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A NEW METHOD FOR SOLVING FUZZY SHORTEST PATH PROBLEMS

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Abstract. To the best of our knowledge, there is no method, in the literature, to find the fuzzy optimal solution of fully fuzzy shortest path (FFSP) problems i.e., shortest path (SP) problems in which all the parameters are represented by fuzzy numbers. In this paper, a new method is proposed to find the fuzzy optimal solution of FFSP problems. Kumar and Kaur [Methods for solving unbalanced fuzzy transportation problems, Operational Research-An International Journal, 2010 (DOI 10.1007/s 12351-010-0101-3)] proposed a new method with new representation, named as JMD representation, of trapezoidal fuzzy numbers for solving fully fuzzy transportation problems and shown that it is better to solve fully fuzzy transportation problems by using proposed method with JMD representation as compare to proposed method with the existing representation. On the same direction in this paper a new method is proposed to find the solution of FFSP problems and it is shown that it is also better to solve FFSP problems with JMD representation as compare to existing representation. To show the advantages of proposed method with this representation over proposed method with other existing representations. A FFSP problem solved by using proposed method with JMD representation as well as proposed method with other existing representations and the obtained results are compared.

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1. Introduction

The SP problem concentrates on finding the path with minimum distance. To find the SP from a source node to the other nodes is a fundamental matter in graph theory. In conventional SP problems, it is assumed that decision maker is certain about the parameters (distance, time etc.) between different nodes.

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But in real life situations, there always exist uncertainty about the parameters between different nodes. In such cases, the parameters are represented by fuzzy numbers [28].

Klein [10] presented new models based on fuzzy shortest paths (FSP) and also given a general algorithm based on dynamic programming to solve the new models. Lin and Chern [14] considered the case that the arc lengths are fuzzy numbers and proposed an algorithm for finding the single most vital arc in a network. Okada and Gen [22] discussed the problem of finding the SP from a fixed origin to a specified node in a network with arcs represented as intervals on real line. Li et al. [13] introduced the neural networks for solving FSP problems. Gent et al. [5] investigated the possibility of using genetic algorithms to solve SP problems. Shih and Lee [25] investigated multiple objective and multiple hierarchies minimum cost flow problems with fuzzy costs and fuzzy capacities in the arcs. Okada and Soper [23] concentrated on a SP problem on a network in which a fuzzy number, instead of a real number, is assigned to each arc length. Okada [21] concentrated on a SP problem on a network in which a fuzzy number, instead of a real number, is assigned to each arc length and introduced the concept of "degree of possibility" in which an arc is on the SP. Liu and Kao [16] investigated the network flow problems in that the arc lengths of the network are fuzzy numbers. Seda [24] dealed with the steiner tree problem on a graph in which a fuzzy number, instead of a real number, is assigned to each edge.

Takahashi [26] discussed the SP problem with fuzzy parameters. He proposed a modification Okada's algorithm [21], using some properties observed by other authors. He also proposed a genetic algorithm to seek an approximated solution for large scale problems. Chuang and Kung [2] represented each arc length as a triangular fuzzy set and a new algorithm is proposed to deal with the FSP problems. Nayeem and Pal [20] considered a network with its arc lengths as imprecise number, instead of a real number, namely, interval number and triangular fuzzy number. Ma and Chen [17] proposed an algorithm for the on-line FSP problems, based on the traditional SP problem in the domain of the operations research and the theory of the on-line algorithms. Kung and Chuang [12] proposed a new algorithm composed of FSP length procedure and similarity measure to deal with the FSP problem. Gupta and Pal [6] presented an algorithm for the SP problem when the connected arcs in a transportation network are represented as interval numbers.

