대한수학교육학회지 수학교육학연구 제 22 권 제 1 호
Journal of Educational Research in Mathematics
Vol. 22, No. 1, 39-52. Feb 2012

Review on Relation between Knowledge for Teaching Mathematics and Student Learning

Park, Jungeun*

Research in teachers' knowledge for teaching mathematics (KTM) has been gradually growing for the past several years. With various conceptualizations about what teachers need to teach mathematics, there have been studies to find out the relationship between teachers' KTM and their students' achievement. In this paper, I reviewed various conceptualizations of teacher's mathematical knowledge for teaching, and existing qualitative and quantitative studies about this relationship. Based on the review, I identified the challenges to studying this relationship mainly focusing on the existence of a phase－teaching practice－between teachers' KTM and students' learning. Considering the challenges that have been identified in the literature review, I proposed topics for future studies that would contribute to our understanding how the teachers' KTMare related to their students' achievement, and investigate further about whether and how teacher-student interaction in classroom is related to changes in teachers' KTM.

I. Introduction

Teachers' knowledge for teaching mathematics (KTM) has been an important topic in mathematics education in recent years. Researchers in math education have conceptualized this knowledge (e.g., Hill, Ball, & Schilling, 2008; Ma, 1999) and have investigated the relationship between KTM and students' achievement on mathematics. Recently, National Mathematics Advisory Panel (2008b) has reviewed existing studies on these connections and concluded that the research on this area "supports the importance of teachers'content knowledge in students learning" (p. 5-xxi), but there is a need for more research on characteristics of teachers' knowledge, which are strongly connected, to what students learn. The panel also stated that because a direct relationship between conventional mathematical study (i.e., the mathematics courses or majors completed) and teacher effectiveness is not supported by a review of high-quality research, future research should uncover those aspects of teacher knowledge and understanding that are most strongly related to student learning (2008, p. 22).

In the panel's (2008) review, most existing studies on the relationship between teachers' knowledge for teaching mathematics and students' achievement were not classified as high quality studies. A lack of high quality research in this area may come from various challenges to conducting studies on this relationship. For example, measuring teachers' mathematical knowledge is not an easy endeavor, and there exist various factors affecting students' learning in addition to their teachers' KTM. Moreover,

* University Central Arkansas (jungeunp@uca.edu)

[Figure I-1] Teaching as a Phase between Teacher Knowledge and Student Learning

the emphasis on connecting teacher knowledge directly to student learning neglects the most important phase, *teaching* (Figure I-1).

Based on these observations, this paper reviews what has been known about the connection between teachers' knowledge and students' learning in the existing studies, discusses the challenges to studying this connection, and proposes topics for future studies that would advance our understanding about this connection. In this paper, the term *knowledge for teaching mathematics* (KTM) is used for knowledge that teachers use for planning, instructional practice, and reflection. It includes content knowledge and specific knowledge for teaching mathematics.

