

# Parameter Estimation Method of Low-Frequency Oscillating Signals Using Discrete Fourier Transforms

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**Abstract** – This paper presents a DFT (Discrete Fourier Transform) based estimation algorithm for the parameters of a low-frequency oscillating signal. The proposed method estimates the parameters, i.e., the frequency, the damping factor, the mode amplitude, and the phase, by fitting a discrete Fourier spectrum with an exponentially damped cosine function. Parameter estimation algorithms that consider the spectrum leakage of the discrete Fourier spectrum are introduced. The multi-domain mode test functions are tested in order to verify the accuracy and efficiency of the proposed method. The results show that the proposed algorithms are highly applicable to the practical computation of low-frequency parameter estimations based on DFTs.

**Keywords:** Discrete Fourier transform, Fourier spectrum, Time series signal, Low-frequency, Parameter estimation

## 1. Introduction

Recently, the WAMS (Wide Area Measurement System) related technologies are rapidly being developed in power system in order to enhance the power system stability [1-5]. In the WAMS, an accurate and fast estimation of the low-frequency oscillation parameters is a very important part in analyzing, controlling and operating a wide area power system. In general, the real-time synchronous parameters of the power system are measured by using synchrophasor technology [6-9]. The key elements of synchrophasor technology are the estimation technique of the low-frequency oscillating parameters, i.e., the phasor, the frequency, and the damping factor.

Prony based estimation algorithms [10-22] have been developed and applied to this area. In practice, the Prony based estimation algorithms have been widely applied to the analysis of the dynamics in power systems. However, it is well known that this method requires computational burdens.

DFT (Discrete Fourier Transform) based estimation algorithms [23-29] have been developed and applied to this area. The DFT algorithms are very widely used in the area of modern digital signal processing. In practice, FFT (Fast Fourier Transform) algorithms are applied to estimate reliable estimation results and to enhance the overall computational speed. In [23], a stability analysis method was proposed that estimates the frequency and the damping factor using the Fourier spectrum. In [24], a methodology

was introduced for the monitoring and on-line analysis of power system dynamic performances based on the time frequency distribution of the energy of electromechanical oscillations obtained by a STFT (Short-Time Fourier Transform). In [25], a methodology was introduced that excites the system with a low-level pseudo-random probing signal and identifies the oscillatory modes using a spectral density estimation and frequency domain transfer function identification. In [26], a Fourier transform based algorithm was introduced that estimates the parameters of the oscillating modes that arise after a system disruption. In [27], a stability monitoring method was proposed that estimates the frequency and the damping factor from the Fourier spectrum. In [28], a hybrid method was proposed which use DFT to start the low-frequency oscillation parameter estimation process. In [29], fast and efficient parameter estimation method based on DFT was proposed. This method adopts simplistic algorithms that estimates the oscillating low-frequency parameters at around 0.2~2.0Hz in power systems. The DFT based parameter estimation method is very fast and computationally efficient. However, in some cases, the errors of the estimated parameters are not acceptable due to the frequency resolution and the rapid changes in the phase value near the peak amplitude value of the discrete Fourier spectrum.

In this paper, the extended parameter estimation algorithms found in [29] are introduced. In the proposed method, the damping factor is approximated from the peak amplitude and its left/right amplitude in the amplitude Fourier spectrum to compensate the spectrum leakage. The mode frequency is estimated from the peak amplitude and the estimated damping factor in order to reduce the estimation errors found in DFT based parameter estimation method. The mode phase is then estimated from the estimated mode frequency in the phase Fourier spectrum.

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The computation speed of the proposed method is very fast because it uses only a simple arithmetic process except for the DFT process for the original signal. Test signals having two or three mode are used to prove the efficiency of the proposed method. From the results, it can be seen that proposed method is highly applicable to low-frequency oscillating parameter estimation.

## 2. The Parameter Estimation in the Continuous Fourier Transform

The amplitude and the phase of the Fourier spectrum can be obtained by the Fourier transform of the original signal. In general, the periodic signals without damping can be transformed into a line spectrum by Fourier transformation. But the periodic signals with damping can be transformed to band spectrum by Fourier transformation. In the continuous spectrum, the Fourier spectrum near the peak value can easily transform into an exponentially damped cosine function. Hence, the mode parameters, i.e., damping factor, frequency, and mode amplitude and phase, of the low-frequency oscillating signal can be directly extracted from the exponentially damped cosine function through the simple arithmetic process. The conceptual signal processing flow of the Fourier transform based parameter estimation is shown in Fig. 1.

