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Development of Covariational Reasoning in a LOGO-Based JavaMAL Microworld[1]

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This paper explores gifted students' reasoning abilities. Three tests were developed in order to assess and analyze their reasoning abilities building on previous research on covariational reasoning. Giving consideration to the arising problems in the tests, we constructed a LOGO-based JavaMAL microworld environment which engages students in an active learning environment. This environment was designed by applying 'instrumental approach' in microworld. Based upon the post test results, the role of activity in microworld environment as 'instrument mediated activity' is also discussed.

Keywords: JavaMAL microworld, covariational reasoning, instrument mediated activity, artifact, sign
MESC Classification: U73
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INTRODUCTION

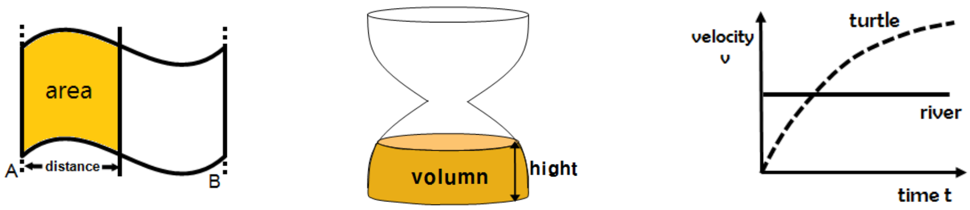
Since the concept of function is introduced in a manner of correspondence in school mathematics and student are also imposed to solve the problem through algebraic manipulation, little attention has been paid to the property of variant function. This approach hindered students from enhancing their abilities to develop the function conception in their learning, so students had difficulties viewing function as covariantional relationship between independent variables and dependent variables (Berry, 2003). Also Monk (1992) classified the view on the concept of function into 'pointwise view' and 'across-time view', and he found the students especially had difficulties to do 'across-time question' which requires an understanding about covariational relation.

However, convariational reasoning is an essential ability to interpret dynamic situations (Kaput, 1994; Rasmussen, 2000), and this perspective also has found to be fundamental for understanding concepts of calculus (Thompson, 1994). In this regard, Carlson (2002) proposed a framework for the study of mental actions of convariational reasoning in dynamic situations. He stressed a great emphasis on the ability to analyze students' understanding about dynamic situations involving two simultaneously changing variables. There have been researches which help students view function as a covariational relationship between the two variables and lessen students' difficulties in understanding a concept of variables. Most of the researches are based on computer utilization and, at times, design a new environment to accomplish an educational objective (Stroup 2002; Berry 2003; Swidan 2009). In his research on teaching the concept of function, Falcade (2007) supposed learning function in a dynamic geometry environment may take an important role in understanding a concept of variables. Particularly, he applied the functions included in Cabri such as 'Dragging tool' and 'Trace tool' to his teaching experiment according to Vygotskian perspective of semiotic mediation.

On the basis of previous research studies on covariational reasoning and the recent studies on computer technology, we propose tools which contribute to the improvement of covariational reasoning abilities. First of all, we designed some tests to evaluate students' covariational reasoning abilities, and identified their understandings and difficulties using these tests. We want mental action which students feel difficult to be developed by providing them a LOGO-based JavaMAL microworld environment which was developed the command "move Δx, Δy" used in Cho (2004). After that we conducted a post-test to see the effect of this new environment on students. The designed environment was created on the basis of instrumental approach that has recently been used in the study related to microworld. The given instrument as a action command language is so easy that even young learners can easily manipulate and change it. Before engaging students in a learning environment where they acquire the knowledge about function and variables as concepts, this command facilitates students understanding about the function as covariational relationships between two variables. In addition, it helps students understand the rate of change by visualizing the objects with simple manipulation in a new environment instead of algebraic and abstract approach.