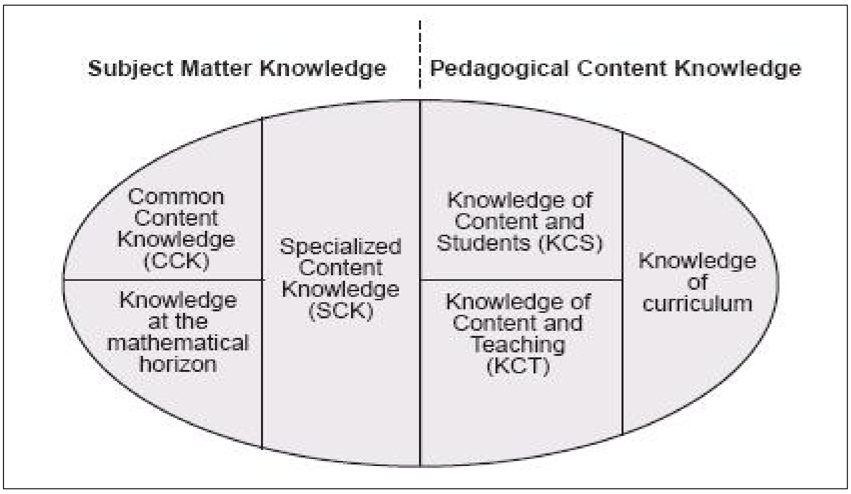
II. Existing Research

Many researchers have argued that teachers' KTM plays an important role in students' learning (e.g., Ball, Lubienski, & Mewborn, 2001; Fennema & Franke, 1992 Hill, Rowan, & Ball, 2005). They have conceptualized KTM in many ways, and explored the relationships (a) between teachers'mathematical knowledge and their instruction, and (b) between instruction and students' learning (e.g., Hill, Blunk, Charalambous, Lewis, Phelps, Sleep & Ball, 2008; Lobato, Ellis, & Munoz, 2003). This section addresses various conceptualizations of KTM, relationships between such knowledge and students' learning, factors affecting the quality of teaching other than teachers' knowledge, and effects of instruction on students' learning.

1. Conceptualizations of teachers' mathematical knowledge

There have been various conceptualizations of teachers' KTM (Shulman, 1986; Hill, Ball, & Schilling, 2008 Ma, 1999). Shulman (1986) discussed the three types of knowledge needed in teaching: subject matter *content knowledge* (CK), *pedagogical content knowledge* (PCK), and curricular knowledge (p.9). CK was defined as "the amount and organization of knowledge…in the mind of teachers" which includes understanding the structure of a subject matter (p. 9). The main idea of PCK lies in *teachability*, which includes knowing how to represent the content for others to understand and what aspects of a certain concept make learning the concept difficult. Curricular knowledge consists of lateral knowledge of a variety of programs for a subject, and vertical knowledge of the order of topics in the subject. Lateral curricular knowledge helps teachers evaluate curriculum effectiveness, and vertical curricular knowledge helps teachers make connections between current topics and the related topics that students learned previously or will learn in the future (Shulman, 1986).

Hill, Ball, et al., (2008) applied Shulman's three



[Figure II-2] Domain Map for Mathematical Knowledge for Teaching

types of teacher knowledgeto mathematics and elaborated them as shown in Figure II-2 (Hill, Ball, et al., 2008, p. 376). Teachers' subject matter knowledge is broken into three subcategories: common content knowledge (CCK), knowledge at the mathematical horizon, and specialized content knowledge (SCK). SCK emphasizes that teachers are expected to know more than what well-educated adult know about a mathematical topic, CCK. For example, to help elementary students who might have trouble understanding the standard algorithms, elementary teachers are supposed to have knowledge about alternative algorithms which are not necessarily a part of well-educated adults'knowledge. Knowledge at the mathematical horizon includes being aware of more general cases of a specific mathematical topic they are currently teaching (e.g., whole number multiplication as a special case of binomial multiplication) (Ball, 2003). In pedagogical content knowledge (PCK), the knowledge of content and students (KCS) contains knowing why students give specific types of wrong answers. Knowledge of content and teaching (KCT) includes knowing advantages of teaching a mathematical topic with various representations. This picture and their descriptions clearly show that having only CCK is not sufficient to teach students by separatingCCK from SCK and Knowledge at the mathematical horizon.

Ma (1999) also argued that teachers' knowledge should go beyond the subject matter knowledge of mathematics. She considered elementary mathematics as fundamental mathematics and argued that elementary school teachers should have a *Profound Understanding of Fundamental Mathematics* (PUFM) to teach children mathematics. Her concept of PUFM is aligned with Shulman's and Hill, Ball, et al.'s (2008) concept of PCK in the sense that PUFM exceeds the ability to compute correctly and give rationales for algorithms. PUFM includes the ability to teach "the conceptual structure and basic attitudes of mathematics inherent in elementary mathematics" elementary school students appropriately (p. 124).

In their book, *the Mathematical Education of Teachers*, the Conference Board of the Mathematical Sciences (CBMS) (2001) listed components of knowledge that mathematics teachers are supposed to have. Similar to other conceptualizations of KTM, CBMS (2001) includedknowledge of curricula, and "deep understanding" of concepts and procedures in school mathematics, similar to CK, and "mathematical knowledge for teaching," similar to PCK (p. 13). Their "mathematical knowledge for teaching" includes recognizing sources of the students'errors and their understanding of mathematical concepts, encouraging the creative responses made by talented students, and making students aware of links between mathematical topics. However, in contrast to other conceptualizations of teachers' knowledge of mathematics, CBMS (2001) emphasized teachers'grade specific knowledge as well as broader knowledge of each cluster: elementary, middle, and high school.

