



# Attribute-based Proxy Re-encryption with a Constant Number of Pairing Operations

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## Abstract

Attribute-based encryption (ABE) is an encryption scheme in which the user is able to decrypt a ciphertext with associated attributes. However, the scheme does not offer the capability of decryption to others when the user is offline. For this reason, the attribute-based proxy re-encryption (ABPRE) scheme was proposed, which combines traditional proxy re-encryption with ABE, so a user is able to empower designated users to decrypt the re-encrypted ciphertext with the associated attributes of designated users. However, previous ABPRE schemes demands a number of pairing operations that imply huge computational overhead. To reduce the number of pairing operations, we reduce the pairing operations with exponent operations. This paper provides a novel approach to an ABPRE scheme with constant pairing operation latency.

**Index Terms:** Attribute based encryption, Proxy re-encryption, Key delegation

## I. INTRODUCTION

Identity-based cryptography encrypts the message with the user's identity including the name and e-mail address. An identity-based encryption (IBE) scheme can restrict an authority to indicate the identity of the recipient [1, 2]. Attribute-based encryption (ABE) was published by Sahai and Waters [3]. Unlike IBE, the message is encrypted with attributes, such as gender, age, and affiliation, so ABE schemes can designate the recipients by assigning common attributes of others. ABE consists of two policies. The first is the key policy ABE (KP-ABE) in which each private key is associated with an access structure [4]. The other scheme is the ciphertext policy ABE (CP-ABE), in which a ciphertext is associated with an access structure [5-7]. The attribute-based proxy re-encryption scheme (ABPRE) extends traditional proxy re-encryption to its attribute-based counterpart. Therefore, the capability of delegation is transferred to the designated users [8]. However, the

previous ABPRE does not provide a constant length of message and number of pairing operations. Recently, a constant ciphertext length based CP-ABE was proposed by Emura et al. [9]. The computation cost and ciphertext length were reduced significantly compared to previous papers. However, the encryption scheme over ABPRE has not been proposed yet. For this reason, in the present paper, we propose a new constant computation-based ABPRE. The scheme utilizes the capability of delegation with constant pairing operations.

The rest of this paper is organized as follows. In section II, we describe ABPRE. In section III, preliminaries such as bilinear map and complexity assumptions are introduced to support the construction and the security proof. In section IV, our new ABPRE scheme is introduced and then we analyze security model and computation performance. Finally the conclusion is drawn in section V.

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## II. DEFINITION OF ABPRE

In ABPRE, a user can re-encrypt the ciphertext using a re-key. Detailed relationships are shown in Fig. 1. First  $U_1$  generates the ciphertext  $C_1$ , which can be decrypted with  $U_1$ 's attributes. One day,  $U_1$  is absent, but we need to read the data from  $C_1$ . In this case,  $U_1$  will store the re-encrypted ciphertext with attributes that represent the specific recipient or groups into a proxy-server. After that,  $U_2$  can access to the proxy-server and then gain the re-encrypted data.

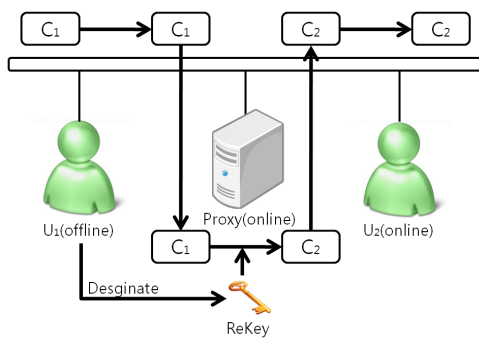


Fig 1. The attribute-based proxy re-encryption scheme system.

### A. ABPRE Model

The ABPRE scheme is illustrated in [8, 10]. An ABPRE scheme is comprised of six steps including SETUP, KEYGEN, RE-GEN, ENC, RE-ENC, and DEC.

**SETUP:** The setup algorithm takes no input other than the implicit security parameter. It outputs the public parameters  $\rho p$  and a master key  $mk$ .

**KEYGEN** ( $S, mk$ ): The key generation algorithm takes as input the master key  $mk$  and a set of attributes  $S$  that describe the key. It outputs a private key  $usk$ .

**RE-GEN** ( $usk, AS$ ): The re-key generation algorithm takes as input the secret key  $usk$  and access structure  $AS$ . It outputs a re-key  $rk$ .

**ENC** ( $m, AS$ ): The encryption algorithm takes as input the public parameters  $\rho p$ , a message  $m$ , and an access structure  $AS$  over the universe of attributes. The algorithm will encrypt  $m$  and produce a ciphertext  $C$  such that only a user that possesses a set of attributes that satisfies the access structure will be able to decrypt the message. The ciphertext implicitly contains  $AS$ .

