system for life testing under Weibull distribution, *Industrial Engineering and Management Systems*, **9**(1), 54-59.

Aslam, M., Jun, C. H., Lee, H., Ahmad, M., and Rasool, M. (2011), Improved group sampling plans based on time-truncated life tests, *Chilean Journal of Statistics*, **2**(1), 85-97.

Baklizi, A. and El Qader El Masri, A. (2004), Acceptance sampling based on truncated life tests in the Birnbaum Saunders model, *Risk Analysis*, **24**(6), 1453-1457.

Balakrishnan, N., Leiva, V., and Lopez, J. (2007), Acceptance sampling plans from truncated life tests based on the generalized Birnbaum-Saunders distribution, *Communications in Statistics: Simulation and Computation*, **36**(3), 643-656.

Birnbaum, Z. W. and Saunders, S. C. (1969), A new family of life distributions, *Journal of Applied Probability*, **6**(2), 319-327.

Calvin, T. W. (1977), TNT zero acceptance number sampling, *Transactions of the 31st ASQC Annual Technical Conference*, Philadelphia, PA, 35-39.

Desmond, A. (1985), Stochastic models of failure in random environments, *Canadian Journal of Statistics*, **13**(3), 171-183.

Gupta, R. D. and Kundu, D. (1999), Theory and methods: generalized exponential distributions, *Australian and New Zealand Journal of Statistics*, **41**(2), 173-188.

Johnson, N. L., Kotz, S., and Balakrishnan, N. (1994), *Continuous Univariate Distributions* (2nd ed.), Wiley, New York, NY, **1**.

Jun, C. H., Balamurali, S., and Lee, S. H. (2006), Variables sampling plans for Weibull distributed lifetimes under sudden death testing, *IEEE Transactions on Reliability*, **55**(1), 53-58.

Lio, Y. L., Tsai, T. R., and Wu, S. J. (2009), Acceptance sampling plans from truncated life tests based on the Birnbaum-Saunders distribution for percentiles, *Communications in Statistics-Simulation and Computation*, **39**(1), 119-136.

Muthuraj, M. and Senthilkumar, D. (2006), Designing and construction of tightened-normal-tightened variables sampling scheme, *Journal of Applied Statistics*, **33**(1), 101-111.

Pascual, F. G. and Meeker, W. Q. (1998), The modified sudden death test: planning life tests with a limited number of test positions, *Journal of Testing and Evaluation*, **26**(5), 434-443.

Schiling, E. G. (1982), *Acceptance Sampling in Quality Control*, CRC Press, New York, NY.

Senthilkumar, D. and Muthuraj, D. (2010), Construction and selection of tightened-normal-tightened variables sampling scheme of type TNTVSS (n_1, n_2; k), *Journal of Applied Statistics*, **37**(3), 375-390.

Shoaib, M., Aslam, M., and Lio, Y. L. (2011), Acceptance decision rule for multiple items under the truncated life test for Birnbaum-Saunders distribution percentiles, *World Applied Sciences Journal*, **12**(10), 1745-1753.

Soundararajan, V. and Vijayaraghavan, R. (1992), Construction and selection of tightened-normal-tightened sampling inspection scheme of type TNT-(n_1, n_2; c), *Journal of Applied Statistics*, **19**(3), 339-349.

Vijayaraghavan, R. and Soundararajan, V. (1996), Procedures and tables for the selection of tightened-normal-tightened (TNT-(n; c_1, c_2)) sampling schemes, *Journal of Applied Statistics*, **23**(1), 69-80.

Vlcek, B. L., Hendricks, R. C., and Zaretsky, E. V. (2003), Monte Carlo simulation of sudden death bearing testing, *NASA/TM-2003-212506*, National Aeronautics and Space Administration, Washington, DC.