These conceptualizations of teachers' knowledge by Shulman (1986), Hill, Ball, et al., (2008), Ma (1999), and CBMS (2001) share a same assumption that teachers should possess knowledge beyond mathematical content knowledge although they are different from each other in terms of components of knowledge for teaching mathematics and levels of specificity in each component.

2. Relationships between teachers' knowledge and students' achievement

Studies on the relationship between teachers' knowledge and their students' achievement on mathematics has adopted various research methods. Qualitative research provided evidence of the impact of teachers' knowledge on students' learning and tried to explain how this relationship plays out in practice (e.g., Carpenter et al., 1989). Quantitative research has foundhow strong this relationship is when controlling other factors that might influence students' learning (e.g., Felter, 1999; Hill, Rowan, & Ball, 2005). This section reviews some quantitative or qualitative studies about the relationships.

A. Qualitative research.

Carpenter and colleagues (1989) conducted research on how teachers' knowledge about children's thinking about addition and subtraction impacts their instructional practice and students'performance on items about these operations, using a mixed research method with a qualitative component. They assigned 40 first-grade teachers into experimental and control groups. The experimental group teachers participated in a four-week workshop in which they read and discussed research on how children develop problem-solving skills in addition and subtraction whereas the control group teachers participated in two 2-hour workshops about non-routine problems to motivate students without making connections to children's cognition. The students of those teachers were tested before and after the addition and subtraction lessons. After the workshop, the experimental group teachers focused on problem solving more than number factors, and predictedwhich strategies their students would use on particular problems in their teaching better than the control group teachers. The two groups of students' performance on the post-test was also different. Although they did not significantly differ in computation skills and advanced problems, the experimental group students significantly outperformed their counterparts on complex word problems. The low-achieving experimental group students (pre-test) significantly outperformed the low-achieving control group in the posttest.

Qualitative research done in classroom can show the impact of teachers' knowledge on students' learning and possible mechanisms of how this impact happens in practice. However, Carpenter et al. (1989) cannot provide a valid scale showing whether relationshipsbetween teachers' knowledge and students' learning are significant or differ in different grade levels.

B. Quantitative research

Existing quantitative studies on the relationshipbetween teachers' KTM and students' learning are featured by large scale of data and multivariate modeling. Three measures have been used to estimate KTM: teacher certificate, mathematical coursework and/or degree, and test scores on assessment of KTM. This section summarize the Panel's (2008) review on the quantitative studies on this relationship with the details of two studies included in the review.

The Panel (2008a) reported that the studies, which used teacher certificates, mathematics coursework, and their degrees as proxies of teachers' mathematical knowledge for teaching showed inconsistent results. Some of these studies were classified as lesser-quality because of flaws in their research design. For example, Felter (1999) conducted a study on the relationship between teachers' certificate and their students' achievement. He used teacher certification as a proxy of teachers'mathematical knowledge and calculated the percentage of mathematics teachers who possess an emergency permit[1] in each school. He showed that these percentages were negatively related to school means of students' scores on Standard Achievement Test Series (SATS) while controlling students' poverty, and their teachers' average number of years of teaching. However, the author failed to control other important factors such as students' mathematics ability before they entered high schools, which usually are reflected in pre-test scores in other studies, or students' motivation to study mathematics.

This inconsistency, however, does not seem to come only from flawed research design because studies classified as high-quality by the Panel (2008a) also reported inconsistent results. Of the five high-quality studies, which used teacher certificate status as a proxy, three reported its positive relationship with students' achievement whereas two reported that the relationship was not significant. Similarly, of seven high-quality studies, which used teachers' course work and/or degree as measures of KTM, four reported a positive relationship between these measures and students' achievement, and three reported a negative relationship (National Mathematics Advisory Panel, 2008a). This inconsistency of results may come from inappropriateness of teachers' certificate and course work/degree as proxies of KTM. For example, there are various ways of being certified as high school teachers, which also vary across states; besides taking certain amount of mathematics classes in their undergraduate coursework, teachers can take a examinationor complete a subject matter program (Hill et al., 2005), or can have emergency permit (Felter, 1999). These various ways of being certified as teacher do not seem to be always related to their knowledge used in teaching mathematics. Teachers'coursework and degree may also not be good proxies because pre-service teachers have various choices of mathematics courses to

1) Emergency permit is a way that school districts can allow less qualified teachers to teach high school mathematics with their bachelor's degree, passing test of basic skill, and taking certain amount of upper leval mathematics courses (Felter, 1999).

complete teacher education programs, and knowledge they obtained from these course might not be the same as the knowledge activated in teaching practice. In addition, some studies showed that teachers' mathematical knowledge for teaching changes during their instruction (e.g., Leikin, 2005).