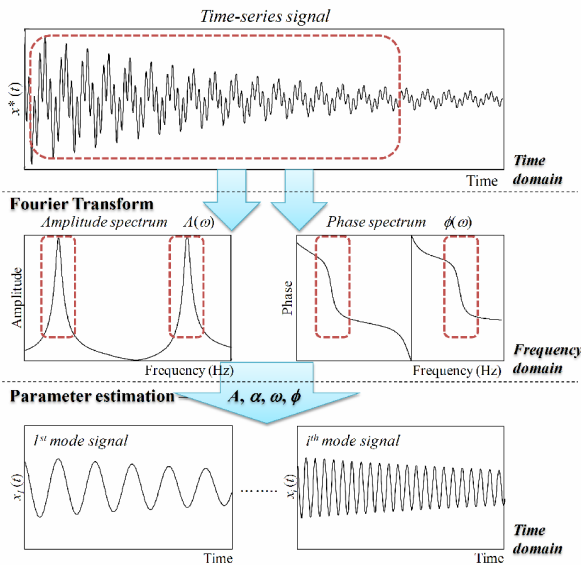


Fig. 1. The conceptual signal processing flow of the Fourier transform based parameter estimation

### 2.1 The Fourier transform of the exponentially damped cosine function

The exponentially damped cosine function  $x(t)$  can be expressed as:

$$x(t) = Ae^{-\alpha t} \cos(\omega_0 t + \phi_0) \tag{1}$$

where  $A$  is the mode amplitude,  $\alpha$  is the damping factor,  $\omega_0$  is the mode frequency, and  $\phi_0$  is the mode phase

The Fourier spectrum  $F(\omega)$  can be obtained from the Fourier transform in (1). It can be expressed as a complex function as:

$$\begin{aligned} F(\omega) &= \frac{A(\alpha + j\omega) \cos \phi_0 - A\omega_0 \sin \phi_0}{\alpha^2 + \omega_0^2 - \omega^2 + j2\alpha\omega} \\ &= \text{Re}(\omega) + j \text{Im}(\omega) = A(\omega) \angle \phi(\omega) \end{aligned} \tag{2}$$

where  $A(\omega)$  is the amplitude spectrum, and  $\phi(\omega)$  is the phase spectrum.

Typically, the Fourier spectrum of a signal with a high-frequency band can be transformed into a cosine/sine functions because the frequency has a larger value than the damping factor in the high-frequency band. The Fourier spectrum of a signal with a low-frequency band, however, can be transformed into an exponentially damped cosine/sine functions according to the value of damping factor.

In (2), the small damping factor results in a narrow spectrum bandwidth for the mode frequency  $\omega_0$ . When  $\omega_0 \gg \alpha$ , the Fourier spectrum becomes:

$$F(\omega_0) \cong \frac{A}{2\alpha} (\cos \phi_0 + j \sin \phi_0) = A(\omega_0) \angle \phi(\omega_0) \tag{3}$$

### 2.2 The mode frequency estimation in the continuous Fourier spectrum

The Fourier transform is widely used in various industry applications. Specifically, the Fourier transform is the most popular method used for frequency analysis. The Fourier spectrum can be taken from the time series data of the signal by a Fourier transform. By using the Fourier transform, the dominant frequency of the signal can be extracted.

The frequency corresponding to the peak spectrum indicates the dominant frequency of the time series data of the signal. Therefore, the frequency corresponding to the peak amplitude of the Fourier spectrum can be chosen as the estimated mode frequency

$$\hat{\omega}_0 = \max_{\omega} A(\omega) \tag{4}$$

### 2.3 The damping factor estimation in the continuous Fourier spectrum

In (2) the following relationship is established between the peak amplitude  $A(\hat{\omega}_0)$  and the  $A(\hat{\omega}_0 + \alpha)$  :

$$A(\hat{\omega}_0) = \sqrt{2} A(\hat{\omega}_0 + \alpha) \tag{5}$$

From (5), the damping factor can be extracted from the frequency of the amplitude corresponding to  $1/\sqrt{2}$  times the peak amplitude. This relationship indicates that the damping factor can be directly calculated from the amplitude of the Fourier spectrum.