**RE-ENC** ( $rk, C$ ): The re-encryption algorithm takes as input the re-key  $rk$  and a ciphertext  $C$ . First the algorithm checks the index set in  $rk$ . If  $rk$  satisfies the access structure of  $C$ , it outputs a re-encrypted ciphertext  $C'$ ; otherwise, it rejects the re-key.

**DEC** ( $\rho p, C, usk$ ): The decryption algorithm takes as input the public parameters  $\rho p$ , a ciphertext  $C$ , which contains an access policy  $AS$ , and a private key  $usk$ , which is a private key for a set  $S$  of attributes. If the set  $S$  of attributes satisfies the access structure  $AS$  then the algorithm will decrypt the ciphertext and return a message  $m$ .

## III. PRELIMINARIES

In this section, some definitions which are the basis of attribute-based encryption schemes, including our scheme, are presented.

### A. Bilinear Groups

**Definition 1.** Bilinear groups and a bilinear map are defined as follows:  $G$  and  $G_T$  are finite cyclic groups of prime order  $p$ .  $e$  is an efficiently computable bilinear map or pairing  $e: G \times G \rightarrow G_T$ , with the following properties;

**Bilinearity:** For any  $g, h \in G$ , and  $a, b \in \mathbb{Z}_p$ , we have

$$e(g^a, h^b) = e(g, h)^{a \cdot b}.$$

**Non-degeneracy:**  $e(g, g) \neq 1$ .

Note that  $e(*, *)$  is symmetric since

$$e(g^a, g^b) = e(g, g)^{a \cdot b} = e(g^b, g^a).$$

### B. Complexity Assumptions

**Definition 2.** Complex Triple Diffie-Hellman (CTDH) problem. Let  $e: G \times G \rightarrow G_T$  be a bilinear map, where  $G$  has prime order  $p$  and  $g$  is a generator of  $G$ , random numbers  $n, a, b, c, d, R \in \mathbb{Z}_p$ . Given a tuple

$$\langle g, n, g_b = g^b, g_c = g^c, g_d = g^d, g_1 = g^{\frac{c}{b}}, g_2 = g^{bc}, g_3 = g^{ac},$$

$$g_4 = g^{abc - Rc}, g_5 = g^{c(R+nd)}, g_6 = g^{\frac{c(R+nd)}{b}} \rangle \text{ as inputs, output } g^{abc}.$$

**Definition 3.** Augment Diffie-Hellman (ADH) problem. Let  $e: G \times G \rightarrow G_T$  be a bilinear map, where  $G$  has prime order  $p$  and  $g$  is a generator of  $G$ , random

numbers  $a, b \in \mathbb{Z}_p$ . Given a tuple  $\langle g, g^a, g^b, g^{b^2} \rangle$  as inputs, output  $g^{ab}$ .

**Definition 4.** The CTDH assumption holds in  $G$  if no probabilistic polynomial time adversary is able to output  $g^{abc}$  from

$$\begin{aligned} &\langle g, n, g_b = g^b, g_c = g^c, g_d = g^d, g_1 = g^{\frac{c}{b}}, g_2 = g^{bc}, \\ &g_3 = g^{ac}, g_4 = g^{abc-Rc}, \\ &g_5 = g^{c(R+nd)}, g_6 = g^{\frac{c(R+nd)}{b}} \rangle \end{aligned}$$

with non-negligible advantage, where random numbers  $n, a, b, c, d, R \in \mathbb{Z}_p$  and generator  $g \in G$  are chosen independently and uniformly at random.

The CTDH problem is more difficult than the ADH problem; if given the input of the ADH problem  $\langle g, g^a, g^b, g^{b^2} \rangle$ , we could select  $R + nd = 1, R = ab, b = c$  and outputs  $g^{Rc}$  from the CTDH oracle with inputs

$$\langle g, n, g^c, g^c, g^{(1-R)n^{-1}}, g, g^{c^2}, g^R, 1, g^c, g \rangle.$$

The CTDH problem is the basis of the master key security in our scheme.

**Definition 5.** Augment decisional bilinear Diffie-Hellman (ADBDH) problem. Let  $e: G \times G \rightarrow G_T$  be a bilinear map, where  $G$  has prime order  $p$  and  $g$  is a generator of  $G$ , random numbers  $a, b, c \in \mathbb{Z}_p$ . Given a

tuple  $\langle g, A = g^a, B = g^b, C = g^c, B' = g^{\frac{1}{b}}, Z \rangle$  as inputs, output 1 if  $Z = e(g, g)^{abc}$ ; otherwise, output 0.