Teachers' test scores seem to be a better measure of their mathematical knowledge than their certificate, course work, and degree if the test is well developed. In the Panel's (2008a) review, all three high-quality studies, which used teachers', test scores as a proxy, reported the positive relationship between these scores and students' achievement although one result was not significant. For example, one high-quality study conducted by Hill et al., (2005) included all three proxies of teachers' mathematical knowledge: their certificate, coursework, and teachers'scores on 30 mathematical knowledge for teaching item about numbers, operations, and pre-algebra. They reported a significant positive relationship between teachers' scores and their students' gain scores but no significant relationship between teachers'certificate or coursework by controlling other factors such as students' socioeconomic status, ethnicity, and absence rate. These results indicate that mathematical knowledge for teaching, which is related to students' learning, was not caught by teachers' certificate or coursework but by their test scores in items specifically designed to measure their knowledge used in teaching (Hill et al., 2005).

3. Factors that affect the quality of teaching

In the studies about the relationship between teachers' KTM and students' learning, researchers have identified two types of factors affecting the quality of teaching other than teachers' mathematical knowledge: (a) teacher factors such as teachers' beliefs and their mathematical language uses (Hill, Blunk, et al., 2008; Kahan, Cooper, & Bethea, 2003), and (b) external factors outside the teacher's control factors includinglearners' motivation or enthusiasm, and opportunities to learn (Fenstermacher & Richardson, 2005). Since existing studies have controlled some of external variables, this paper will focus on teacher factors.

Kahan, Cooper, and Bethea (2003) explored the role of mathematics teachers' content knowledge in teaching by gathering information abouttheir mathematical knowledge using paper-pencil tests and observing their teaching. In this study, they defined mathematics teachers' knowledge as content knowledge (CK) consisting of ″a deep foundation of factual knowledge,″ ″understanding of the 'facts and ideas in the context of a conceptual framework,'″ and ″organization of knowledge 'in ways that facilitate retrieval and application'″ and excluding pedagogical content knowledge (Kahan et al., 2003, p. 225). Kahan, Cooper, and Bethea (2003) conducted a test designed to assess the teachers' CK, analyzed their teaching practice and interviewed them after teaching. After comparing teachers' test scores and their instructions, they concluded that teachers' content knowledge cannot explain all critical components of their teaching,and some these components were explained by other factors such as their beliefs and expectations of levels of students' knowledge. For example, in lessons about prime numbers, although one of the observed teachers knew that the number1 is not a prime or composite number, she did not mention it in class because she believed that from the definition of a prime number, it is obvious enough for students to understand.

Hill, Blunk, et al. (2008) also found factors affecting the quality of instruction and their relationships with teachers' MKT. In contrast to Kahan and his colleagues (2003), Hill, Blunk, et al. defined MKT more broadly including CCK and PCK (see Figure I-1). They measured these kinds of knowledge using paper-and pencil assessment on number and operations, geometry, and algebra. They defined the mathematical quality of teaching as "rigor and richness of the mathematics of lessons," and rated the quality by analyzing the videotapes of the lessons using the rubric which includes "connecting classroom activities to mathematics," "responding [to] students appropriately or…inappropriately," "mathematical language use," "presence or absence of mathematical errors," "richness of mathematics" (e.g., "multiple representations, connections between representations, mathematical explanation and justification") (Hill, Blunk, et al., 2008, p.437). The correlation coefficients between teachers' MKT scores and their video scales have shown that only some of the factors that affect mathematical quality of teaching were significantly related to teachers' MKT scores. "Responding [to]students appropriately" and "presence or absence of mathematical errors" were significantly related to teachers' MKT scores, but some other factors such as "connecting classroom activity to mathematics," and "richness of the mathematics" were not significantly related to teachers' MKT scores (Hill, Blunk, et al., 2008, p. 442).

