한국수학사학회지 *The Korean Journal for History of Mathematics*
제25권 제2호 (2012년 5월), 1–9

Chosun Mathematics in the early 18th century

18世紀 初 朝鮮 算學

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After disastrous foreign invasions in 1592 and 1636, Chosun lost most of the traditional mathematical works and needed to revive its mathematics. The new calendar system, ShiXianLi(時憲曆, 1645), was brought into Chosun in the same year. In order to understand the system, Chosun imported books related to western mathematics. For the traditional mathematics, Kim Si Jin(金始振, 1618–1667) republished SuanXue QiMeng(算學啓蒙, 1299) in 1660. We discuss the works by two great mathematicians of early 18th century, Cho Tae Gu(趙泰耉, 1660–1723) and Hong Jung Ha(洪正夏, 1684–?) and then conclude that Cho's JuSeoGwanGyun(籌書管見) and Hong's GuIlJib(九一集) became a real breakthrough for the second half of the history of Chosun mathematics.

1592년과 1636년 양대 전란으로 전통적인 조선 산학의 결과는 거의 소멸되어, 17세기 중엽 조선 산학은 새로 시작할 수밖에 없었다. 조선은 같은 시기에 청으로 부터 도입된 시헌력(時憲曆, 1645)을 이해하기 위하여 서양수학에 관련된 자료를 수입하기 시작하였다. 한편 전통 산학을 위하여 김시진(金始振, 1618–1667)은 산학계몽(算學啓蒙, 1299)을 중간(重刊)하였다. 이들의 영향으로 이루어진 조태구(趙泰耉, 1660–1723)의 주서관견(籌書管見)과 홍정하(洪正夏, 1684–?)의 구일집(九一集)을 함께 조사하여 이들이 조선 산학의 발전에 새로운 전기를 마련한 것을 보인다.

Keywords: Revival of Chosun Mathematics in the 18th century, Hong Jung Ha(洪正夏), GuIlJib(九一集), Cho Tae Gu(趙泰耉), JuSeoGwanGyun(籌書管見).

0 Introduction

Chosun suffered devastating foreign invasions from the late 16th to early 17th century, and consequently lost most of its mathematics books and mathematical works. The first attempt to revive its mathematics was the republication of SuanXue QiMeng(算學啓蒙, [3]) in 1660. And the new calendar system, ShiXianLi(時憲曆) was

MSC: 01A13, 01A20, 01A50, 12-03, 12D10, 12E12
Receiveded on 3월 20일, revised on 4월 25일, accepted on 5월 7일.

brought into Chosun in the middle of the 17th century. ShiXianLi was, as is well known, initiated by western mathematics and astronomy introduced by Jesuit missionaries. The revival of traditional mathematics and the influx of western mathematics at the same time had a great influence on Chosun mathematics in the early 18th century which may be called modern mathematics in Chosun.

The development of this modern mathematics owes much to two great Chosun mathematicians, Hong Jung Ha(洪正夏, 1684–?) and Cho Tae Gu(趙泰耉, 1660–1723). Cho was from Yang Ban(兩班), the noble class of Chosun and served as the highest official, YoungEiJung(領議政). He wrote a book entitled JuSeoGwanGyun(籌書管見, 1718, [1]) which is strongly tied with western mathematics. Hong, on the other hand, was a commoner from a Jung In(中人) family of an illegitimate descent of a noble family, who worked in the government ministry HoJo(戶曹). He wrote GuIlJib(九一集, 1713–1724, [2]) which is the culmination of traditional mathematics in Chosun.

The purpose of this paper is to show that Hong and Cho successfully revive Chosun mathematics in such a short period after a total catastrophe of Chosun mathematics. This is a first report on this subject, in particular Cho Tae Gu's work. Cho's work will be detailed in the ensuing papers. Part of this paper was presented in The 2nd International Conference on the History of Modern Mathematics, Xi'an, China, May 17–20, 2012.

The paper divides into two parts, namely Hong and Cho's mathematical works respectively. Although GuIlJib was published later than JuSeoGwanGyun, its main part was completed before Cho's book and hence we discuss Hong's work in the first section and then Cho's work in the second section.