Moazeni [19] discussed the SP problem from a specified node to every other node on a network in which a positive fuzzy quantity with finite support is assigned to each arc as its arc length. Chuang and Kung [3] pointed out that there are several methods reported to solve this kind of problem in the open literature. In these methods, they can obtain either the fuzzy shortest length or the SP. In their paper, a new algorithm was proposed that can obtain both of them. The discrete fuzzy shortest length method is proposed to find the fuzzy shortest length, and the fuzzy similarity measure is utilized to get the SP. Ji et al. [8] considered the SP problem with fuzzy arc lengths. According to different decision criteria, the concepts of expected SP, a-SP and the SP in fuzzy environment are originally proposed, and three types of models are formulated. In order to solve these models, a hybrid intelligent algorithm integrating simulation and genetic algorithm is provided and some numerous examples are given to illustrate its effectiveness.

Hernandes et al. [7] proposed an iterative algorithm that assumes a generic ranking index for comparing the fuzzy numbers involved in the problem, in such a way that each time in which the decision-maker wants to solve a concrete problem (s)he can choose (or propose) the ranking index that best suits that problem. Yu and Wei [27] proposed a simple linear multiple objective programming to deal with the FSP problem. The proposed algorithm does not need to declare 0-1 variables to solve the FSP problem because it meets the requirements of the network linear programming constraints. Mahdavi et al. [18] proposed a dynamic programming approach to solve the fuzzy shortest chain problem using a suitable ranking method.

In this paper, a new method is proposed to find the solution of FFSP problems and it is shown that it is better to solve FFSP problems with JMD representation as compare to existing representation. To show the advantages of proposed method with this representation over proposed method with other existing representations. A FFSP problem solved by using proposed method with JMD representation as well as proposed method with other existing representations.

This paper is organized as follows: In Section 2, some basic definitions, existing representations of trapezoidal fuzzy numbers, arithmetic operations and an existing method for comparing fuzzy numbers are presented. In Section 3, linear programming formulation of crisp shortest path (CSP) problems are reviewed and also the linear programming formulation of FFSP problems are proposed. In Section 4, a new method is proposed to find the fuzzy optimal solution of FFSP problems. In Section 5, a new representation of trapezoidal fuzzy number is proposed. Advantages of new representation of trapezoidal fuzzy numbers over existing representations of trapezoidal fuzzy numbers are discussed in Section 6. To illustrate the proposed method, numerical example is solved in Section 7. Conclusions are discussed in Section 8.

2. Preliminaries

In this section some basic definitions, existing representations of trapezoidal fuzzy numbers, arithmetic operations between trapezoidal fuzzy numbers and method for comparing fuzzy numbers are presented.

2.1. Basic definitions. In this subsection, some basic definitions are presented.

Definition 1 ([9])**.** The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\widetilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\widetilde{A}} : X \to [0,1]$. The assigned value indicate the membership grade of the element in the set A. The function $\mu_{\widetilde{A}}$ is called the membership function and the set $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)); x \in X\}$ defined by $\mu_{\widetilde{A}}(x)$ for each $x \in X$ is called a fuzzy set.

Definition 2 ([9]). A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:
(i) $\tilde{A}$ is convex i.e., $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq$ minimum $(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))\ \forall\ x_1, x_2 \in R,\ \forall\ \lambda \in [0,1]$.
(ii) $\tilde{A}$ is normal i.e., $\exists\ x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$.
(iii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 3 ([4]). A fuzzy number $\tilde{A}$ is called non-negative fuzzy number iff $\mu_{\tilde{A}}(x) = 0,\ \forall\ x < 0$.

2.2. Existing representations of trapezoidal fuzzy numbers. In the literature, trapezoidal fuzzy number are represented as follows:

2.2.1. (a, b, c, d) **representation of trapezoidal fuzzy numbers.**

Definition 4 ([9]). A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ 1\ , & b \leq x \leq c \\ \frac{(x-d)}{(c-d)}, & c \leq x \leq d \end{cases}$$

where, $a, b, c, d \in R$

Definition 5 ([9]). A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be zero trapezoidal fuzzy number iff $a = 0, b = 0, c = 0, d = 0$.

