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SOLVING THE GENERALIZED FISHER'S EQUATION BY DIFFERENTIAL TRANSFORM METHOD

M. MATINFAR*, S. R. BAHAR, M. GHASEMI

ABSTRACT. In this paper, differential transform method (DTM) is considered to obtain solution to the generalized Fisher's equation. This method is easy to apply and because of high level of accuracy can be used to solve other linear and nonlinear problems. Furthermore, is capable of reducing the size of computational work. In the present work, the generalization of the two- dimensional transform method that is based on generalized Taylor's formula is applied to solve the generalized Fisher equation and numerical example demonstrates the accuracy of the present method.

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1. Introduction

Mathematical modeling of many physical systems leads to linear and nonlinear differential equations in various fields of physics and engineering. The numerical and analytical approximations of such systems have been intensively studied. Recently, several mathematical methods including the Adomian decomposition method [1, 2, 3], variational iteration method [4, 5, 6] and homotopy perturbation method [7, 8, 9] have been developed to obtain exact and approximate analytic solutions. Some of these methods use transformation in order to reduce equations into simpler equations or systems of equations and some other methods give the solution in a series form which converges to the exact solution. In this work, we develop a semi-numerical method based on the two-dimensional differential transform method [10, 11, 12] called (DTM) to solve the Generalized Fisher's equation. The concept of the differential transform was first proposed by Zhou [13], and its main applications therein are to solve both linear and nonlinear initial value problems in electric circuit analysis. This method constructs

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a semi-analytical numerical technique that uses Taylor series for the solution of differential equations in the form of a polynomial. It is different from the high-order Taylor series method which requires symbolic computation of the necessary derivatives of the data functions. The Taylor series method is computationally time-consuming especially for high order equation. The differential transform is an iterative procedure for obtaining analytic Taylor series solutions of differential equations. It can be said that Differential transform method is a universal one, and is able to solve various kinds of functional equations. For example, it was applied to two point boundary value problems [14], to differential-algebraic equations [15], to the KdV and mKdV equations [16], to the Schrodinger equations [17] and to fractional differential equations [18].
Generalized Fisher's equation is the following form

$$u_y = u_{xx} + u(1 - u^{\alpha}), \tag{1}$$

where $u_y = \frac{\partial u}{\partial y},\ u_{xx} = \frac{\partial^2 u}{\partial x^2}$. Fisher proposed equation of $u_y = u_{xx} + u(1-u)$ as a model for the propagation of a mutant gene, with u denoting the density of an advantageous. This equation is encountered in chemical kinetics [19] and population dynamics which includes problems such as nonlinear evolution of a population in a nuclear reaction and branching. Moreover, the same equation occurs in logistic population growth models [20], flame propagation, neurophysiology, autocatalytic chemical reaction, and branching Brownian motion processes. A lot of works have been done in order to find the numerical solution of this equation. For example, variational iteration method and modified variational iteration method for solving the generalized Fisher equation [21], an analytical study of Fisher equation by using Adomian decomposition method [22], numerical solution for solving Burger-Fisher equation [23]. In this paper, we develop (DTM) to solve the generalized Fisher equation and numerical result shows that the applied method is reliable and has an excellent agreement with the exact solution .

2. Generalized two-dimensional differential transform method

In this section we shall derive the generalized two-dimensional differential transform method that we have developed for the numerical solution of partial differential equations. Consider a function of two variables $u(x,y)$, and suppose that it can be represented as a product of two single variable functions, i.e. $u(x,y) = f(x)g(x)$. On the basis of the properties of generalized two-dimensional differential transform [24], the function $u(x,y)$ can be represented as

$$\begin{aligned} u(x,y) &= \sum_{k=0}^{\infty} F(k)x^k \sum_{h=0}^{\infty} G(h)y^h \\ &= \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} U(k,h)x^k y^h, \end{aligned} \tag{2}$$

where $U(k,h) = F(k)G(h)$ is called the spectrum of $u(x,y)$.
The generalized two-dimensional differential transform of the function $u(x,y)$ is as follows:

$$U(k,h) = \frac{1}{k!h!}\left[\frac{d^k d^h u(x,y)}{dx^k dy^h}\right]_{(x_0,y_0)} \tag{3}$$