1 Hong Jung Ha's Mathematics

Hong Jung Ha's excellent mathematics book, GuIlJib(九一集) is composed of 9 books. The first eight books were written before 1713 when he met He Guo Zhu(何國柱) to discuss mathematics, and they contain 20 chapters with 493 problems. Book 9 was added in 1724 as an appendix. We discuss a few important contributions that GuIlJib made to Chosun mathematics. Since the mid-17th century when ShiXianLi(時憲曆) was first introduced into Chosun, pretty many mathematics books dealing with western mathematics were brought into Chosun. But they were studied mainly by the officials in the state observatory (觀象監) and Hong Jung Ha didn't have any information on those books. And hence his approach to mathematics was restricted to the traditional one and his main references were SuanXue QiMeng(算學啓蒙, 1299), YangHui SuanFa(楊輝算法, 1274–1275), XiangMing SuanFa(詳明算法, 1373) and SuanFa TongZong(算法統宗, 1592)([3]).

Almost half of Guiljib is devoted to theory of equations. The chapter GuGoHoEunMun(句股互隱門) in Book 5 deals with equations related to right triangles (78 problems) and Book 6, 7 and 8 also involve the theory of equations, GaeBangGakSulMun(開方各術門)(166 problems), which show that Hong's main interest in GuIlJib is theory of equations.

He uses TianYuanShu(天元術) extensively in constructing equations. To this end, he needs the Jia Xian(賈憲) 's triangle, also known as Pascal's triangle. In his introductory remark(凡例), he includes the tables for $(x+1)^n (n \leq 10)$ and $(x-1)^n (n \leq 12)$. Furthermore, he expands $a(x+b)^n$ by the synthetic divisions in ZengChengKaiFangFa(增乘開方法).

Indeed, one has coefficients $a_0, a_1, a_2, \ldots, a_n$ of

$$ay^n = a_n(y-b)^n + a_{n-1}(y-b)^{n-1} + \cdots + a_1(y-b) + a_0$$

and then by $x = y - b$, or $y = x + b$, one can have coefficients of $a(x+b)^n$.

We note that the above method for $(x+1)^n$ was first introduced by Yang Hui and later transmitted to Wu Jing(吳敬) 's JiuZhang SuanFa BiLei DaQuan(九章算法比類大全, 1450, [3]). Furthermore, if Hong gets the expansion of $(x-1)^n (n \leq 12)$ by the above synthetic divisions, then he may be the first mathematician to use the synthetic division to divide a polynomial $p(x)$ by $x + b$ for a positive b.

TianYuanShu was introduced only in SuanXue QiMeng among the books mentioned above, and it was used in the last two chapters, namely FangCheng ZhengFuMen(方程正負門) and KaiFang ShiSuoMen(開方釋鎖門). Hong used TianYuanShu for problems related with finite series and hexahedrons in the chapters BuByungTweTaMun(缶瓶堆垜門) and ChangDonJukSokMun(倉囤積粟門) of his Guiljib Book 4. In the chapter GuGoHoEunMun(句股互隱門) it is most likely that Hong used

TianYuanShu in the sense of Li Ye(李冶)' s CeYuan HaiJing(測圓海鏡, 1248, [3]), i.e., the method to represent $\sum_{k=-m}^{n} a_k x^k (0 \leq m, n)$ and hence he could construct equations much easier than those with the TianYuanShu which represents only polynomials and hence is the same with JieGenFang(借根方) as Mei Jue Cheng (梅瑴成, 1681–1763) claimed([4]).

Hong notices that in the GouGuShu(句股術), there is a symmetry between Gou and Gu, and then makes problems in a coherent order according to conditions given by three sides with four basic operations except those with sum of three sides. But his friend Yu Su Suk(劉壽錫) deals with these problems together with the same kind of problems dealt by Hong in his YuSi GuGoSulYo(劉氏句股術要). Thus Hong and Yu obtained one of the most advanced theory of right triangles in the early 18th century([6]).