Definition 6 ([9]). A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be non-negative trapezoidal fuzzy number iff $a \geq 0$.

Definition 7 ([9]). Two trapezoidal fuzzy numbers $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ are said to be equal i.e., $\tilde{A}_1 = \tilde{A}_2$ iff $a_1 = a_2, b_1 = b_2, c_1 = c_2, d_1 = d_2$.

2.2.2. (m, n, α, β) **representation of trapezoidal fuzzy numbers.**

Definition 8 ([9]). A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \frac{m-x}{\alpha}, & m-\alpha \le x \le m \\ 1\ , & m \le x \le n \\ 1 - \frac{x-n}{\beta}, & n \le x \le n+\beta \end{cases}$$

where, $n - m \ge 0, \alpha \ge 0, \beta \ge 0$

Definition 9 ([4])**.** A trapezoidal fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be zero trapezoidal fuzzy number iff $m = 0, n = 0, \alpha = 0, \beta = 0$.

Definition 10 ([4])**.** A trapezoidal fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be non-negative trapezoidal fuzzy number iff $m - \alpha \ge 0$.

Definition 11 ([4])**.** Two trapezoidal fuzzy numbers $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)$ are said to be equal i.e., $\tilde{A}_1 = \tilde{A}_2$ iff $m_1 = m_2, n_1 = n_2, \alpha_1 = \alpha_2, \beta_1 = \beta_2$.

2.3. Arithmetic operations. In this subsection addition and multiplication operations between two trapezoidal fuzzy numbers are reviewed.

2.3.1. Arithmetic operations between (a, b, c, d) type trapezoidal fuzzy numbers ([9])**.** Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$

(ii) $\tilde{A}_1 \otimes \tilde{A}_2 \simeq (a, b, c, d)$, where $a =$ minimum $(a_1a_2, a_1d_2, a_2d_1, d_1d_2)$, $b =$ minimum $(b_1b_2, b_1c_2, c_1b_2, c_1c_2)$, $c =$ maximum $(b_1b_2, b_1c_2, c_1b_2, c_1c_2)$, $d =$ maximum $(a_1a_2, a_1d_2, a_2d_1, d_1d_2)$

2.3.2. Arithmetic operations between (m, n, α, β) type trapezoidal fuzzy numbers ([4])**.** Let $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)$ be two trapezoidal fuzzy numbers then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)$

(ii) $\tilde{A}_1 \otimes \tilde{A}_2 = (m', n', \alpha', \beta')$,
where $m' =$ minimum $(m_1m_2, m_1n_2, n_1m_2, n_1n_2)$,
$n' =$ maximum $(m_1m_2, m_1n_2, n_1m_2, n_1n_2)$,
$\alpha' = m' -$ minimum $((m_1 - \alpha_1)(m_2 - \alpha_2), (m_1 - \alpha_1)(n_2 + \beta_2), (n_1 + \beta_1)(m_2 - \alpha_2), (n_1 + \beta_1)(n_2 + \beta_2))$,
$\beta' =$ maximum $((m_1 - \alpha_1)(m_2 - \alpha_2), (m_1 - \alpha_1)(n_2 + \beta_2), (n_1 + \beta_1)(m_2 - \alpha_2), (n_1 + \beta_1)(n_2 + \beta_2)) - n'$

2.4. Comparison of fuzzy numbers. An efficient approach for comparing the fuzzy numbers is by the use of a ranking function [15] $\Re : F(R) \to R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists.
Let (a, b, c, d) be a trapezoidal fuzzy number then $\Re(a, b, c, d) = \frac{a+b+c+d}{4}$.
Let (m, n, α, β) be a trapezoidal fuzzy number then $\Re(m, n, \alpha, \beta) = \frac{m+n}{2} + \frac{\beta-\alpha}{4}$.

3. Linear programming (LP) formulation of CSP and FFSP problems

The SP problem concentrates on finding the path with minimum distance. To find the SP from a source node to the other nodes is a fundamental matter in graph theory. In this section the LP formulation of CSP problems is presented and also the LP formulation of FFSP problems is proposed.