**2.4 The mode amplitude estimation in the continuous Fourier spectrum**

In (3), the following relationship is established between the peak amplitude  $A(\hat{\omega}_0)$  and the mode amplitude  $\hat{A}$ .

$$\hat{A} = 2\alpha A(\hat{\omega}_0) \tag{6}$$

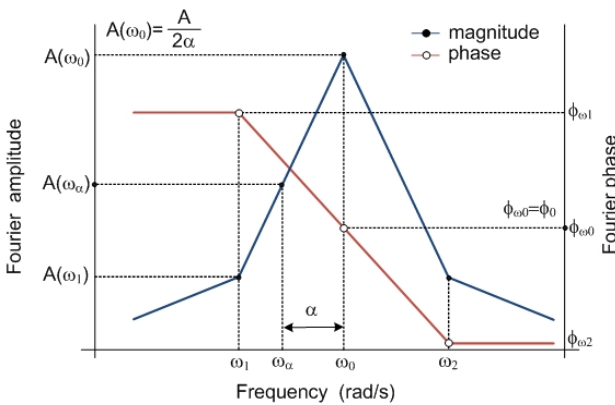
From (6), it can be seen that the mode amplitude can be directly calculated from the product of the peak amplitude of the Fourier spectrum and damping factor. The mode amplitude can be extracted from the amplitude spectrum because the damping factor can be directly calculated from the amplitude spectrum.

**2.5 The mode phase estimation in the continuous Fourier spectrum**

When  $\phi(\hat{\omega}_0)$  is the phase of the Fourier spectrum at the mode frequency  $\hat{\omega}_0$ , the mode phase  $\hat{\phi}_0$  in (3) is equal to the phase of the Fourier spectrum, and so the mode phase becomes:

$$\hat{\phi}_0 = \phi(\hat{\omega}_0) \tag{7}$$

Therefore, the mode phase can be easily extracted from the phase spectrum. It should be noted that the phase near the peak value is sensitive to frequency variations because the phase curve is expressed as an arctangent function. Fig. 2 illustrates the relationship between the dominant oscillating parameters and the Fourier spectrum.



**Fig. 2.** The relationship between the mode parameters and the continuous Fourier spectrum

**3. The Practical Computation Algorithms in Discrete Fourier Transforms**

Mathematically, it can be seen that the mode parameters of the low-frequency oscillating signal are easily estimated from the continuous Fourier spectrum. However, in practice, the parameter estimations found by using (4) ~ (7) contain errors due to the spectrum leakage of the discrete Fourier spectrum. In the mode phase estimation, the phase is very sensitive to the frequency changes. For example, the mode phase varies 360° per second for a 1 Hz signal. Therefore, the estimation results of the phase found using (7) may include large errors according to the frequency intervals of the discrete Fourier spectrum. A small sampling time and a large time interval are required in order to estimate accurate parameters from the Fourier spectrum found using the DFT. However, it requires computational burdens to guarantee acceptable parameter estimation results.

In the DFT, the Fourier spectrum has a discrete value with constant frequency intervals. The frequency resolution (frequency interval) of the discrete Fourier spectrum becomes:

$$\Delta f = \frac{1}{NT} [Hz] \tag{8}$$

where  $\Delta f$  is the frequency resolution (interval),  $N$  is the data number in the sampling interval and  $T$  is the sampling time.

A small frequency resolution for the discrete Fourier spectrum enhances the accuracy of the parameter estimation results. However, this requires a smaller sampling time and/or a larger sampling interval.

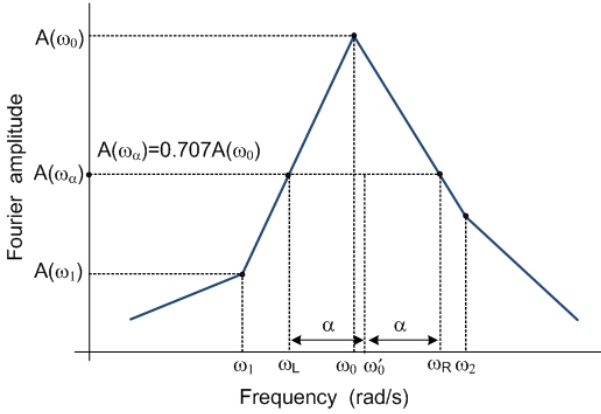
In this paper, a practical computation algorithm for the DFT based parameter estimation is proposed. It uses a simple arithmetic calculation process for the discrete Fourier spectrum. In addition, it enhances parameter estimation accuracy with a limited sampling time and sampling interval.