THE COMPLEXITY OF THE COVARIATIONAL REASONING

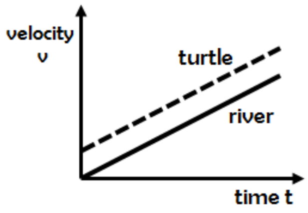
The test items were developed based on the previous research, and they are real life experiences regarding covariational reasoning. There are three parts in the test: Parts 1 and 2 are about covariational reasoning based on the previous research; Part 3 asks students to track the turtle's movement in the river by giving the conditions about turtle and river's velocity graph and their directions. In fact, the turtle moves from south to north and the river flows from west to east. Most previous studies on covariational reasoning were possible to fix one variable, infer the amount of change of the other variable. However, we attempt to find out how students comprehend the rate of change when given variables which cannot fix both of them.



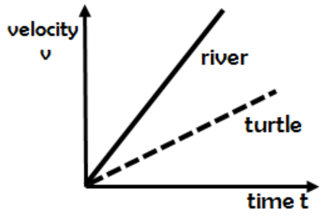
a. Part 1 b. Part 2 c. Part 3

Figure 1. The test items of three parts

A pre-test regarding covariational reasoning was conducted on general middle/high school students, middle school gifted students and college students majoring in mathematics. Test results revealed that the average score of college students and gifted middle school students was 9 points in Parts 1 and 2. On the other hand, general middle/high school students scored average of 5.5 points and they seemed to have a hard time to infer their thoughts. Interestingly, all students showed significant poor performance on the certain items in Part 3. We analyzed the reasons of their poor performance using students' descriptions and interview data. Below are the two representative "tricky" items, which troubled students.



a. Item 8 of Part 3



b. Item 9 of Part 3

Figure 2. Velocity vs. time

As well as interview data, Students' mental actions which derived from pre-test were analysed by employing Covariation framework by Carlson (2002). In particular, we focused on stages 3 and 4, and the framework can be found below (Table 1).

Table 1. Mental Actions of the Covariation Framework

Mental action	**Description of mental action**	**Behaviors**
Mental Action 3 (MA3)	Coordinating the amount of change of one variable with changes in the other variable	• Plotting points/constructing secant lines • Verbalizing an awareness of the amount of change of the output while considering changes in the input
Mental Action 4 (MA4)	Coordinating the average rate-of-change of the function with uniform increments of change input variable	• Constructing contiguous secant lines for the domain • Verbalizing an awareness of the rate of change of the output (with respect to the input) while considering uniform increments of the input

As student interview reflected, most students who made errors in Items 8 and 9 tried to find answers only considering the velocity distinction of both turtle and river. By doing so, they could only figure out the straight movement of the turtle since their velocity distinction change regularly in Item 8; because of the increasing distinction between the two velocities, they inferred movement line would be curve shape in Item 9. This kind of students' responses can be characterized as the 3rd stage of mental action. However, the above items in Figure 2 obviously cannot be solved with this level of mental actions. Therefore, the need of comprehending the rate of change which is above the 4th stage of mental action has been called for. On the basis of the studies on instrumental approach, this study intends to design an instrument that helps students internalize the mental action 4 and apply this tool to the students.

THEORETICAL BACKGROUND

Unlike other psychological approaches that clearly separate the technological and concrete artifacts from signs, Vygotskian perspective claims an analogy between them.

> The invention and use of signs as auxiliary means of solving a given psychological problem (to remember, compare something, report, choose, and so on) is analogous to the invention and use of tools in one psychological respect. The sign acts as an instrument of psychological activity in a manner analogous to the role of a tool in labour (Vygotsky, 1978, p. 52).

Studies on psychological tools were initiated by Vygotsky (*cf.* Drijvers & Trouche, 2008). Vygotsky (1978) viewed instrument as a newly created intermediary element which was between objects and psychic operation, and the instrument not only includes concrete objects but also immaterial objects such as languages and symbols. He classified instruments into technical tools and psychological tools, and psychological tools can function as a semiotic mediator for the internalization process which can transform externally oriented operations into internally oriented operations.

The model based on the seminal idea of semiotic mediation introduced by Vygotsky (1978) aims to describe and explain the process that starts with the students' use of an artifact to accomplish a task and leads to the students' appropriation of a particular mathematical content. The learning process, centered on the use of an artifact, is often expressed in terms of mediation (Radford, 2003), referring to the potentiality that a specific artifact has with respect to fostering the education process.