3.1. LP formulation of CSP problems ([1])**.** Let us consider a directed and connected network $G = (N, A)$ consisting of a finite set $N = \{1, 2, ..., n\}$ of n nodes and A is the set of arcs $(i,\ j)$. c_{ij} is the cost per unit flow through arc $(i,\ j)$ and x_{ij} is the decision variable denoting the flow through arc $(i,\ j)$. The LP formulation of CSP problems is as follows:

$$\text{Minimize} \sum_{(i,j)\in A} c_{ij}x_{ij}$$

subject to

$$\sum_{j:(1,j)\in A} x_{ij} = \sum_{k:(k,1)\in A} x_{ki} + 1$$

$$\sum_{j:(i,j)\in A} x_{ij} = \sum_{k:(k,i)\in A} x_{ki},\ i \neq 1, n$$

$$\sum_{j:(n,j)\in A} x_{ij} + 1 = \sum_{k:(k,n)\in A} x_{ki}$$

x_{ij} is a non-negative real number $\forall (i, j) \in A$

3.2. Proposed LP formulation of FFSP problems. In conventional SP problems, it is assumed that decision maker is certain about the parameters (distance, time etc.) between different nodes. But in real life situations, there always exist uncertainty about the parameters between different nodes. In such cases, the parameters are represented by fuzzy numbers. When the values associated with the arcs are fuzzy numbers, then we have a FSP problem. Suppose the parameters c_{ij} and x_{ij}, $(i, j) \in A$ are imprecise and are represented by fuzzy numbers $\tilde{c}_{ij}$ and $\tilde{x}_{ij}$, $(i, j) \in A$ respectively. Then the FFSP problems may be formulated into the following fuzzy linear programming (FLP) problem:

$$\text{Minimize} \sum_{(i,j)\in A} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$$

subject to

$$\sum_{j:(1,j)\in A} \tilde{x}_{ij} = \sum_{k:(k,1)\in A} \tilde{x}_{ki} \oplus 1$$

$$\sum_{j:(i,j)\in A} \tilde{x}_{ij} = \sum_{k:(k,i)\in A} \tilde{x}_{ki},\ i \neq 1, n$$

$$\sum_{j:(n,j)\in A} \tilde{x}_{ij} \oplus 1 = \sum_{k:(k,n)\in A} \tilde{x}_{ki}$$

$$\tilde{x}_{ij} \text{ is a non-negative fuzzy number } \forall (i,j) \in A$$

Remark 1. In this paper, at all places $\sum_i^n \tilde{x}_i$ and $\sum_i^n x_i$ represents the fuzzy and crisp addition respectively. i.e., $\sum_i^n \tilde{x}_i = \tilde{x}_1 \oplus \tilde{x}_2 \oplus \ldots \oplus \tilde{x}_n$ and $\sum_i^n x_i = x_1 + x_2 + \ldots + x_n$, where $\tilde{x}_i$ and x_i are fuzzy number and real number respectively.

4. Proposed method

In this section, a new method is proposed to find the fuzzy optimal solution of FFSP problems. The steps of the proposed method are as follows:

Step 1 Represent all the parameters of FFSP problem by a particular type of trapezoidal fuzzy number and formulate the given problem, as proposed in Section 3.2.

Step 2 Convert the fuzzy objective function into the crisp objective function form by using appropriate ranking formula.

Step 3 Convert all the fuzzy constraints and restrictions into the crisp constraints and restrictions by using the arithmetic operations.

Step 4 Find the optimal solution of obtained crisp linear programming (CLP) problem by using software (TORA, LINGO, LINDO etc.).

Step 5 Find the fuzzy optimal solution using the crisp optimal solution, obtained in Step 4.

Step 6 Find the FSP and the corresponding fuzzy shortest distance using the fuzzy optimal solution, obtained from Step 5.