**3.1 The damping factor estimation algorithm in the discrete Fourier spectrum**

The damping factor can be directly extracted from the Fourier spectrum by the Fourier transform. However, in the DFT, the discrete Fourier spectrum is expressed in a discrete value with constant frequency intervals. Thus, spectrum leakage exists in the discrete Fourier spectrum.

In the proposed method, an interpolation technique is used to correct the estimation error of the damping factor as shown in Fig. 3. The damping factor can be calculated by a linear approximation through a simple arithmetic expression from the amplitude and the frequency of the peak spectrum and the left/right spectrums. The left and

right frequency corresponding to  $1/\sqrt{2}$  times the peak amplitude in a discrete amplitude spectrum can be obtained by:



**Fig. 3.** The illustration of the proposed damping factor and frequency estimation in the discrete Fourier spectrum

$$\omega_L = \omega_0 - \frac{(\omega_0 - \omega_1)(A(\omega_0) - A(\omega_\alpha))}{A(\omega_0) - A(\omega_1)} \quad (9)$$

$$\omega_R = \omega_0 + \frac{(\omega_2 - \omega_0)(A(\omega_0) - A(\omega_\alpha))}{A(\omega_2) - A(\omega_0)} \quad (10)$$

where  $A(\omega_0)$  is the peak amplitude,  $\omega_0$  is the peak frequency,  $A(\omega_1)$  is the first left-side amplitude corresponding to  $A(\omega_\alpha)$  in the left-side amplitude to the peak amplitude,  $\omega_1$  is the frequency corresponding to  $A(\omega_1)$ ,  $A(\omega_2)$  is the first right-side amplitudes corresponding to  $A(\omega_\alpha)$  in the right-side amplitudes to the peak amplitude,  $\omega_2$  is the frequency corresponding to  $A(\omega_2)$ , and  $A(\omega_\alpha)$ : is the  $1/\sqrt{2}$  times  $A(\omega_0)$ .

From (9) and (10), the damping factor in the discrete Fourier spectrum with spectrum leakage can be approximated by

$$\alpha = \left| \frac{\omega_R - \omega_L}{2} \right| \quad (11)$$

### 3.2 The frequency estimation algorithm in the discrete Fourier spectrum

In the discrete Fourier spectrum, the distances between the peak amplitude and its first left and right-side amplitude are not same due to the spectrum leakage. Therefore, the mode frequency corresponding to the peak amplitude of the discrete amplitude spectrum contains errors. If there is no spectrum leakage, then the distances between the peak amplitude and its first left and right-side amplitude must be same.

In the proposed method, the mode frequency in a discrete Fourier spectrum with spectrum leakage can be

approximated by using (11):

$$\hat{\omega}_0' = \omega_L + \alpha = \omega_R - \alpha \quad (12)$$

### 3.3 The phase estimation algorithm in the discrete Fourier spectrum

The phase spectrum is expressed by using an arctangent function of the frequency. Therefore, it is sensitive to the frequency changes near the peak frequency band. It is expected that the phase value estimation from the discrete phase spectrum may include large estimation errors. Thus, accurate frequency estimation is very important to the phase estimation in the DFT based parameter estimation.

Therefore, the mode phase in the discrete Fourier spectrum with spectrum leakage can be estimated by using (12):

$$\hat{\phi}_0' = \phi(\hat{\omega}_0') \quad (13)$$

### 3.4 The mode magnitude estimation algorithm in the discrete Fourier spectrum

When the discrete Fourier transform is applied to the same time interval, the amplitude of the Fourier spectrum is calculated differently according to sampling. Therefore, the Fourier spectrum needs to be normalized so that the estimated values may not be different according to time interval and sampling time. Because the discrete amplitude spectrum is proportional to the number of the data, the normalized amplitude can be calculated by:

$$A_{norm} = A(\hat{\omega}_0') \left( \frac{T_0}{N} \right) = A(\hat{\omega}_0') T \quad (14)$$

Then, the mode amplitude can be determined by:

$$\hat{A} = 2\alpha A_{norm} \quad (15)$$

## 4. The Numerical Examples

The test signals having the multi-domain modes were tested in order to verify the accuracy and efficiency of the proposed method. The test signal consisted of the sum of an exponential damped cosine functions with the multi-mode. The generalized formula becomes:

$$x(t) = \sum_{i=1}^n A_i e^{-\alpha_i t} \cos(\omega_i t + \phi_i) \quad (16)$$

Test signals having two or three modes were used to prove the efficiency of the proposed method. Two test signals were considered. Test signal 1 has two modes. Test

signal 2 has three modes. The detailed parameters of the test signals are listed in Table 1. The Fourier spectrum (amplitude and phase spectrum) was obtained through an FFT (Fast Fourier Transform).