The construction of knowledge relative to the use of the artifact is thus explicitly connected to helping students become conscious of the personal meanings and linking them to mathematical shared meanings. Therefore, any artifact may offer a valuable semiotic potential with respect to particular educational goals (Bartolini Bussi & Mariotti, 2008).

However, the mediating function of instrument is not a property we can realize intrinsically from the instrument as Hegedus & Moreno-Armella (2008) expresses it:

> The tool (axe) reveals the idea of softness when we integrate it into a scheme of intentional action. [...] Then, in the hands of an expert agent, the axe is more than a device to cut.

Rabardel (2005) had similar perspective and said that the instrument cannot be reduced to the artifact, the technical object or the machine, depending on the terminology employed. In his perspective, the instrument is a composite entity made up of an artifact component and a scheme component. Scheme component of instrument means specific use of artifact and it is called ‘utilization scheme’. Artifact and utilization scheme can be linked various way, the same utilization scheme can be applied to a range of artifacts and conversely, an artifact is liable to be integrated into a range of utilization schemes. The reorganization and restructuring of instrument-linked is called an ‘instrumental genesis’. The first direction of instrumental genesis is ‘instrumentation’ which involves the construction of schemes of instrumented actions to be integrated into the individual's awn cognitive structure. Second direction is ‘instrumentalisation’ which refers to how the functionalities and affordances of the artifact in question are adjusted and transformed for specific uses (Healy, 2010). Rabardel (2005) named this approach ‘instrument-mediated activity approach’. Instrumental approach can be very important in educational research, in particular the pedagogy of use of CAS environment, because it provide a frame to analyze the cognitive processes related to the use of a specific computer based artifact (Bartolini Bussi & Mariotti, 2008).

Another approach that uses the specific mediating function of instrument, semiotic mediation, is currently being erected on the terrain of technology and mathematics education.(Mariotti, 2001; Falcade, 2007; Bartolini Bussi & Mariotti, 2008). In this approach, the artifact is based on the computer program and students are placed in specific task. The artifact and its use are related to the task and mathematical meaning that student generate through solving the task. Since this double relationship may be expressed by signs, it will be named ‘the semiotic potential of an artifact’ and in this sense, the artifact is called ‘an instrument of semiotic mediation’ (Bartolini Bussi & Mariotti, 2008). Because of this relationship, the artifact may function as a mediator of this process, but a mediation function is not automatically activated, as we said above, and so teacher has the awareness of the semiotic potential of an artifact and foster and guide the process of production and evolution of signs centered on the use of an artifact.

MEHOD

Fourteen gifted students who are second graders of middle school have participated in this study. Their results of pre-test and interview revealed MA3 but no one showed MA4 except for one student. For this reason, we think they are suitable for our study. A pre-test as well as individual interview has been conducted and their problem solving strategies and thinking patterns have been analyzed and then we designed lessons based upon our analysis. Three lessons were conducted in a designed microworld environment and the students were divided into four groups. In addition to microworld environment, textbooks including writing tasks which developed to reflect their learning process were also provided for students.

The post-test was conducted after they took lesson. Several analysis have been processed in the post-test such as checking answers of multiple choice items and short answer items, comparative analysis of pre-test and post-test and individual interview data. Student interviews were conducted before the lessons, with the aim of examining the effect of lessons on their behavior changes and whether they used or not during the post-test. The author interviewed each student individually and these interview data were recorded and then transcribed.

Instrument mediated activity: Move Δx, Δy

Based on the above theoretical background, a LOGO-based microworld environment as instrument was developed in order to improve covariational reasoning abilities in this study. The amount of change within graph expression can be represented by the command “move Δx, Δy” in this environment

Command: move Δx, Δy:
Moves turtle horizontally Δx steps and vertically Δy steps

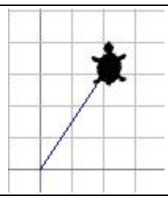


Figure 3. Move command and move 2, 3

Students used ‘move Δx, Δy’ in a microworld environment, so microworld and ‘move Δx, Δy’, which were originally artifacts, can be transformed into instruments of mediated activity to develop covariational reasoning ability. For this reason, we set utilization schemes toward “move Δx, Δy” as below.