**Definition 6.** The ADBDH assumption holds in  $G$  if no probabilistic polynomial time adversary is able to distinguish the tuples

$D_{rand} = \langle g, g^a, g^b, g^c, g^{\frac{1}{b}}, e(g, g)^z \rangle$  with non-negligible advantage, where  $a, b, c, z \in \mathbb{Z}_p$  and a generator  $g \in G$  are chosen independently and uniformly at random.

## IV. ATTRIBUTE-BASED PROXY RE-ENCRYPTION WITH CONSTANT PAIRING OPERATION LATENCY

### A. Our Techniques

We adapt a constant number of pairing operations to a previous ABPRE scheme [8]. Whenever the attribute is

decided in the scheme, to reflect the attributes, we need to compute the pairing operation with its designated attributes. To reduce the number of pairing operations, we reduce the pairing operation by using an exponential operation which can easily calculate the summation of the exponent. Therefore, we calculate the exponent and then compute the pairing operation just once.

### B. Satisfying an Access Structure

In this scheme we consider the access structure consisting of AND gates between positive and negative attributes. Denote the index set of all the attributes as  $\tau$ . The access structure is represented as  $\wedge(+d_i, -d_j)_{i \in \tau}$ , which are the positive attribute and the negative attribute, respectively. Any user receives a secret key associated with an attribute set  $S \subseteq \tau$  from the authority. The users can decrypt the ciphertext, if the following conditions of the attribute are met:

If  $+d_i$  appears in  $AS$ , then  $i \in S$ ;

If  $-d_j$  appears in  $AS$ , then  $i \notin S$ .

### C. Main Construction

**SETUP**( $1^k$ ): A bilinear group  $G$  of prime order  $p$ , with bilinear map  $e: G \times G \rightarrow G_T$  is generated. Next, it selects elements  $k, y, z, t_i (1 \leq i \leq 3n)$  in  $\mathbb{Z}_p$  and two generators  $g, h$  of  $G$  at random. Let  $Y := e(g, h)^y$  and  $T_i := g^{t_i}$  for each  $1 \leq i \leq 3n$ . The public parameter  $\rho p$  includes  $\langle e, g^z, h, Y^{k \cdot z}, \{T_i, \frac{t_i}{k \cdot z}\}_{1 \leq i \leq 3n} \rangle$ . The master key  $mk$  is  $\langle k, y, z, \{t_i\}_{1 \leq i \leq 3n} \rangle$ .

**KGEN**( $S, mk$ ): Let  $S$  denote an index set of attributes. It chooses a random  $r_1, \dots, r_n$  from the  $\mathbb{Z}_p$  and sets  $r = r_1 + r_2 + \dots + r_n$ . It computes  $\hat{D} = (h^{y-r})^k$ , and for each  $i \in N (N = \{1, 2, \dots, n\})$ :  $(D_{i,1} = h^{r_i})_{i \in N}$ . This outputs a user's secret key  $usk = \langle S, (D_{i,1})_{i \in N}, \hat{D}, k \cdot z \rangle$ .

**ENC**( $m, AS$ ): Let  $AS$  denote an access structure. To encrypt a message  $m \in G_T$ , it selects a random  $s \in \mathbb{Z}_p$  and computes  $\tilde{C} = m \cdot Y^{s \cdot k \cdot z}$ ,  $\hat{C} = g^{s \cdot z}$ ,  $\check{C} = h^{s \cdot k \cdot z}$ . For  $i \in N$ : if  $+d_i$  appears as  $AS$ ,  $C_i = T^{s_i}$ ; if  $-d_i$  appears as  $AS$ ,  $C_i = T^{s_{n+i}}$ ; otherwise,  $C_i = T^{s_{2n+i}}$ . It outputs  $C = \langle AS, \tilde{C}, \hat{C}, \check{C}, (C_i)_{i \in N} \rangle$ .

**RKGEN**( $usk, AS^i$ ): Let  $usk$  denote a valid secret key consisting of  $\langle S, (D_{i,1})_{i \in N}, \hat{D}, k \cdot z \rangle$  and let  $AS^i$  denote an access structure. It selects a random  $d \in \mathbb{Z}_p$  and set  $\mathfrak{S} = g^d, \hat{D}^i = \hat{D}$ . For  $i \in N$   $D'_{i,1} = D_{i,1} \cdot h^d$ ;  $C^i$  is the

ciphertext of  $\mathfrak{S}$  under the access structure  $AS^i$ .