Since Yang Hui's XiangJie JiuZhang SuanFa(詳解九章算法, 1261, [3]) was never brought into Chosun, mathematicians didn't know the name of ZengChengKaiFangFa in Chosun. Chosun mathematician in the 18th century studied methods of solving equations only through SuanXue QiMeng and YangHui SuanFa. The former includes how to extract the nth roots $\sqrt[n]{a}$ and the latter contains detailed comments on the method to find roots of quadratic equations given by Liu Yi(劉益) in his YiGu GenYuan(議古根源) and one quartic equation which is solved simply by a single synthetic division. Just with this limited informations Hong figured out the structure of ZengChengKaiFangFa for equations of any degrees and found the exactly same method as the modern one([8]).

Although greatest common divisors were introduced in the first chapter FangTian(方田) of JiuZhang SuanShu(九章算術, [3]), least common multiples were rather neglected in eastern mathematics. However, Hong characterizes least common multiples as follows in the first chapter GuiChunChaBunMun(貴賤差分門) in Book 2 of GuIlJib([7]).

For natural numbers $a_1, a_2, \ldots, a_n$, their greatest common divisor d and least common multiple l, he first notices that $\frac{a_i}{d}(1 \leq i \leq n)$ are relatively prime and then shows that l is given by $l = a_i c_i (1 \leq i \leq n)$ for some relatively prime $c_i (1 \leq i \leq n)$. He also obtains a method to find those relatively prime c_i by greatest common divisor and HoSeung(互乘).

In Book 9, he detailed his meeting with A Qi Tu(阿齊圖) and He Guo Zhu who visited Chosun as Chinese emperor KangXi(康熙)'s envoys. We note that A Qi Tu worked as KaoCe(考測) and He Guo Zhu as JiaoSuan(校算) for LiXiang KaoCheng (曆象考成, 1723). Since A and He were both eminent scholars in mathematics and astronomy, they must have been interested in mathematics in Chosun and asked to meet some local mathematicians. Hong Jung Ha had already finished 8 books of GuIlJib and was picked as a representative albeit his low rank. In fact, Hong was serving as the lowest official Hoe Sa(會士, 從9品) in the Ministry Ho Jo(戶曹) in 1713. A and He were very much impressed by his mathematics, in particular TianYuanShu and counting rod calculations. Hong was also impressed by mathematics influenced by western mathematics, in particular trigonometry which could not be explained fully without the table. We will illustrate further development of their visit in the next section. One can easily find that Hong Jung Ha did not simply follow the old methods of other mathematicians but tried to reveal mathematical structures in the traditional problems and then established his own mathematics.

2 Cho Tae Gu's Mathematics

Cho Tae Gu passed the national examination for the civil services(文科 科擧) in 1686 and visited Yanjing(燕京) in 1709 as the chief envoy of DongJiSa(冬至使 正使) and he could collect mathematics books. From 1711, he served as the ministers of the 6 Ministries, IJo(吏曹), HoJo, YeJo(禮曹), ByungJo(兵曹), HyungJo(刑曹), GongJo(工曹). In particular he served as the minister of HoJo altogether 5 times where mathematical officials worked as Hong Jung Ha did. In 1713, he was the minister of HoJo and so Cho Tae Gu might have recommended Hong Jung Ha to meet with He Guo Zhu. The following paragraph of the Annals of the Chosun Dynasty(朝鮮王朝實錄, [9]) on July 30, King SukJong(肅宗) 39th year(1713) indicates that He Guo Zhu also met another official Heo Won(許遠) who served at the national observatory and learned about mathematics and astronomical devices from He. Cho got the king's permission that in the future mission to China, Heo should be a member of envoys to contact with He. In 1715, Heo managed to meet He during his visit to Yanjing and brought back books RiShi BuYi(日食補遺), JiaoShi ZhengBu(交食證補), LiCao PianZhi(曆草騈枝) and astronomical devices(朝鮮王朝實錄, April 18, 1715, [9]). The above records show that Cho Tae Gu had a strong interest in mathematics and astronomy.