**Table 1.** Detailed parameters of the test signals

Test signal 1				
Mode	$\alpha$	$f$ (Hz)	$A$	$\varphi$ (deg)
1	0.10000	0.69230	0.80000	60.00000
2	0.10000	0.99950	1.00000	30.00000
Test signal 2				
Mode	$\alpha$	$f$ (Hz)	$A$	$\varphi$ (deg)
1	0.10000	0.71230	0.80000	0.00000
2	0.07000	1.02350	1.00000	60.00000
3	0.15000	1.34320	0.70000	120.00000

When the sampling interval was set to 100 s with the sampling time of 0.1 s, the estimated mode parameters of the test signals are listed in Table 2 and Table 3. It can be seen that the estimated parameters values of the proposed method are close to the actual parameter values. Obviously, the accuracy of the parameter estimation mainly depends on the frequency resolution and/or spectrum leakage. The frequency resolution and spectrum leakage should be dominated by the sampling interval and sampling times.

**Table 2.** Parameter estimation results for test signal 1

Estimated parameter – [29]				
Mode	$\alpha$	$f$ (Hz)	$A$	$\varphi$ (deg)
1	0.09755	0.70000	0.71768	37.81384
2	0.10018	1.00000	1.02985	25.76731
Estimated parameter – proposed method				
Mode	$\alpha$	$f$ (Hz)	$A$	$\varphi$ (deg)
1	0.10285	0.69090	0.83440	67.64939
2	0.10199	1.00080	1.03755	23.27019

**Table 3.** Parameter estimation results for test signal 2

Estimated parameter – [29]				
Mode	$\alpha$	$f$ (Hz)	$A$	$\varphi$ (deg)
1	0.09635	0.72000	0.64994	-27.00641
2	0.06775	1.02000	0.89754	76.34754
3	0.12191	1.34000	0.48359	126.69380
Estimated parameter – proposed method				
Mode	$\alpha$	$f$ (Hz)	$A$	$\varphi$ (deg)
1	0.10362	0.71165	0.75672	3.44003
2	0.07885	1.02384	1.01752	58.68497
3	0.15929	1.34595	0.62306	110.44924

In order to evaluate the accuracy and efficiency of the proposed estimation algorithms, the parameter estimation errors of the conventional and proposed method are compared with different sampling intervals and sampling times.

The parameter estimation errors for damping factor, frequency and amplitude are calculated as follows:

$$Error = \frac{y_{est} - y_0}{y_0} \times 100(\%) \quad (17)$$

where  $y_{est}$  is the estimated value and  $y_0$  is the actual value. It is note that degree is used as the unit of the phase error instead of percentage.

The parameter estimation errors for test signal 1 with different sampling intervals and sampling times are listed in Table 4, Table 5 and Table 6. From the estimation errors of the parameter for test signal 1, in most cases, it was shown that the estimation errors of the proposed method are smaller than those of the method in [29]. And it is note that estimation errors are limited within a relatively small range by applying the proposed linear approximation algorithms in spite of small sampling intervals and large sampling times. Thus, the overall estimation errors using the proposed method are more accurate than those found using the method in [29] under the small sampling intervals and sampling times.

**Table 4.** Parameter estimation errors for test signal 1 (Sampling interval: 100 s)

Sampling time (sec.)	Method	Mode	Estimation error			
			$\Delta\alpha$ (%)	$\Delta f$ (%)	$\Delta A$ (%)	$\Delta\varphi$ (deg)
0.1	[29]	1	<b>2.4500</b>	<b>1.1122</b>	<b>10.290</b>	<b>22.1862</b>
		2	0.1800	0.0500	2.9850	4.2327
	proposed	1	<b>2.8500</b>	0.2022	4.3000	7.6494
		2	1.9900	0.1301	3.7550	6.7298
0.05	[29]	1	1.4100	0.3322	0.3762	11.0896
		2	2.6700	0.0500	0.9070	4.1556
	proposed	1	2.8300	0.2340	<b>4.7037</b>	8.8486
		2	1.9300	0.1231	3.7920	6.4307
0.01	[29]	1	0.2900	0.3322	1.0725	11.4401
		2	2.3400	0.0500	1.2410	4.0405
	proposed	1	2.8400	<b>0.2600</b>	4.2487	<b>9.7794</b>
		2	1.8900	0.1171	3.0380	6.1420