It outputs  $rk = \langle S, AS^i, (D_{i,1})_{i \in N}, \hat{D}^i, k \cdot z, C^i \rangle$

REENC( $rk, C$ ): Let  $rk$  denote a valid re-key consisting of  $\langle S, AS^i, (D_{i,1})_{i \in N}, \hat{D}^i, k \cdot z, C^i \rangle$  and  $C$  denote a well-formed ciphertext  $\langle AS, \tilde{C}, \hat{C}, \tilde{C}, (C_i)_{i \in N} \rangle$ . This step checks whether  $S$  satisfies  $AS$ ; if not, output  $\perp$ ; otherwise, for  $i \in N$ :

$$+d_j \text{ appears in } AS, T_j = \frac{t_j}{k \cdot z};$$

$$-d_j \text{ appears in } AS, T_j = \frac{t_{n+i}}{k \cdot z}.$$

$$\text{Otherwise, } T_j = \frac{t_{2n+i}}{k \cdot z};$$

$$\text{It computes } T = \frac{1}{\prod_{i \in N} T_i} = \frac{k \cdot z}{\sum_{j \in S} t_j} = \frac{k \cdot z}{t},$$

$$C = \prod_{i \in N} C_i = g^{S \cdot \sum_{j \in S} t_j} = g^{S \cdot t} \text{ and}$$

$$D = \prod_{i \in N} D_i = h^{d + \sum_{i \in N} r_i} = h^{n \cdot d + r}.$$

$$\text{Next it computes } E = e(C, D^T) = e(g, h)^{(n \cdot d + r)(k \cdot s \cdot z)}.$$

It then computes

$$\begin{aligned} \bar{C} &= e(\hat{C}, \hat{D}^i) \cdot E = e(g^{S \cdot z}, h^{k \cdot (y-r)}) \cdot e(g, h)^{(n \cdot d + r)(k \cdot s \cdot z)} \\ &= e(g, h)^{(k \cdot s \cdot z \cdot y) + (n \cdot d \cdot k \cdot s \cdot z)}; \end{aligned}$$

It outputs a re-encrypted ciphertext

$$C_{re} = \langle AS^i, \tilde{C}, \bar{C}, \tilde{C}, C^i \rangle$$

Note that  $C_{re}$  can be re-encrypted iteratively. Thus we would obtain  $C'_{re} = \langle AS^n, \tilde{C}, \bar{C}, \tilde{C}, C^n \rangle$  where  $C^n$  is obtained from the REENC algorithm with the input of another  $rk^i$  and  $C^i$ . The size of the ciphertext and re-encryption times increase linearly.

DEC( $C, usk$ ): Let  $usk$  denote a valid secret key  $\langle S, (D_{i,1})_{i \in N}, \hat{D}, k \cdot z \rangle$ . It checks whether  $S$  satisfies  $AS$ ; if not, it outputs  $\perp$ ; otherwise, decrypt.

If  $C$  is an original well-formed ciphertext consisting of  $\langle AS, \tilde{C}, \hat{C}, \tilde{C}, (C_i)_{i \in N} \rangle$ , for  $i \in N$ :

$$+d_j \text{ appears in } AS, T_j = \frac{t_j}{k \cdot z};$$

$$-d_j \text{ appears in } AS, T_j = \frac{t_{n+i}}{k \cdot z};$$

$$\text{Otherwise, } T_j = \frac{t_{2n+i}}{k \cdot z}.$$

$$\text{It computes } T = \frac{1}{\prod_{i \in N} T_i} = \frac{k \cdot z}{\sum_{j \in S} t_j} = \frac{k \cdot z}{t},$$

$$C = \prod_{i \in N} C_i = g^{S \cdot \sum_{j \in S} t_j} = g^{S \cdot t} \text{ and}$$

$$D = \prod_{i \in N} D_i = h^{\sum_{i \in N} r_i} = h^r.$$

$$\text{Next it computes } E = e(C, D^T) = e(g, h)^{k \cdot r \cdot s \cdot z}.$$

It outputs

$$\frac{\tilde{C}}{e(\hat{C}, \hat{D}) \cdot E} = \frac{m \cdot e(g, h)^{k \cdot s \cdot y \cdot z}}{e(g^{S \cdot z}, h^{k \cdot (y-r)}) \cdot e(g, h)^{k \cdot r \cdot s \cdot z}} = m.$$

Otherwise, if  $C$  is a re-encrypted well-formed ciphertext consisting of  $\langle AS^i, \tilde{C}, \bar{C}, \tilde{C}, C^i \rangle$ , then it decrypts  $C^i$  using  $usk$  and obtains  $\mathfrak{S} = g^d$ . Then it outputs  $\frac{\tilde{C} \cdot e(\mathfrak{S}, \bar{C})^n}{\bar{C}} = \frac{m \cdot e(g, h)^{k \cdot s \cdot y \cdot z} \cdot e(g, h)^{n \cdot d \cdot s \cdot k \cdot z}}{e(g, h)^{(k \cdot s \cdot z \cdot y) + (n \cdot d \cdot k \cdot s \cdot z)}} = m$ .