4.1. Proposed method with (a, b, c, d) representation of trapezoidal fuzzy numbers. If all the parameters of FFSP problems are represented by (a, b, c, d) type trapezoidal fuzzy numbers then the steps of the proposed method are as follows:

Step 1 Suppose all the parameters $\tilde{c}_{ij}$ and $\tilde{x}_{ij}$ are represented by (a, b, c, d) type trapezoidal fuzzy numbers $(c_{ij}, c'_{ij}, c''_{ij}, c'''_{ij})$ and $(x_{ij}, y_{ij}, z_{ij}, w_{ij})$ respectively, then the LP formulation of FFSP problems, presented in Section 3.2, can be written as:

Minimize $\sum_{(i,j)\in A} (c_{ij}, c'_{ij}, c''_{ij}, c'''_{ij}) \otimes (x_{ij}, y_{ij}, z_{ij}, w_{ij})$

subject to

$$\sum_{j:(1,j)\in A} (x_{ij}, y_{ij}, z_{ij}, w_{ij}) = \sum_{k:(k,1)\in A} (x_{ki}, y_{ki}, z_{ki}, w_{ki}) \oplus (1, 1, 1, 1)$$

$$\sum_{j:(i,j)\in A}(x_{ij},y_{ij},z_{ij},w_{ij}) = \sum_{k:(k,i)\in A}(x_{ki},y_{ki},z_{ki},w_{ki}),\;\; i\neq 1,n$$

$$\sum_{j:(n,j)\in A}(x_{ij},y_{ij},z_{ij},w_{ij})\oplus(1,1,1,1) = \sum_{k:(k,n)\in A}(x_{ki},y_{ki},z_{ki},w_{ki})$$

$(x_{ij},y_{ij},z_{ij},w_{ij})$ is a non-negative trapezoidal fuzzy number $\forall(i,j)\in A$

Step 2 Using ranking formula, presented in Section 2.4, the LP formulation of FFSP problems can be written as:

Minimize $\Re[\sum_{(i,j)\in A}(c_{ij},c'_{ij},c''_{ij},c'''_{ij})\otimes(x_{ij},y_{ij},z_{ij},w_{ij})]$

subject to

$$\sum_{j:(1,j)\in A}(x_{ij},y_{ij},z_{ij},w_{ij}) = \sum_{k:(k,1)\in A}(x_{ki},y_{ki},z_{ki},w_{ki})\oplus(1,1,1,1)$$

$$\sum_{j:(i,j)\in A}(x_{ij},y_{ij},z_{ij},w_{ij}) = \sum_{k:(k,i)\in A}(x_{ki},y_{ki},z_{ki},w_{ki}),\;\; i\neq 1,n$$

$$\sum_{j:(n,j)\in A}(x_{ij},y_{ij},z_{ij},w_{ij})\oplus(1,1,1,1) = \sum_{k:(k,n)\in A}(x_{ki},y_{ki},z_{ki},w_{ki})$$

$(x_{ij},y_{ij},z_{ij},w_{ij})$ is a non-negative trapezoidal fuzzy number $\forall(i,j)\in A$

Step 3 Using the arithmetic operations, described in Section 2.3.1 and Definitions 6, 7, FLP problem, obtained in Step 2, is converted into the following CLP problem:

Minimize $\Re[\sum_{(i,j)\in A}(c_{ij},c'_{ij},c''_{ij},c'''_{ij})\otimes(x_{ij},y_{ij},z_{ij},w_{ij})]$

subject to

$$\sum_{j:(1,j)\in A}x_{ij} = \sum_{k:(k,1)\in A}x_{ki}+1,\quad \sum_{j:(1,j)\in A}y_{ij} = \sum_{k:(k,1)\in A}y_{ki}+1$$

$$\sum_{j:(1,j)\in A}z_{ij} = \sum_{k:(k,1)\in A}z_{ki}+1,\quad \sum_{j:(1,j)\in A}w_{ij} = \sum_{k:(k,1)\in A}w_{ki}+1$$