In 1718 he completed a mathematics book JuSeoGwanGyun(籌書管見). It consists of three parts : Introductory Remark, GuJang(九章) and GuJang MunDab(九章問答). Cho studied the books mentioned in the first section together with TongWen SuanZhi(同文算指, 1613, [3]), Choe Suk Jung's GuSuRyak(九數略) and some other books influenced by western mathematics, probably JiHe YuanBen(幾何原本, [3]). The first part, Introductory Remark includes basic terminologies, methods of multiplications and divisions, theory of fractions in TongWen SuanZhi, proportions(異乘同除 = 四率法, 同乘異除) and areas of circles and regular polygons. For this part, he used $\sqrt{3} = 1.75$, or $\frac{7}{4}$, and term BiLi(比例) and HuShi(互視) which may be an abbreviation of HuXiangShiZhiXing(互相視之形) that appeared in JiHe YuanBen.

The second part, GuJang begins with the quote of chapters' names(九章名義) in JiuZhang SuanShu. JiuZhang SuanShu was not brought into Chosun until the middle of the 19th century. As in XiangMing SuanFa, he used SuBu(粟布) instead of SuMi(粟米). He just used chapters' names and then distributed problems according to JiuZhang MingYi(九章名義) and his problems are completely disjoint from the original problems in JiuZhang SuanShu.

We take the chapter ShaoGuang(少廣), where the author deals with the extraction of the nth roots ($n \leq 5$) by ZhengChengKaiFangFa and general equations. For the extraction of the nth roots, he also used the interpolation and noticed that for $\sqrt{a} = \alpha\frac{a-\alpha^2}{1+2\alpha}(= \alpha\frac{n}{m})$, $\sqrt{a+(m-n)} = \alpha+1$. Furthermore, the interpolation is precisely obtained by the coefficients given in the final stage of ZhengChengKaiFangFa. For the general quadratic equations, he deals with problems of two sides of rectangles of two sides a, b with conditions ab and $a-b$ and then shows that problems with ab and $a+b$ can be reduced to the above problems. Since Cho didn't use the TianYuanShu, his construction of equations is rather complicated.

The chapter ShangGong(商功) includes the theory of finite series and chapter GuoGu(句股) the problems in HaiDao SuanJing(海島算經, [3]), where he uses XiangShi(相似) for the similarity and the method to find the height of a triangle with three sides.

The most important feature of JuSeoGwanGyun is that the book discloses explicitly mathematical structures and contains the process to reach them by proofs. This feature appears in the last part GuJang MunDab(九章問答), literally translated into questions and answers of JiuZhang. He repeatedly uses the terms methods(法) and principles(理).

Traditional mathematics books used the same units for the length and area but by the relation between chi(尺) and cun(寸), he explains chi and chi squared, where he also introduces line(線) and face(面) as the western mathematicians put them(西士所謂).

He also introduces the earliest concept of limits in Chosun as he explaines the method of circle divisions (割圓之法). As division process tends to infinite, the chord, i.e., the side of polygon approaches the arc(弦亦極小極短 而弦之直者與弧之彎者同線矣).

Cho treats approximation or possibly earlier stage of irrational numbers in GuGyungJiRon(究竟之論) by $\sqrt{2}$ and $\sqrt{3}$. Starting from 1.4(方5斜7), he states $\frac{99}{70}$ and $\frac{239}{169}$ for approximations of $\sqrt{2}$, where the latter is introduced in the problem of the area of a regular octagon. We note that they are terms of the sequence $(\frac{y_n}{x_n})$, where $x_{n+1} = x_n + y_n, y_{n+1} = 2x_n + y_n$ and $\lim \frac{y_n}{x_n} = \sqrt{2}$. For $\sqrt{3}$, starting from (正6面7, i.e., $\sqrt{3} \approx \frac{12}{7}$), he states $\frac{168}{97}$ which is the term of the sequence $(\frac{y_n}{x_n})$, where $x_{n+1} = x_n + y_n, y_{n+1} = 3x_n + y_n$ and $\lim \frac{y_n}{x_n} = \sqrt{3}$, or possibly $x_{n+1} = 2x_n + y_n, y_{n+1} = 3x_n + 2y_n$. He also uses $\sqrt{3} \approx \frac{97}{56}$ where the sequence begins with $\frac{1}{1}$.