On the basis of the definitions (2) and (3) we have the following results:

Theorem 1. *Suppose that $U(k,h), G_1(k,h), G_2(k,h), G_3(k,h)$, and $G_4(k,h)$ are the differential transformations of the functions $u(x,y), g_1(x,y), g_2(x,y), g_3(x,y)$ and $g_4(x,y)$, respectively;*

(a) If $u(x,y) = g_1(x,y) \pm g_2(x,y)$, then $U(k,h) = G_1(k,h) \pm G_2(k,h)$.

(b) If $u(x,y) = ag(x,y); a \in \mathbb{R}$, then $U(k,h) = aG(k,h)$.

(c) If $u(x,y) = g_1(x,y)g_2(x,y)g_3(x,y)g_4(x,y)$, then

$$\begin{aligned} U(k,h) = \sum_{k_3=0}^{k}\sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_2}\sum_{h_3=0}^{h}\sum_{h_2=0}^{h_3}\sum_{h_1=0}^{h_2} & G_1 \quad (k_1, h-h_3)G_2(k_2-k_1, h_3-h_2) \\ \bigotimes & G_3 \quad (k_3-k_2, h_2-h_1)G_4(k-k_3, h_1) \end{aligned}$$

(d) If $u(x,y) = x^n y^m$, then $U(k,h) = \delta(k-n)\delta(h-m)$, δ is the Kronecker delta.

(e)If $u(x,y) = \frac{d^n u(x,y)}{dx^n}$, then $U(k,h) = \frac{(k+n)!}{k!}U(k+n,h)$.

3. Applications and results

Consider the generalized Fisher's equation as follows:

$$u_y = u_{xx} + u(1-u^\alpha), \tag{4}$$

subject to the initial condition

$$u(x,0) = \phi^2(x;\alpha),$$

where

$$\phi(x;\alpha) = \frac{1}{(1+e^{\frac{\alpha}{\sqrt{2\alpha+4}}x})^{\frac{1}{\alpha}}}.$$

This equation with $\alpha = 3$ was solved by the Modified Variational Iteration Method [5]. Our purpose in this section is solving the generalized Fisher's equation (4) with $\alpha = 3$ by (DTM):

$$u_y = u_{xx} + u(1-u^3), \tag{5}$$

subject to the initial condition

$$u(x,0)=\phi^2(x;3),$$

where

$$\phi(x;3)=\frac{1}{(1+e^{\frac{3}{\sqrt{10}}x})^{\frac{1}{3}}}.$$

The exact solution of (5) is given by Wang [25] as:

$$u(x,y)=\left\{\frac{1}{2}\tanh\left[-\frac{\alpha}{2\sqrt{2\alpha+4}}\left(x-\frac{\alpha+4}{\sqrt{2\alpha+4}}y\right)\right]+\frac{1}{2}\right\}^{\frac{2}{3}}.$$

Applying the generalized two-dimensional differential transform to both sides of (5) the generalized Fisher's equation (5) transforms to

$$\begin{aligned} U(k,h+1) &= \frac{1}{h+1}[(k+2)(k+1)U(k+2,h)+U(k,h)\\ &- \sum_{k_3=0}^{k}\sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_2}\sum_{h_3=0}^{h}\sum_{h_2=0}^{h_3}\sum_{h_1=0}^{h_2}U(k_1,h-h_3)U(k_2-k_1,h_3-h_2)\\ &\times U(k_3-k_2,h_2-h_1)U(k-k_3,h_1). \end{aligned} \tag{6}$$

The generalized two-dimensional differential transform of the initial condition

$$\phi(x;3)=\frac{1}{(1+e^{\frac{3}{\sqrt{10}}x})^{\frac{2}{3}}}$$

is:

$$\begin{aligned} U(k,0) &= 0.63\,\delta(k)-0.1992\,\delta(k-1)-0.0158\,\delta(k-2)\\ &+ 0.0116\,\delta(k-3)+0.0014\,\delta(k-4)\\ &- 0.00014\,\delta(k-5)-0.00012\,\delta(k-6)\\ &+ 0.000007\,\delta(k-7)+0.00001\,\delta(k-8). \end{aligned} \tag{7}$$

Utilizing the recurrence relation (6) and the transformed initial condition (7), gives the first few components of $U(k,h)$ and the results are given in Table 1.