$$\sum_{j:(i,j)\in A}x_{ij} = \sum_{k:(k,i)\in A}x_{ki},\quad \sum_{j:(i,j)\in A}y_{ij} = \sum_{k:(k,i)\in A}y_{ki},\qquad i\neq 1,n$$

$$\sum_{j:(i,j)\in A}z_{ij} = \sum_{k:(k,i)\in A}z_{ki},\quad \sum_{j:(i,j)\in A}w_{ij} = \sum_{k:(k,i)\in A}w_{ki},\qquad i\neq 1,n$$

$$\sum_{j:(n,j)\in A}x_{ij}+1 = \sum_{k:(k,n)\in A}x_{ki},\quad \sum_{j:(n,j)\in A}y_{ij}+1 = \sum_{k:(k,n)\in A}y_{ki}$$

$$\sum_{j:(n,j)\in A} z_{ij} + 1 = \sum_{k:(k,n)\in A} z_{ki}, \quad \sum_{j:(n,j)\in A} w_{ij} + 1 = \sum_{k:(k,n)\in A} w_{ki}$$

$$y_{ij} - x_{ij} \geq 0, \;\; z_{ij} - y_{ij} \geq 0, \quad w_{ij} - z_{ij} \geq 0$$

$$x_{ij}, y_{ij}, z_{ij}, w_{ij} \geq 0 \;\forall (i,j) \in A$$

Step 4 Find the optimal solution $x_{ij}, y_{ij}, z_{ij}, w_{ij}$ by solving the CLP problem, obtained in Step 3.

Step 5 Find the fuzzy optimal solution $\tilde{x}_{ij}$ by putting the values of x_{ij}, y_{ij} , z_{ij} and w_{ij} in $\tilde{x}_{ij} = (x_{ij}, y_{ij}, z_{ij}, w_{ij})$.

Step 6 Find the fuzzy shortest distance by putting the values of $\tilde{x}_{ij}$ in $\sum_{(i,j)\in A} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$.

Step 7 Find the FSP by combining all the (i, j) arcs such that $\tilde{x}_{ij} = (1, 1, 1, 1)$

4.2. Proposed method with (m, n, α, β) representation of trapezoidal fuzzy numbers. If all the parameters of FFSP problems are represented by (m, n, α, β) type trapezoidal fuzzy numbers then the steps of the proposed method are as follows:

Step 1 Suppose all the parameters $\tilde{c}_{ij}$ and $\tilde{x}_{ij}$ are represented by (m, n, α, β) type trapezoidal fuzzy numbers $(c'_{ij}, c''_{ij}, \gamma_{ij}, \delta_{ij})$ and $(y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})$ respectively, then the LP formulation of FFSP problems, proposed in Section 3.2, can be written as:

$$\text{Minimize} \sum_{(i,j)\in A} (c'_{ij}, c''_{ij}, \gamma_{ij}, \delta_{ij}) \otimes (y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})$$

subject to

$$\sum_{j:(1,j)\in A} (y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij}) = \sum_{k:(k,1)\in A} (y_{ki}, z_{ki}, \alpha_{ki}, \beta_{ki}) \oplus (1, 1, 0, 0)$$

$$\sum_{j:(i,j)\in A} (y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij}) = \sum_{k:(k,i)\in A} (y_{ki}, z_{ki}, \alpha_{ki}, \beta_{ki}), \;\; i \neq 1, n$$

$$\sum_{j:(n,j)\in A} (y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij}) \oplus (1, 1, 0, 0) = \sum_{k:(k,n)\in A} (y_{ki}, z_{ki}, \alpha_{ki}, \beta_{ki})$$

$(y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})$ is a non-negative trapezoidal fuzzy number $\forall (i, j) \in A$