4. Teaching practice that shapes students' learning

Studies about how teachers'mathematical knowledge is related to students' learning also need to investigate the relationship between teaching practice and students' learning. There have been several studies about how different lessons shape students' learning differently (e.g., Bingolbali, 2004; Lobato, Ellis, & Munoz, 2003). By exploring instructional settings that affect students' conceptualization of the slope, Lobato et al. (2003) have shown how the instructional environment supports students' tendency to think about slope as a difference rather than a ratio (p 1). By analyzing the videotapes of the slope lessons taught by a teacher and the interviews with her students, the authors have identified four instructional settings: (a) a teachers' iterative use of a phrase "goes up by," (b) using examples of well-ordered tables of x and y values where the difference between any adjacent x values is 1, (c) using a graphing calculator on whose screen the scale of x is shown as the change in x, and (d) using uncoordinated sequences, i.e. sequences and differences where the change in x is not 1 (p. 23), whose change in x is bigger than 1 in order to calculate the slope or connect to the slope formula. Lobato et al.(2003) have found that these settings support students' generalizations of the slope concept as the change in y, the change in x, or the scale of x. In the first and second instructional settings, they have identified that the teacher's questions and explanations of the slope, using the phrase "how much it goes up by" with a well-ordered table where the change in adjacent x values is 1, drew students'attention to the change in y rather than the ratio between the change in y and the change in x (p. 18). Although the teacher asked how often y value increased by a certain amount of x, students did not realize the conceptual difference between the change in y and the rate of change of y with respect to x. They found this tendency when students were asked to calculate the slope from tables of x and y values where the difference between adjacent x values was not 1. Students consistently gave the change in adjacent y values without considering the change in x (Lobato et al., 2003, p. 18). In the third and fourth settings, the teacher explained the slope using a graphing calculator and with uncoordinated sequences and differences. These settings tended to draw students' attention to the change or scale in x because they set up the scale of x in a graphing calculator, and calculated the change in x in order to obtain the value of slope from an uncoordinated graph or table and further connect the value to the slope formula, which is the change in y over the change in x (Lobato et al., 2003).

To explain the difference in conceptual development of the derivative between Mathematics (M) and Mechanical Engineering (ME) students, Bingolbali (2004) explored calculus lectures' interpretations of the derivative and corresponding instructional settings. He interviewed lecturers who taught in both M and ME departments and observed calculus courses, and collected their students' scores on pre- and post-tests in the beginning and end of the semester. Whereas there was no difference between M and ME students' pretest scores, there was a significant difference between their answers in the posttest. ME students outperformed M students with problems involving the rate of change as an interpretation of the derivative while the M students outperformed ME students with problems about the slope of a tangent line as an interpretation of the derivative. Interviews with the lecturers and observation of lessons showed that the lecturers focused on different aspects of the derivative when they taught calculus in the different department. With ME students, they spent more time on explaining the applications of the derivative in physical contexts using the rate of change whereas they spent more time on interpreting the derivative as the slope of a tangent line with M students. The lectures also used different types of examples in different departments in ME classes, they solved more examples involving rate of change of moving objects or energy, but in M class, but worked on examples involving graphical notations of the derivative. Bingolbali (2004) concluded that this emphasis on different interpretations of the derivative affected students' understanding of the derivative.

5. Challenges to Studying Connection between Teacher Knowledge and Student Learning

Due to the middle phase between teachers' KTM and students' learning (Figure I-1), one can find various factors affecting students' learning besides teachers' mathematical knowledge, which make studying this relationship difficult. These factors may not be controlled with a complicated modeling. Challenges to studying the relationship between teacher knowledge and student learning can be found in both parts of the model in Figure I-1: (a) the connection between teachers' mathematical knowledge for teaching and (b)the connection between teaching practice and students' learning.

A. Challenges to Studying Relationship between Teachers' MKT and Teaching

The first type of challenges to studying the relationships between teachers' mathematical knowledge and their students' learning can be found in teaching practice. Two types of challenges exist: (a) existence of various conceptualizations of teachers' mathematical knowledge which gives many choices to researchers and limits the generalizabilityof the result of the study, and (b) teachers' mathematical knowledge which is used in their instruction is hard to estimate or measure, and moreover, this knowledge changes over time (e.g., Leikin, 2005).

1) Various conceptualizations of teachers' mathematical knowledge.