Table 1: The first few components of $U(k,h)$ for the generalized Fisher's equation with $\alpha=3$

	$U(0,h)$	$U(1,h)$	$U(2,k)$	$U(3,k)$
U(k,0)	0.63	-0.1992	-0.0158	0.0116
U(k,1)	0.4409	0.0696	-0.0777	-0.0178
U(k,2)	-0.0777	0.1557	0.0862	-0.0659
U(k,3)	-0.0968	-0.0590	$*$	$*$

Therefore from (2) and using MATLAB software, the approximate solution of the generalized Fisher's equation (5) can be derived as

$$
\begin{aligned}
u(x,y) &= 0.63 - 0.1992x - 0.0158x^2 + 0.4409y + 0.0696xy \\
&- 0.0777y^2 + 0.1557xy^2 + 0.0862x^2y^2 - 0.0968y^3 \\
&- 0.0590xy^3 - 0.0178x^3y - 0.0659x^3y^2 + \cdots
\end{aligned}
$$

As we can see, the obtained results are in good agreement with the exact solution.

4. Conclusion

In this paper, the differential transform method has been successfully applied to finding the approximate solution of the generalized Fisher's equation. The small size of computation in comparison with the computational size required in other numerical methods, and the rapid convergence show that the method is reliable and introduces a significant improvement in solving differential equations over existing method. So, it can be employed to a wide variety of physical problems.

References

1. A.K. Khalifa, K.R. Raslan, H.M. Alzubaidi, *Numerical study using the ADM for the modified regularized long wave equation* , Appl. Math. Modelling , vol. 32, no. 12, pp. 2962-2972, 2008.
2. S. Abbasbandy, M.T. Darvishi, *A numerical solution of Burgers' equation by modified Adomian method*, Appl. Math. Comput., vol. 163, pp. 1265-1272, 2005.
3. S. Momani, Z. Odibat, *Analytical solution of a time-fractional NavierStokes equation by Adomian decomposition method*, Appl. Math. Comput., vol. 177, pp. 488494, 2006.
4. Z. Odibat, S. Momani, *Application of variational iteration method to nonlinear differential equations of fractional order*, Int. J. Non. Sci. Numer. Simul., vol. 7, no. 1, pp. 1527, 2006.
5. Hakan K. Akmaz, *Variational Iteration Method for elastodynamic Green's functions*, Non. Analysis. Teory and Applications, vol. 71, no. 12, pp. 218-223, 2009.
6. D. Altintan, O. Ugur, *Variational Iteration Method for Sturm-Lioville differential equations*, Compu. and Math. with Application, vol. 58, no. 2, pp. 322-328, 2009.
7. J.Biazar, H.Ghazvini, *Numerical solution for special nonlinear Fredholm integral equation by HPM*, Appl. Math. and Compu., vol. 195, no. 2, pp. 681-687, 2008.
8. B. Ganjavi, H. Mohammadi, D. D. Ganji, A. Barari, *Homotopy pertubration method and variational iteration method for solving Zakharov-Kuznetsov equation*, Am. J. Appl .Sci., vol. 5, no. 7, pp. 811-817, 2008.
9. M. Gorji, D.D. Ganji and S. Soleimani, *Homotopy Perturbation Method for solving boundary value problems*, Phy. Lett., A, vol. 350, no. 1-2, pp. 87-88, 2006.
10. N. Bildik, A. Konuralp, F. Bek, S. Kucukarslan, *Solution of different type of the partial differential equation by differential transform method and Adomians decomposition method*, Appl. Math. Comput., vol. 172, pp. 551567, 2006.
11. Z. Odibat, S. Momani, *A generalized differential transform method for linear partial differential equations of fractional order*, App. Math. Lett., vol. 21, pp. 194-199, 2008.
12. J. Biazar, M. Eslami, *Differential Transform Method for Quadratic Riccati Differential Equation*, Int. J. Non. Sci., vol. 9, no. 4, pp. 444-447, 2010.
13. J. K. Zhou, *Differential Transformation and its Applications for Electrical Circuits*, Huzhong Univ. Press, Wuhan, China, 1986.