He includes the ruler and compass constructions to get the inscribed square and circle in a right triangle. He includes proofs for the problem in HaiDao SuanJing and the formula of the height in a triangle. In the former, he uses the similarity of triangles and compares it with Yang Hui's proof which is also a consequence of the similarity. In the latter, he proves it by an algebraic approach.

In the postscript, he mentions TianYuanShu and trigonometry(平三角) and spherical trigonometry(弧三角). He considers TianYuanShu as an extension of ShaoGuang (少廣之演也) and trigonometry as a deeper theory of right triangles(三角句股之奧也).

In many occasions, he states that he can not figure out the processes to reach the answer or to reveal structures.

3 Conclusion

Although Cho Tae Gu and Hong Jung Ha produced most remarkable works contemporarily, they failed to integrate their works together probably because of the great difference of their perspectives on mathematics and social statuses. We recall that for the construction of the inscribed square in a right triangle, Cho Tae Gu said "It is very difficult to explain the process in words but easy to clarify it by a diagram(此難用言喻 可以圖明之)" and whenever he proved any statement, he used diagrams. Such process gives mathematicians the universal quantifier as the ancient greek mathematics did. On the other hand, the traditional eastern mathematicians used the process implicitly and they showed intrinsic mathematical structure mostly by specific cases as Hong Jung Ha disclosed the structure of least common multiples just by three problems. Hong and Cho's works show that both kinds of mathematics coexisted in the early 18th-century Chosun and renewed Chosun mathematics successfully.

However, Cho had an ill fate that his official positions were completely deprived after his death and hence his work was forgotten even in the Yang Ban's society. GuIlJib was read only by his descendants who formed a great family of Jung In mathematicians totalled more than 100. Jung In, Lee Sang Hyuk(李尙爀, 1810–?) and Yang Ban, Nam Byung Gil(1820–1869) later appreciated and studied GuIlJib which effected their mathematical development([5]). Unfortunately, in all, their great works were not more widely studied and further developed by the mathematicians of the following generation and eventually failed to help materialize greater leap of Chosun mathematics.

References

1. 趙泰耉,《籌書管見》, 1718, 韓國科學技術史資料大系, 數學編, 2卷, 驪江出版社, 1985.
2. 洪正夏,《九一集》, 1713–1724, 韓國科學技術史資料大系, 數學編, 2卷, 驪江出版社, 1985.
3. 《中國科學技術典籍通彙》數學卷 全五卷, 河南教育出版社, 1993.
4. 梅瑴成, 赤水遺珍, 1761.
5. Hong Sung Sa, Hong Young Hee, Chosun Mathematics book SuanXue QiMeng Commentary(朝鮮 算書 算學啓蒙註解), The Korean Journal for History of Mathematics(한국수학사학회지) 22(2009), No. 2, pp. 1–12.
6. Hong Sung Sa, Hong Young Hee, Kim Chang Il, GouGuShu in the 18th century Chosun(18世紀 朝鮮의 句股術), The Korean Journal for History of Mathematics(한국

수학사학회지) 20(2007), No. 4, pp. 1–22.

7. Hong Sung Sa, Hong Young Hee, Kim Chang Il, Hong Jung Ha's Number Theory(洪正夏의 數論), The Korean Journal for History of Mathematics(한국수학사학회지) 24(2011), No. 4, pp. 1–6.
8. Hong Sung Sa, Hong Young Hee, Kim Young Wook, Liu Yi and Hong Jung Ha's KaiFang-Shu(劉益과 洪正夏의 開方術), The Korean Journal for History of Mathematics(한국수학사학회지) 24(2011), No. 1, pp. 1–13.
9. The Annals of the Chosun Dynasty(朝鮮王朝實錄), `http://sillok.history.go.kr`

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