**Table 5.** Parameter estimation errors for test signal 1 (Sampling interval: 75 s)

Sampling time (sec.)	Method	Mode	Estimation error			
			$\Delta\alpha$ (%)	$\Delta f$ (%)	$\Delta A$ (%)	$\Delta\varphi$ (deg)
0.1	[29]	1	20.190	<b>0.1488</b>	23.206	0.9678
		2	2.4800	0.0500	0.2140	<b>4.2712</b>
	proposed	1	3.6700	0.2311	6.2125	6.9608
		2	3.3600	0.1431	<b>6.2240</b>	6.9305
0.05	[29]	1	22.030	0.1488	24.067	0.5617
		2	2.0800	0.0500	0.2900	4.1484
	proposed	1	3.8100	0.2701	5.5275	8.2263
		2	3.3000	0.1361	5.1760	6.6150
0.01	[29]	1	<b>23.480</b>	0.1488	<b>24.788</b>	0.2453
		2	1.6800	0.0500	0.5650	4.0333
	proposed	1	<b>3.9400</b>	<b>0.3004</b>	5.0650	<b>9.2108</b>
		2	3.2600	0.1291	4.4060	6.3098



**Table 6.** Parameter estimation errors for test signal 1 (Sampling interval: 50 s)

Sampling time (sec.)	Method	Mode	Estimation error			
			$\Delta\alpha$ (%)	$\Delta f$ (%)	$\Delta A$ (%)	$\Delta\varphi$ (deg)
0.1	[29]	1	96.120	<b>1.1122</b>	79.873	<b>25.3352</b>
		2	8.3700	0.0500	5.5080	4.4521
	proposed	1	36.510	0.2586	<b>24.953</b>	7.7568
		2	2.4600	0.1441	0.3530	6.5293
0.05	[29]	1	99.040	1.1122	80.166	21.8848
		2	8.0300	0.0500	6.0290	4.1094
	proposed	1	37.530	0.3019	24.416	8.4408
		2	2.5400	0.1371	0.6410	6.2647
0.01	[29]	1	<b>101.34</b>	1.1122	<b>80.443</b>	21.6378
		2	7.7000	0.0500	6.3310	3.9940
	proposed	1	<b>38.340</b>	<b>0.3366</b>	24.062	<b>9.5194</b>
		2	2.5900	0.1311	1.3710	6.0084

The parameter estimation errors for test signal 2 with different sampling intervals and sampling times are listed in Table 7, Table 8 and Table 9. It is note that the test signal 2 contains three dominant modes.

**Table 7.** Parameter estimation errors for test signal 2 (Sampling interval: 100 s)

Sampling time (sec.)	Method	Mode	Estimation error			
			$\Delta\alpha$ (%)	$\Delta f$ (%)	$\Delta A$ (%)	$\Delta\varphi$ (deg)
0.1	[29]	1	3.6500	<b>1.0810</b>	18.757	<b>27.0064</b>
		2	3.2143	0.3420	10.246	16.3475
		3	18.726	0.2382	<b>30.915</b>	6.6938
	proposed	1	3.6200	0.0913	5.4100	3.4400
		2	<b>12.642</b>	0.0332	1.7520	1.3150
		3	6.1933	<b>0.2047</b>	<b>10.991</b>	<b>9.5508</b>
0.05	[29]	1	6.6800	0.3229	14.886	9.5764
		2	<b>21.228</b>	0.3420	28.314	18.2256
		3	16.246	0.2382	28.495	7.4966
	proposed	1	3.7900	0.0884	5.3163	3.3649
		2	12.414	0.0244	1.9980	0.6740
		3	4.6933	0.1340	10.545	6.0964
0.01	[29]	1	6.5700	0.3229	15.436	9.6189
		2	20.814	0.3420	28.088	18.4073
		3	14.286	0.2382	26.575	8.0069
	proposed	1	3.9100	0.0884	5.9488	3.3622
		2	12.257	0.0186	1.6110	0.2085
		3	4.2733	0.0916	10.661	3.9945