Otherwise, if  $C$  is a multi-time re-encrypted well-formed ciphertext, then decryption is similar with the above phases.

### D. Security Proof for ABPRE

Since our scheme is an expansion of a previous ABPRE scheme in [8], the security proof is based on previous proof in [8] and we also extend it to our scheme.

Theorem 1: When the ADBDH assumption holds in  $(G, G_T)$ , the ABPRE scheme ensures selective-structure chosen plaintext secure in the standard model.

Proof: In this section, we show that SS-CPA-ABPRE meets the requirements in terms of the ADBDH assumption.

We suppose that an adversary wins the SS-CPA-ABPRE game with a non-negligible advantage  $\epsilon$ . A simulator  $S$  can be constructed to distinguish  $D_{adbdh}$  from  $D_{rand}$

with the non-negligible advantage  $\frac{\epsilon}{2}$ .

We first suppose the challenger set the groups  $G$  and  $G_T$  with bilinear map  $e$  and a generator  $g$ . The challenger randomly selects a side of a fair coin  $c$ , without  $S$ 's intervention. If  $c = true$  the challenger sets  $\langle g, A, B, C, B^i, Z \rangle \in D_{adbdh}$ ; otherwise it sets

$$\langle g, A, B, C, B^i, Z \rangle \in D_{rand}.$$

Init:  $S$  receives a challenge access structure  $AS^*$ , and names  $I_+^*, I_-^*$  the index set of positive and negative attributes, respectively. Then  $S$  chooses  $x, y, \alpha_j, \beta_j, \gamma_i$  at random from  $Z_p$  for  $i \in N$  and generates the public key  $Y = e(A, B)^y$ . Then  $S$  outputs the public parameters as follows:

$$i \in I_+^*, T_j = g^{\alpha_j}, T_{n+i} = B^{\beta_j}, T_{2n+i} = B^{\gamma_i};$$

$$i \in I_-^*, T_j = B^{\alpha_j}, T_{n+i} = g^{\beta_j}, T_{2n+i} = B^{\gamma_i};$$

$$\text{Otherwise, } T_j = B^{\alpha_j}, T_{n+i} = B^{\beta_j}, T_{2n+i} = g^{\gamma_i}.$$

Phase 1: An adversary  $A$  makes several queries to the key generation oracle  $O_{kg}$ , the re-key generation oracle  $O_{rkg}$ , and the re-encryption oracle  $O_{ree}$ .

$A$  makes several queries to the  $O_{kg}$  with an index set  $I_q$ . According to the security game, if  $I_q$  satisfies  $AS^*$ , it outputs  $\perp$ .

Otherwise,  $S$  queries  $usk = \langle l_q, (D_{i,1})_{i \in N}, \hat{D}, k \cdot z \rangle$  from the oracle and outputs  $usk$ .

$A$  makes a query to  $O_{rkq}$  with an index set  $l_q$  and an access structure  $AS$ . According to the security game, if  $l_q$  satisfies  $AS^*$ , it outputs  $\perp$ . Otherwise,  $S$  submits  $l_q$  to  $O_{kg}$  and obtains a secret key  $usk = \langle l_q, (D_{i,1})_{i \in N}, \hat{D}, k \cdot z \rangle$ .  $S$  executes the following procedure:

It selects a random  $d \in Z_p$  and set  $\mathfrak{S} = g^d, \hat{D}' = \hat{D}$ .

For  $i \in N$   $D'_{i,1} = D_{i,1} \cdot h^d$ ,

it outputs  $rk = \langle l_q, AS', (D'_{i,1})_{i \in N}, \hat{D}', k \cdot z, C' \rangle$ , in which  $C'$  is the ciphertext of  $\mathfrak{S}$  under the access structure  $AS'$ .

$A$  makes a query to  $O_{ree}$  with an index set  $l_q$ , an access structure  $AS'$ , and a ciphertext  $C = \langle AS, \tilde{C}, \hat{C}, (C_i)_{i \in N} \rangle$ . According to the security game, if  $l_q$  satisfies  $AS^*$ , it outputs  $\perp$ . If  $l_q$  does not satisfy  $AS$ , it outputs  $\perp$ . Then  $S$  submits  $(l_q, AS')$  to the re-key generation oracle and obtains  $rk = \langle l_q, AS', (D'_{i,1})_{i \in N}, \hat{D}', k \cdot z, C' \rangle$ .  $S$  uses  $rk$  to re-encrypt the ciphertext  $C$ . For  $i \in N$ :

$+d_j$  appears in  $AS$ ,  $T_j = \frac{t_j}{k \cdot z}$ ;

$-d_j$  appears in  $AS$ ,  $T_j = \frac{t_{n+i}}{k \cdot z}$ ;

Otherwise,  $T_j = \frac{t_{2n+i}}{k \cdot z}$ .