Step 2 Using ranking formula, presented in Section 2.4, the LP formulation of FFSP problems can be written as:

$$\text{Minimize } \Re[\sum_{(i,j)\in A} (c'_{ij}, c''_{ij}, \gamma_{ij}, \delta_{ij}) \otimes (y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})]$$

subject to

$$\sum_{j:(1,j)\in A}(y_{ij},z_{ij},\alpha_{ij},\beta_{ij}) = \sum_{k:(k,1)\in A}(y_{ki},z_{ki},\alpha_{ki},\beta_{ki}) \oplus (1,1,0,0)$$

$$\sum_{j:(i,j)\in A}(y_{ij},z_{ij},\alpha_{ij},\beta_{ij}) = \sum_{k:(k,i)\in A}(y_{ki},z_{ki},\alpha_{ki},\beta_{ki}), \quad i \neq 1,n$$

$$\sum_{j:(n,j)\in A}(y_{ij},z_{ij},\alpha_{ij},\beta_{ij}) \oplus (1,1,0,0) = \sum_{k:(k,n)\in A}(y_{ki},z_{ki},\alpha_{ki},\beta_{ki})$$

$(y_{ij},z_{ij},\alpha_{ij},\beta_{ij})$ is a non-negative trapezoidal fuzzy number $\forall (i,j) \in A$

Step 3 Using the arithmetic operations, described in Section 2.3.2 and Definitions 10, 11, FLP problem, obtained in Step 2, is converted into the following CLP problem:

Minimize $\Re[\sum_{(i,j)\in A}(c'_{ij},c''_{ij},\gamma_{ij},\delta_{ij}) \otimes (y_{ij},z_{ij},\alpha_{ij},\beta_{ij})]$

subject to

$$\sum_{j:(1,j)\in A} y_{ij} = \sum_{k:(k,1)\in A} y_{ki} + 1, \quad \sum_{j:(1,j)\in A} z_{ij} = \sum_{k:(k,1)\in A} z_{ki} + 1$$

$$\sum_{j:(1,j)\in A} \alpha_{ij} = \sum_{k:(k,1)\in A} \alpha_{ki} + 1, \quad \sum_{j:(1,j)\in A} \beta_{ij} = \sum_{k:(k,1)\in A} \beta_{ki} + 1$$

$$\sum_{j:(i,j)\in A} y_{ij} = \sum_{k:(k,i)\in A} y_{ki}, \quad \sum_{j:(i,j)\in A} z_{ij} = \sum_{k:(k,i)\in A} z_{ki}, \qquad i \neq 1,n$$

$$\sum_{j:(i,j)\in A} \alpha_{ij} = \sum_{k:(k,i)\in A} \alpha_{ki}, \quad \sum_{j:(i,j)\in A} \beta_{ij} = \sum_{k:(k,i)\in A} \beta_{ki}, \qquad i \neq 1,n$$

$$\sum_{j:(n,j)\in A} y_{ij} + 1 = \sum_{k:(k,n)\in A} y_{ki}, \quad \sum_{j:(n,j)\in A} z_{ij} + 1 = \sum_{k:(k,n)\in A} z_{ki}$$

$$\sum_{j:(n,j)\in A} \alpha_{ij} + 1 = \sum_{k:(k,n)\in A} \alpha_{ki}, \quad \sum_{j:(n,j)\in A} \beta_{ij} + 1 = \sum_{k:(k,n)\in A} \beta_{ki}$$

$$y_{ij} - \alpha_{ij} \geq 0, \quad z_{ij} - y_{ij} \geq 0$$

$$y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij} \geq 0 \; \forall (i,j) \in A$$

Step 4 Find the optimal solution $y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij}$ by solving the CLP problem, obtained in Step 3.

Step 5 Find the fuzzy optimal solution $\tilde{x}_{ij}$ by putting the values of y_{ij}, z_{ij} , α_{ij} and β_{ij} in $\tilde{x}_{ij} = (y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})$

Step 6 Find the fuzzy shortest distance by putting the values of $\tilde{x}_{ij}$ in $\sum_{(i,j)\in A}\tilde{c}_{ij}\otimes$

$\tilde{x}_{ij}$.