There exist various conceptualizations of what teachers need to know to teach students (Hill, Ball, et al., 2008 Ma, 1999 Shulman, 1986). Depending on researchers' choice on one of these conceptualizations in their research, they might obtain different results, which could also limit the generalizability of their study. For example, Kahan, Cooper, and Bethea (2003) limited teachers' knowledge as mathematical content knowledge (CK) exclusively, when they conducted research on the roles teachers' knowledge play in their teaching (p. 225). They have concluded that teachers' content knowledge cannot explain some critical parts of their teachingbecause it does not take into account teachers' beliefs or teachers' expectation about levels of students' knowledge; for example, one of the teachers in their study did not teach that 1 is not a prime number not because of she did not know the fact but because she believed that this fact would obvious to her students from the definition of a prime number (Kahan et al, 2003). In contrast, Hill, Blunk, et al.(2008) define mathematical knowledge for teaching (MKT) more broadly including common content knowledge (CCK) and pedagogical content knowledge (PCK) in their research on the relationships between MKT and the mathematical quality of instruction. During interviews with teachers, Hill, Blunk, et al. (2008) found that teachers' MKT were closely related to scales of the quality of teaching although some scales were not significantly related to teachers' scores on the MKT test. As seen in these two studies, different conceptualizations of teachers' mathematical knowledge for teaching can identify different relationshipsto their teaching although they explain a similar phenomenon in which teachers' CK is not enough to explain all the aspects of teaching but its combination with PCK explains more aspects.

2) Measuring teachers' mathematical knowledge.

Mathematical knowledge that teachers use in instruction is likely to be combined with other factors such as their beliefs or expectationsabout their students' level (Kahan et al., 2003). The relationship of MKT and other factors makes it hard to capture MKT with proxies such as teachers' certificates, coursework and/or degrees, and test scores. The first two proxies may not offer proper information about teachers'mathematical knowledge that teachers use in their lessons because there are various ways of being certified and completing coursework and/or degrees. For example, ways of being certified as high school teachers vary; they may have completed a bachelor's degree with a one-year preparation program, or have taken a certification examination (Felter, 1999). Various ways of being certified as teachers might result in different types of mathematical knowledge for teaching, and finding out how they are related to teachers' knowledge used in practice seems to be affected by may other factors. Teachers' coursework and degreesalso might not be good proxies because pre-service teachers have various choices of mathematics courses to complete their teacher education programs. Different choices in completing coursework and/or degrees may allow pre-service teachers to form different levels of mathematical knowledge, which can be activated differently when they are teaching children.

Teachers'test scores, therefore, seem to be a better proxy of mathematical knowledge for teaching than other proxies. However, developing reliable and valid assessment is not an easy endeavor. Although items for testing teachers' performance on mathematical procedures may be easily developed, items for PCK or curricular knowledge should be developed with greater care with an analysis of curricula and research on how teachers address mathematical concepts in practice, and how students understand the concepts. In other words, designing a good assessment tool is not possible without support from existing literature. However, not all mathematical concepts have been addressed in mathematics education research with the same level of detail, which makes it hard to develop a test for MKT for some mathematical concepts.

Even with a well-designed test for teachers' KTM, no test can capture all the aspects of teachers' knowledge that they use in teaching. Different components of their knowledge may or may not be activated during the lesson depending on other factorssuch as students' questions and reactions to the problems teachers presented. Moreover, some studies have shown that teachers' mathematical knowledge changes while they interact with students (e.g., Leikin, 2005 Tzur, 2007). Leikin (2005) conducted research on how teachers'knowledge changes with their experience of teaching an unfamiliar topic. By analyzing how teachers planned, taught, and reflected on their teaching, Leikin (2005) has shown that teachers' knowledge changes while dealing with unforeseen students'answers or conjectures during the class, and reflecting on them after class. Similarly, Tzur (2007) argued that teachers learn mathematics by being aware of unexpected ways which their students or peers suggested while implementing their lesson plans. These changes from teachers' learning through teaching cannot be measured by one administration of assessment even with a well-designed assessment.