It computes  $T = \frac{1}{\prod_{i \in N} T_i} = \frac{k \cdot z}{\sum_{j \in S} t_j} = \frac{k \cdot z}{t}$ ,

$C = \prod_{i \in N} C_i = g^{s \cdot \sum_{j \in S} t_j} = g^{s \cdot t}$ , and

$D = \prod_{i \in N} D_i = h^{d + \sum_{i \in N} r_i} = h^{n \cdot d + r}$ . Next, it computes

$E = e(C, D^T) = e(g, h)^{(n \cdot d + r)(k \cdot s \cdot z)}$ .

Finally, it computes

$\bar{C} = e(\hat{C}^{k \cdot z}, \hat{D}') \cdot E = e(g^{k \cdot s \cdot z}, h^{y-r}) \cdot e(g, h)^{(n \cdot d + r)(k \cdot s \cdot z)}$   
 $= e(g, h)^{(k \cdot s \cdot z \cdot y) + (n \cdot d \cdot k \cdot s \cdot z)}$ ; It outputs a re-encrypted ciphertext  $C_{re} = \langle AS', \tilde{C}, \bar{C}, \hat{C}, C' \rangle$ .

Challenge:  $A$  submits two equal messages  $M_0$  and  $M_1$  in length.  $S$  produces a challenge ciphertext:

$C^* = \langle \tilde{C} = M_\mu \cdot Z^k, C, C^k, (C^{\alpha_i})_{i \in L_+^*}, (C^{\beta_i})_{i \in L_-^*}, (C^{\gamma_i})_{i \in L_+^* \cup L_-^*} \rangle$ .

Phase 2: Same procedure as Phase 1.

Guess: A tuple was given from  $D_{adbdb}$  when  $S$  outputs  $c' = 1$  and  $A$  gives a correct guess  $\mu' = \mu$ ; otherwise a tuple was given from  $D_{rand}$  when  $S$  outputs

$c' = 0$ .

According to [3], the advantage of the simulator to output a correct  $v' = v$  is  $\Pr[v = v'] - \frac{1}{2}$   
 $= \Pr[v = v', v = 0] + \Pr[v = v', v = 1] - \frac{1}{2} = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot (\frac{1}{2} + \epsilon) - \frac{1}{2} = \frac{\epsilon}{2}$

Theorem 2: If the CTDH assumption holds in  $G$ ,  $G_T$ , then the ABPRE scheme has master key security.

Proof: The simulator  $S$  receives a tuple

$\langle g, n, g_b = g^b, h = g^c, g_d = g^d, g_1 = g^{\frac{c}{b}}, g_2 = g^{bc},$

$g_3 = g^{ac}, g_4 = g^{abc - Rc},$

$g_5 = g^{c(R+nd)}, g_6 = g^{\frac{c(R+nd)}{b}} \rangle$  and a challenge index set

$I^*$ . To output  $g^{abc}$ , the simulator  $S$  does as follows:

Init:  $S$  chooses  $\alpha_i, \beta_j, \gamma_i$  at random from  $Z_p$  for  $i \in N$  and generates the public key and set  $Y = e(g, h)^{ab} = e(g_b, g_3)$ . Then it computes public keys as follows:

$i \in I^*$ ,  $T_i = g^{\alpha_i}, T_{n+i} = g_b^{\beta_i}, T_{2n+i} = g_b^{\gamma_i}$ ;

$i \notin I^*$ ,  $T_i = g_b^{\alpha_i}, T_{n+i} = g^{\beta_i}, T_{2n+i} = g_b^{\gamma_i}$ .

$S$  outputs public parameter

$pp := \langle e, g^z, h, Y^{k \cdot z}, \{T_i, T_{n+i}, T_{2n+i}, \frac{t_j}{k \cdot z}\}_{i \in N} \rangle$ .

Key generation oracle:  $A$  makes a query to the key generation oracle with an index set  $l_q \subseteq N$  where  $l_q \neq I^*$ . An index  $j$  must belong to the following conditions  $((j \in l_q) \wedge (j \notin I^*))$  or  $((j \notin l_q) \wedge (j \in I^*))$ . For this reason, we just analyze the case of  $(j \notin l_q) \wedge (j \in I^*)$ .