- Scheme 1: while Δx is fixed, change Δy to move the turtle
- Scheme 2: while Δy is fixed, change Δx to move the turtle
- Scheme 3: while sequence of Δx or Δy which change within a certain range, change another variable to move the turtle

Students are expected to find mathematical knowledge and meanings as bellowing two notions with the above utilization schemes:

- Students conclude that graphs can be constructed using scheme 1 and scheme 2, while fixing the amount of change of one variable with the quantitative change of another variable. Furthermore, they can be associated with the concavity of the graph and the quantitative change of another variable. But when they explore scheme 3, they could understand that, unlike the other two schemes, concept of ratio of variation of two variables is needed to interpret the relation of two variables and construct the graph. In other word, they can see the relationship between the ratio and graph when they solve the problems which pertain to scheme 3.

- While observing by connecting sequence of ‘move $\triangle$x, $\triangle$y’ and the turtle’s moving track in scheme 1, scheme 2 and scheme 3, student can find the turtle’s movements are contiguous secant lines which are close to smooth curves, using move command which is a symbol of action. From these observation results, they can internalize the facts that discretized movements gathering together to become close to continuous movement and conversely, smooth curves can be explained as contiguous secant line made by discretized movements. Finally, we expect that they realize graph can be constructed through the change of amount of one variation or the change of the average rate of change in each intervals formed by dividing the domain of independent variable or parameter.

As can be seen, for example, students represented the graph of $y = 0.25x^2$ using scheme 1 (Figure 4). While $\triangle$x is fixed at 4, construct contiguous secant lines which is close smooth curves by only changing $\triangle$y. Also, they found $y = 0.25x^2$ using the fact

$$1 + 3 + 5 + \cdots + 2n - 1 = n^2.$$

Moreover, they made a connection to the fact that $\triangle$y is increasing to concavity of the graph and viewed the curve as contiguous secant lines which made by sequence of “move $\triangle$x, $\triangle$y”.

Command : move 4, 1
move 4, 3
move 4, 5

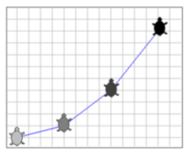


Figure 4. Representing the graph of $y = 0.25x^2$ in term of move $\triangle$x, $\triangle$y

Students’ activity in a JavaMAL microworld environment

Students are supposed to learn utilization scheme in a JavaMAL environment. A parabola, connecting a start point and aiming point, is drawn on the coordinate of unit length 10 in the environment which students encounter. Activities are presented for scheme 1, scheme 2 and scheme 3 in this problem situation. Activity 1 and activity 3 in Figure 3 are provided for scheme 1 and scheme 3 respectively.

Figure 5. JavaMAL microworld environment

Students look at the situation in the picture, enter appropriate values of △y (or △x) into △x (or △y), and check the answers by clicking an execution button. A turtle in the microworld makes contiguous secant lines according to the values entered by students and students can receive feedback immediately. After all activities were done in the microworld, we had a writing task and a group session on the sequence of 'move △x, △y' and the shape of graph students entered.

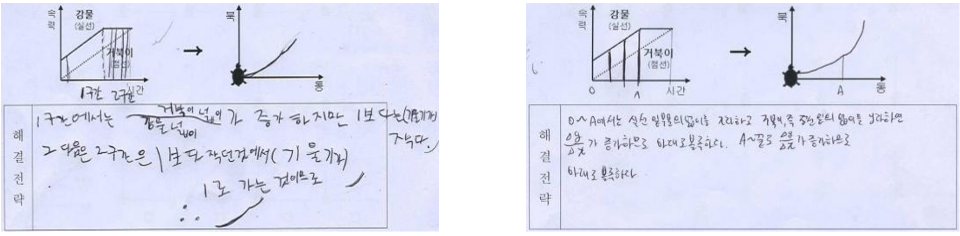
EMPIRICAL DATA ANALYSIS

Fourteen gifted students who participated in the pre-test scored high in Part 1 and Part 2. Also most of them showed relatively good performance to infer their answer with respect to amount of change regarding Parts 1 and 2 in the following pre-interviews. So post-test was only conducted toward Part 3. As a result, the average score of 10 items in Part 3 has been increased from 6.6 points to 8.7 points and they showed a remarkable improvement in two items which they did poorly in the pre-test.