B. Relationship between instruction and students' learning

The second type of challenge to studying how teachers' KTM is related to their students' learning is the difficulty in assessing the relationship between teaching and student learning. Although students' factors (e.g., motivation & mathematical competence), and school factors (e.g., school mean socioeconomic status & sizes) mightbe controlled in modeling, there still exist classroom factors that cannot be controlled. Classroom factors include contents and types of questions that students ask and the extent of teachers' emphasis on a certain aspect of a mathematical concept (Bingolbali, 2004; Lobato et al., 2003). For example, calculus instructors in Bingolbali's (2004) study used different types of examples when they teach in Mechanical Engineering department and Mathematics department. Since the same lecturers taught both ME and M students, one can assume that their MKT is the same, but results of the study have shown that ME and M students develop different conceptualizations of the derivative (Bingolbali, 2004). Moreover, Lobata et al. (2003) stated that construction of this instructional environment is not completely "under the teacher's control" (p. 30).

III. Discussion and Conclusion

As reviewed in this paper, there have been an effort to find out the relationship between teachers' KTM and students' learning in mathematics education research field, and existing studies have contributed a lot to our understanding this relationship. However, it should be noted that teachers' knowledge and students' learning is often not direct due to the middle stage, "teaching" between the point where teachers' KTM are measured and the point where students' learning is assessed (Figure I-1). Studies have identified other factors such as teachers' beliefs and expectations about students' knowledge that affect their teaching than their KTM (Kahan et al., 2003), and some of the factors of the quality of teaching (e.g., teachers' mathematical language use & response to students' questions) are not significantly related to teachers' MKT scores (Hill, Ball, et al., 2008). There also exist studies about how instructional settings shape students' different understanding of the concepts (Bingolbali, 2004; Lobato et al., 2003). The characteristics of KTM, which is hard to measure and changes over time, is also a challenge of studying this relationship. Recent studies (e.g., Hill et al., 2005)used improved research methods with well-designed assessment for teachers' MKT and bycontrolling other factors affecting teachers' instructional practice and students' achievement. However, such research design still does not (a) explain how teacher knowledge changes over time or (b) give suggestions about what teachers can learn from these studies and do better in teaching practice.

A study about the changes in teacher knowledge can be designed to explore what part of teachers' knowledge is activated and how it is related to their teaching practice. The object is whether and how teachers' KTM changes from the experience of dealing with students' reactions to what they plan to teach. Such changes can be investigated by quantitative research involving administration of well-designed pre- and post-tests for teachers' KTM before and after they teach amathematical concept. A follow-up study can be conducted with teachers who showed significant changes in their KTM to explore possible occasions teachers change their KTM while they interact with students. However, one cannot go back in time and explore how the teachers, who showed significant change in pre- and post-test scores, actually taught their students in class. A possible option would be observing the teachers' classes in the following semester after the tests. Teachers' lesson plans can be used as supplementsof their KTM scores on pre-test, and video taped classroom lesson will show how they used their KTM and lesson plans in their class. After collecting video, the researcher can share the class episodes in which teachers taught differently from their lesson plans or teachers end up changing the following lesson plans, and ask for the reasons of change. During this process, the researcher may also investigate the teachers' beliefs about whatis important to cover in class and their assumptions about what their students know, because these are important factors affecting what part of the teacherknowledge is activated in teaching practice (Speer, 2005).

An additional study can be designed to investigate what teachers, whose students show a big achievement gap before and after the teachers taught the topic. A pre- and post-test format for students' performance can be adopted one of existing well-designed studies (Hill et al., 2005). For the similar reason discussed in the previous paragraph, one could observe lessons taught by teachers, whose students showed significant gain scores between pre- and post-tests, and observe their teaching in the following semester.

The two studies described above will provide us the information about what changes teachers' KTM and how implementation of KTM is related to high students' gain from their class although the students in the two studies will not be the students whom the teachers taught when the tests were administered. These additional studies with a well-designed study about the relationship between teachers' KTM and students' achievement will contribute to the current field of mathematics education by providing more complete picture of how significant these two components are related and where this significance comes from. Observing actual lessons and interviewing teachers' about the use of their own knowledge will be crucial component to make an argument that teachers' knowledge for teaching mathematics is a key factor of their students' learning, and thus provide a ground for improving teacher education programs for pre-service teachers and professional development opportunities for in-service teachers.

References

Ball, D. (2003) What mathematical knowledge is needed for teaching mathematics?, *Secretary's Summit on Mathematics*, Department of Education, Washington, DC.

Ball, D.L., Lubienski, S., & Mewborn, D. (2001) Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching*. New York: Mcmillan.