14. C. L. Chen, Y. C. Liu, *Solution of two point boundary value problems using the differential transformation method*, J. Opt. Theory Appl., vol. 99, pp. 23-35, 1998.
15. Fatma Ayaz, *Applications of differential transform method to to differential-algebraic equations*, App. Math. and Compu., vol. 152, pp. 649-657, 2004.
16. Figen Kangalgil and Fatma Ayaz, *Solitary wave solutions for the KdV and mKdV equations by differential transform method*, Chaos Solitons and Fractals, vol.41, no. 1, pp. 464-472, 2009.
17. S. V. Ravi Kanth, K. Aruna *Two-dimensional differential transform method for solving linear and non-linear Schroinger equations*, Chaos Solitons and Fractals, vol. 41, no. 5, pp. 2277-2281, 2009.
18. A. Arikoglu, I. Ozkol, *Solution of fractional differential equations by using differential transform method*, Chaos Solitons and Fractals, vol. 34, pp. 1473-1481, 2007.
19. W. Malfliet, *Solitary wave solutions of nonlinear wave equations*, Am. J. Phys., vol. 7, pp. 650-654, 1992.
20. P. Brazhnik, J. Tyson, *On traveling wave solutions of Fisher's equationin two spatial dimensions*, SIAM, J. Appl. Math., vol. 60, no. 2, pp. 371-391, 1999.
21. M. Matinfar, M. Ghanbari, *The application of the modified variational iteration method on the generalized Fisher's equation*, Springer, J. Appl. Math. Comput., vol. 31, pp. 165-175, 2009.
22. A.M. Wazwaz, A. Gorguis, *An analytic study of Fisher's equation by using Adomian decomposition method*, App. Math. and Compu., vol. 154, no. 3, pp. 609620, 2004.
23. A. Golbabai, M. Javidi, *A spectral domain decomposition approach for the generalized Burger's-Fisher equation*, Chaos Solitons and Fractals, vol. 39, no. 1, pp. 385392, 2009.
24. N. Bildik, A. Konuralp, F. Bek, ,S. Kucukarslan, *Solution of differentent type of the partial differential equation by differential transform method and Adomian's decomposition method*, Appl. Math. Comput., vol. 7, pp. 551-567, 2006.
25. X.Y. Wang, *Exact and explicit solitary wave solutions for the generalized Fisher's equation*, Phys. Lett. A, vol. 131, no.(4/5), pp. 277-279, 1988.

Mashallah Matinfar received his BS from Tarbiat Moallem University of Tehran(TMU), M.Sc. from Isfahan University and Ph.D In applied mathematics from Lomonosov Moscow State University in Moscow. I am University Teaching and my research interests focus on the Numerical Solution of Differential Equations, Integral Equations and Linear Algebra. Also I have done research on the Numerical Solution of the fuzzy Linear system algebra. I have at least 80 paper in journals and conferences. My lectures for MSc are Advanced Numerical Analysis, Numerical Methods in Linear Algebra, Numerical Solution of Differential Equations (ODE), Numerical Solution of Partial Differential Equations (PDE), and Numerical Solution of Integral Equations. My position is the dean of the faculty of engineering (University of Mazandaran) and was member of some of the scientific committee seminars.

Dep. of Mathematics, University of Mazandaran, P.O.Box: 47416-95447, Babolsar, Iran.
e-mail: m.matinfar@umz.ac.ir