Table 2. Comparison of pre-, post test scores of Items 8 and 9 of Part 3

Item Number	Pre-test Score	Post-test Score
8	3.8	6.2
9	3.1	10.0

Follow-up individual interviews also conducted after post-test in order to examine how students' practice functions as cognitive tool and how their views changed. Interestingly, only one student out of fourteen who inferred correctly the turtle's movement in Part 3 from the perspective of the rate of change in the pre-interview, while most students could view the problems in Part 3 from the perspective of the rate of change by manipulating ‘move △x, △y’ in the post-interview.



Oh’s response Yoon’s response

Figure 6. Two students’ responses on Part 3 items in post-test

As can be seen from Oh’s response, he divided the time at regular intervals. He depicted the ratio of the flow distance of river to turtle’s movement had increased and this ratio rate was smaller than 1 in the first section, but it became closer to 1 in the second section. As checking students’ answers in the short answer questions, we were able to confirm that students’ views changed through learning in an experiment environment after the pre-test.

Int: Can you explain about your answer?
Oh (directing what he wrote): As I wrote here, I compared the ratio of turtle graph area and river graph area by dividing intervals.
Int: Did you also approach the change of function by dividing the entire section into smaller intervals in the pre-test as you did here?
Oh: Yes, I also observed the change by dividing the whole into small intervals in the pre-test, but I simply considered the velocity comparison only.
Int: Velocity comparison? How did you compare it?
Oh: I just draw a graph by comparing the velocity distinction of turtle and river.
Int: Were you able to complete all tasks accurately in that way?
Oh: No. Some worked out, but many of them did not.
Int: Then, wasn’t it difficult to find out that the ratio of two variables’ amount of change defines the shape of graph?
Oh: No.

Yoon assumed the turtle’s movement and the flow distance of river as △x &△y respectively. He also explained the graph representation turned to be curves in the first section since the ratio of these two variations had been increased, and the similar tendency was described with the same ratio changes in the second section. In addition, they revealed it became easier to see the most complicated items – Items 8 and 9 from the view of the rate of the change.

CONCLUSIONS AND DISCUSSION

The analysis of tests and interview data indicates most students have developed their views via the activity which involves the command 'move Δx, Δy' in a microworld environment. Their views shifted from adjusting amounts to the rate of change regarding the two dynamic variables. Some students answered they could easily find answers but they could neither draw a correct graphs nor infer the graphs what they drew. More importantly, this study provides some tasks that call for a higher level thinking since they involve two variables which one of them cannot be fixed in a dynamic context. In addition, we provide opportunities for students to experience cognitive struggle when they could not figure out the changing patterns by only adjusting the rate of amount in a microworld environment. In this respect, students realized they need to change their views and began to apply the command-'move Δx, Δy' which functions as cognitive tool to a new environment. Consequently, the tasks introduced in Part 3 are expected to stimulate students develop their mental actions from MA3 to MA4.

Furthermore, the command-'move Δx, Δy' used in this study are symbol actions and it facilitates students uses. Another implication of this study lies on the possible development of 'move Δx, Δy', which can functions as an instrument of approaching and understanding other concepts. Indeed, this is associated with 'instrumentalisation' of instrumental approach. For instance, when students learn about vector, they could understand this conception using the command 'move Δx, Δy'. They could see the result by moving an object to Δx on the x axis, and Δy on the y axis. This can be associated with the vector learning activity using embodied cognition in Poynter's (2004) study of "Action and Effect". Moreover, the command 'move Δx, Δy' can be an instrument of semiotic mediator in this kind of vector learning activities.

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