Bingolbali, E. (2004) The influence of lecturers privileging different aspects of derivative on students' conceptions, *Proceedings of the British Society for Research into Learning Mathematics*, Leeds, 24 (2), 7-13.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C. P., Loef, M. (1989) Using knowledge of children's mathematics thinking in classroom teaching: An experimental study American Educational Research Journal, 26, 499 - 531. Reprinted in T. P. Carpenter, J. Dossey, & J. L. Koehler (Eds.). (2004). *Classics in mathematics education research*. Reston, VA: National Council of Teachers of Mathematics.

Conference Board of mathematical science (2001) *The mathematical education of teachers*, Washington, D.C.

Fennema, E., & Franke, M. L. (1992) Teachers' knowledge and its impact. *Handbook of Research on Mathematics Teaching and Learning*, Macmillan Publishing Company, New York, NY.

Fetler, M. (1999). High school staff characteristics and mathematics test results. Education Policy Analysis Archives, 7(9). Retrieved February 13, 2009, from http://epaa.asu.edu/epaa/v7n9.html.

Hill, H. C., Ball, D., Schilling, S.(2008) Unpacking Pedagogical Content Knowledge: Conceptualizing and Measuring Teacher's Topic-Specific Knowledge of Students, *Journal for Research in Mathematics Education* 39 (4) 372-400.

Hill, H. C., Rowan, B., & Ball, D.L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42, 371 - 406.

Hill, H. C., Blunk, M.L., Charalambous, C., Y., Lewis, J. M., Phelps, G. C., & Sleep, L. (2008) Mathematical knowledge for teaching and the mathematical quality of instruction: an Exploratory study. *Cognition and instruction*, 26, 430-511.

Kahan, J. A., Cooper, D. A., & Bethea, K. A. (2003). The role of mathematics teachers' content knowledge in their teaching: A framework for research applied to a study of student teachers. *Journal of Mathematics Teacher Education*, 6(3), 223-252.

Leikin, R. (2005). *Teachers' learning in teaching: Developing teachers' mathematical knowledge through instructional interactions*. Paper presented at the 15th ICMI Study Conference: The Professional Education and Development of Teachers of Mathematics. Retrieved March 20, 2009 from http://stwww.weizmann.ac.il/G-math/ICMI/log_in.html

Lobato, J., Ellis, A.B., & Munoz, R. (2003). How "focusing phenomena" in the instructional environment afford students' generalizations. *Mathematical Thinking and Learning*, 5, 1-36

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum.

Speer, N.M. (2005). Issues of Methods and Theory in the Study of Mathematics Teachers' Professed and Attributed Beliefs. *Educational Studies in Mathematics,* 58(3), 361-391.

National Mathematics Advisory Panel (2008a) *Reports of the Task Groups and Subcommittees*. Washington, DC: Author.

National Mathematics Advisory Panel (2008b) *The Final Report of National Mathematics Advisory Panel* Retrieved January 11, 2009 from http://www.soe.unich.edu/downloads/MathPanelFialReport.pdf

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.

Tzur, R (2008) What and how might teachers learn via teaching: contributions to closing and an unspoken gap, *Proceedings 31st Conference of the Psychology of Mathematics Education*, 1, 142-149.

교수를 위한 수학적 지식과 학생의 성취도에 관한 문헌 연구

박 정 은 (센트럴 알칸소 대학교)

교사의 교수를 위한 수학적 지식에 관한 연구는 최근 몇년동안 서서히 발전하고 있다. 교사가 수학을 가르치기위해 필요한 지식이 무엇인가에 대한 다양한 해석과 함께, 교사의 교수를 위한 수학적 지식이 어떻게 학생 성취도와 관련되어 있는 지를 밝히려는 연구들도 역시 이루어 지고 있다. 이 논문은 교수를 위한 수학적 지식에 관한 다양한 해석들과 이 지식과 학생들의 성취도간의 관계를 조사한 연구들을 고찰하고, 이 주제를 연구하는데 어려움들이 무엇인지 밝힌다. 또한, 이 연구는 관련 문헌 연구에 기초하여, 교사의 교수를 위한 수학적 지식과 학생들의 성취도에 관한 가능한 연구 주제 및 연구 방법을 제안한다.

*key words : 수학 교수를 위한 지식(Knowledge of Teaching Mathematics), 학생 성취도 (Students' Achievement)

논문접수 : 2011. 11. 23
논문수정 : 2012. 2. 8
심사완료 : 2012. 2. 20