**Table 8.** Parameter estimation errors for test signal 2 (Sampling interval: 75 s)

Sampling time (sec.)	Method	Mode	Estimation error			
			$\Delta\alpha$ (%)	$\Delta f$ (%)	$\Delta A$ (%)	$\Delta\varphi$ (deg)
0.1	[29]	1	20.450	<b>0.7904</b>	29.770	21.7044
		2	40.471	0.3097	29.381	16.6949
		3	6.6400	0.2583	7.8729	11.7707
	proposed	1	12.690	0.0505	0.5763	2.2584
		2	12.514	0.0049	3.7480	2.2774
		3	3.6600	<b>0.2055</b>	10.467	<b>9.8529</b>
0.05	[29]	1	20.370	0.7904	30.427	21.8498
		2	41.857	0.3097	29.830	16.5349
		3	9.9467	0.2583	5.1500	10.8897

0.01	proposed	1	12.890	0.0477	1.5125	2.1616
		2	12.957	0.0049	3.5460	1.5875
		3	3.6067	0.1452	<b>10.641</b>	6.8297
	[29]	1	20.270	0.7904	<b>30.843</b>	<b>21.9668</b>
		2	<b>42.857</b>	0.3097	30.158	16.4149
		3	12.233	0.2583	3.2300	10.3066
proposed	1	13.020	0.0477	2.1825	2.1437	
	2	13.271	0.0127	3.4140	1.0873	
	3	3.6600	0.1057	10.624	4.8445	

**Table 9.** Parameter estimation errors for test signal 2 (Sampling interval: 50 s)

Sampling time (sec.)	Method	Mode	Estimation error			
			$\Delta\alpha$ (%)	$\Delta f$ (%)	$\Delta A$ (%)	$\Delta\varphi$ (deg)
0.1	[29]	1	88.0000	<b>1.0810</b>	<b>57.9737</b>	<b>28.8730</b>
		2	5.6286	0.3420	12.2220	18.3327
		3	18.3467	0.2382	30.5186	7.2176
	proposed	1	34.2900	0.1193	<b>12.6675</b>	2.4475
		2	12.0286	0.1495	2.4520	8.6287
		3	7.9667	<b>0.2293</b>	8.1229	8.8761
0.05	[29]	1	88.0400	1.0810	56.0200	27.2940
		2	5.0429	0.3420	12.0500	16.6116
		3	15.3333	0.2382	27.6643	7.6633
	proposed	1	34.4600	0.1165	11.5350	2.3697
		2	11.7000	0.1593	1.8070	9.2525
		3	7.0667	0.1601	8.5786	5.6775
0.01	[29]	1	<b>88.2000</b>	1.0810	54.6837	27.4800
		2	4.6286	0.3420	11.9100	16.7979
		3	13.0200	0.2382	25.4457	8.1821
	proposed	1	<b>34.6300</b>	0.1165	10.7425	2.3729
		2	11.4857	0.1671	1.3600	<b>9.7038</b>
		3	6.6867	0.1124	8.6414	3.5123

From the parameter estimation errors of test signal 2, it is verified that the estimation errors using the proposed method are more accurate than those using the method in [29] under the all sampling intervals and sampling times. Thus, it is verified that the proposed linear approximation algorithms are very efficiently compensate the spectrum leakage of the discrete Fourier Spectrum. This show that the proposed linear approximation algorithms are efficient in the practical computation of the DFT based parameter estimation.

However, the estimated phase parameters contain considerable errors with respect to the other estimated parameters. The small estimation error of the frequency results in a considerable estimation error for the phase estimation because the phase spectrum is the arctangent function of the frequency.

### 5. Conclusion

This paper described methods used to estimate low-frequency parameters in the discrete Fourier spectrum. The approximated estimation algorithms reduce errors and to enhance the accuracy of DFT based parameter estimation. The proposed estimation algorithms are very simple and

have a very fast overall computation speed compared to the existing method because all of the computation processes use simple arithmetic calculations except the DFT or FFT processing of the signal.

It can be seen that the parameter estimation results of the proposed method for the test signals are very accurate and have reasonable error tolerances. In addition, it is well known that the DFT based parameter estimation method is very robust to noise. As a result, the proposed algorithms are highly applicable to the parameter estimation of low-frequency oscillating signal in the power systems.

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