For every  $i \in N$ ,  $S$  chooses  $r_i \in Z_p$  at random and implicitly sets  $r_i$  in the following ways:  $r_i = br'_i$  ( $i \neq j$ );  $r_j = ab + br'_i$  (otherwise).

Denote  $r = \sum_{i=1}^n r_i = ab + \sum_{i=1}^n r'_i \cdot b$  and

$\hat{D} = (h^{y-r})^k = h^{k(-\sum_{i=1}^n r'_i \cdot b)} = \prod_{i=1}^n g_2^{\frac{k}{r'_i}}$ .

For  $i \in l_q$ ,  $i \neq j$ : if  $i \in I^*$   $D_{i,1} = D_{i,2} = h^{r_i} = g_2^{\frac{k}{r'_i}}$   
else  $i \notin I^*$   $D_{i,1} = D_{i,2} = h^{r_i} = g_2^{\frac{k}{r'_i}}$ .

For  $i \notin l_q$ ,  $i = j$ : if  $i \in I^*$ ,  $D_{i,1} = D_{i,2} = h^{r_i} = g_2^{\frac{k}{r'_i}}$   
else  $i \notin I^*$   $D_{i,1} = D_{i,2} = h^{r_i} = g_2^{\frac{k}{r'_i}}$ .

For,  $i = j :$ ,  $D_{i,1} = D_{i,2} = h^{r_i} = h^{ab+br^i}$ .

It outputs a secret key  $usk = \langle l_q, (D_{i,1})_{i \in N}, \hat{D}, k \cdot z \rangle$ .

Rekey generation oracle:  $A$  makes a query to the key generation oracle with an index set  $l_q \subseteq N$  and access structure  $AS$ , if  $l_q \neq l^*$ , obtain  $usk$  from  $KGEN(l_q)$  and generate  $rk = RKGEN(usk, AS)$ ; else  $l_q = l^*$ .

Select  $j \in l^*$  and  $r_j, r \in Z_p$  at random for each  $i \in N \setminus \{j\}$ ;

Implicitly set  $r_j = r + R + nd - \sum_{i=1, i \neq j}^n r_i$  and  $r_i = r_j - d$  for  $i \in N$ ;

Compute

$$\hat{D} = h^{k(y - \sum_{i=1}^n r_i)} = h^{k(y - \sum_{i=1}^n (r_j - d))} = h^{k(ab - R - r)} = (g_d h^{-r})^k;$$

For  $i \in N$ :

If  $i \neq j, i \in l^*, D_{i,1} = h^{r_i+d} = h^{r_j}$ ;

Else if  $i \neq j, i \notin l^*, D_{i,1} = h^{r_i+d} = h^{r_j}$ ;

Else if

$$i = j, D_{i,1} = h^{r_j+d} = h^{r_j} = h^{r+R+nd - \sum_{i=1, i \neq j}^n r_i} = g_5 h^{r - \sum_{i=1, i \neq j}^n r_i}$$

Output  $rk = \langle S, AS, (D_{i,1})_{i \in N}, \hat{D}, k \cdot z, C \rangle$  where  $C$  is the ciphertext of  $g_d \cdot g^t$  under the access structure  $AS$ .

Re-encryption oracle:  $A$  makes a query to the key generation oracle with an index set  $l_q \subseteq N$  and access structure: if  $l_q \neq l^*$ , obtain  $rk$  from  $RKGEN(usk, AS)$  and generate  $C^1 = REENC(rk, C)$ ; else  $l_q = l^*$ .

-For  $i \in N$ :

-  $+d_j$  appears in  $AS$ ,  $D_{i,1}^{T_j} = h^{\frac{r_j+d}{kz\alpha_j}} = h^{\frac{r_j}{kz\alpha_j}}$  where

$$T_j = \frac{t_j}{k \cdot z};$$

-  $-d_j$  appears in  $AS$ ,  $D_{i,1}^{T_j} = h^{\frac{r_j+d}{kz\beta_j}} = h^{\frac{r_j}{kz\beta_j}}$  where

$$T_j = \frac{t_{n+i}}{k \cdot z};$$

- Otherwise,  $D_{i,2}^{T_j} = h^{\frac{r_j+d}{bkz\gamma_j}} = h^{\frac{r_j}{kz\gamma_j}}$  where  $T_j = \frac{t_{2n+i}}{k \cdot z}$ .

It outputs a secret key  $usk^*$  for  $l^*$  including  $usk = \langle S, (D_{i,1})_{i \in N}, \hat{D}, k \cdot z \rangle$ . If it is a valid secret key,  $usk^*$  satisfies the following equation:  

$$e(g^z, \hat{D}) \prod_{i \in l^*} e(T_j, D_{i,1}^{T_j}) \prod_{i \notin l^*} e(T_{n+i}, D_{i,1}^{T_j}) = e(g, h)^y;$$

The decryption oracle is straightforward, since the secret key and re-key could be correctly generated from the Key generation and Rekey generation oracle.

It outputs  $\hat{D}^z \cdot \prod_{i \in l^*} D_{i,1}^{-\alpha_j} \cdot \prod_{i \in l^*} D_{i,1}^{-\beta_j} = h^y = g^{abc}$  and solves the CTDH problem.

### E. Evaluation

Majority of ABE or ABPRE schemes require computing a number of pairing operations to generate the ciphertext depending on unique attributes but the computational cost of a pairing operation is much higher than other operations. For this reason, a small number of pairing operations is important factor for efficient cryptography algorithms. In 2009, Emura et al. [9] presented a constant length of pairing operations in an ABE scheme. This scheme is the motivation of our algorithm. In our scheme, we propose an expansion algorithm of ABE, ABPRE maintaining a constant pairing operation. A comparison of computation complexity is illustrated in Table 1.

The notation  $kG$  and  $kC_e$  denote the k-times calculation over group  $G$  and pairing, respectively. Let  $u = \{att_1, att_2, \dots, att_n\}$  be the set of attributes.

**Table 1.** Computational time of each algorithm

	Encryption	Decryption	Re-encryption	Re-encrypted decryption
Cheung & Newport [5]	$(n+1)G_1 + 2G_T$	$(n+1)C_e + (n+1)G_T$	-	-
Bethencourt et al. [7]	$(2r_1+1)G_1 + 2G_T$	$2r_1C_e + (2r_1+2)G_T$	-	-
Nishide et al. [11]	$(2N'+1)G_1 + 2G_T$	$(3n+1)C_e + (3n+1)G_T$	-	-
Waters [12]	$(1+3r_1n)G_1 + 2G_T$	$(1+n+r_1)C_e + (3r_1-1)G_1 + 3G_T$	-	-
Emura et al. [9]	$(n+1)G_1 + 2G_T$	$2C_e + 2G_T$	-	-
Liang et al. [8]	$(n+2)G_1 + 2G_T$	$2G_T + (n+2)C_e$	$G_T + (n+1)C_e$	$4G_T + (n+3)C_e$
Our scheme	$(n+2)G_1 + 2G_T$	$(3n+2)G_1 + 2G_T + 2C_e$	$3nG_1 + G_T + 2C_e$	$3nG_1 + 4G_T + 3C_e$

**Table 2.** Some properties of attribute-based encryption schemes

	Policy	Recipient anonymity	Capability of delegation	Assumption
Cheung & Newport [5]	Ciphertext	No	No	DBDH
Bethencourt et al. [7]	Ciphertext	No	No	Generic group model
Nishide et al. [11]	Ciphertext	Yes	No	DBDH, D-linear
Waters [12]	Ciphertext	No	No	DBDH
Emura et al. [9]	Ciphertext	No	No	DBDH
Liang et al. [8]	Ciphertext	No	Yes	CTDH, ADBDH
Our scheme	Ciphertext	No	Yes	CTDH, ADBDH

DBDH: decisional bilinear Diffie-Hellman, CTDH: complex triple Diffie-Hellman, ADBDH: augment decisional bilinear Diffie-Hellman.

Let  $r_1$  and  $r_2$  be a set of attributes associated with the ciphertext

and a set of attributes associated with the secret key, respectively. Let  $N = \sum_{i=1}^n n_i$  be the total number of possible statements of attributes.

Comparison of some properties including policy anonymity, capability of delegation, and security assumption is illustrated in Table 2. All schemes follow the ciphertext policy. Nishide et al. [11] scheme only provides recipient anonymity. Capability of delegation is a strong feature of ABPRE-based schemes. Therefore, we can empower designated users with the capability of decryption. Our security assumption is based on CTDH and ADBDH. This provides a selective-structure chosen to be plaintext secure and master key secure.

## V. CONCLUSIONS

In the paper, we present the ABPRE with constant number of pairing operations. The scheme is motivated from previous CP-ABE by computing constant length and number of pairing operations. Compared to previous ones, our proposal has the strength of its capability of delegation. Through the feature, we can empower designated users to decrypt the ciphertext re-encrypted with a new access structure. The scheme can be adapted to various applications including e-mail forwarding and distributed file systems.

Future work includes how to implement the scheme efficiently in various environments such as traditional computing environments and embedded systems (wireless sensor network, near field communication, and radio frequency identification).

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