Step 7 Find the FSP by combining all the (i, j) arcs such that $\tilde{x}_{ij} = (1, 1, 0, 0)$

5. JMD representation of trapezoidal fuzzy numbers

Kumar and Kaur [11] proposed a new representation, named as JMD representation, of trapezoidal fuzzy numbers and shown that it is better to solve fuzzy transportation problems by representing the parameters as JMD representation of trapezoidal fuzzy numbers instead of existing representation of trapezoidal fuzzy numbers. On the same direction in this section, it is shown that if all the parameters are represented by JMD representation instead of existing representation of trapezoidal fuzzy numbers and proposed method is applied to find the fuzzy optimal solution of FFSP problems then the fuzzy optimal solution is same but the total number of constraints, in converted CLP problem, is less than the number of constraints, obtained by using the existing representation of trapezoidal fuzzy numbers.

Definition 12. Let (a, b, c, d) be a trapezoidal fuzzy number then its JMD representation is $(x, \alpha', \beta', \gamma')_{JMD}$, where $x = a$, $\alpha' = b - a \geq 0$, $\beta' = c - b \geq 0$ and $\gamma' = d - c \geq 0$

Definition 13. A trapezoidal fuzzy number $\tilde{A} = (x, \alpha', \beta', \gamma')_{JMD}$ is said to be zero trapezoidal fuzzy number iff $x = 0, \alpha' = 0, \beta' = 0, \gamma' = 0$

Definition 14. A trapezoidal fuzzy number $\tilde{A} = (x, \alpha', \beta', \gamma')_{JMD}$ is said to be non-negative trapezoidal fuzzy number iff $x \geq 0$

Definition 15. Two trapezoidal fuzzy numbers $\widetilde{A} = (x_1, \alpha'_1, \beta'_1, \gamma'_1)_{JMD}$ and $\widetilde{B} = (x_2, \alpha'_2, \beta'_2, \gamma'_2)_{JMD}$ are said to be equal i.e., $\widetilde{A} = \widetilde{B}$ iff $x_1 = x_2$, $\alpha'_1 = \alpha'_2$, $\beta'_1 = \beta'_2$, $\gamma'_1 = \gamma'_2$

5.1. Ranking function for JMD trapezoidal fuzzy numbers.

The ranking formula, presented in Section 2.4, is converted into the following ranking formula:
Let $(x, \alpha', \beta', \gamma')_{JMD}$ be a trapezoidal fuzzy number then

$$\Re(x, \alpha', \beta', \gamma') = x + \frac{3(\alpha')+2(\beta')+\gamma'}{4}$$

5.2. Proposed method with $(x, \alpha', \beta', \gamma')_{JMD}$ representation of trapezoidal fuzzy numbers.

If all the parameters of FFSP problems are represented by $(x, \alpha', \beta', \gamma')_{JMD}$ type trapezoidal fuzzy numbers, then the steps of the proposed method are as follows:

Step 1 Suppose all the parameters $\tilde{c}_{ij}$ and $\tilde{x}_{ij}$ are represented by trapezoidal fuzzy numbers $(c_{ij}, \delta_{ij}, \zeta_{ij}, \eta_{ij})_{JMD}$ and $(x_{ij}, \alpha'_{ij}, \beta'_{ij}, \gamma'_{ij})_{JMD}$ respectively, then the LP formulation of FFSP problems, proposed in Section 3.2, can be written as:

Minimize $\sum_{(i,j)\in A} (c_{ij},\delta_{ij},\zeta_{ij},\eta_{ij})_{JMD} \otimes (x_{ij},\alpha'_{ij},\beta'_{ij},\gamma'_{ij})_{JMD}$

subject to

$$\sum_{j:(1,j)\in A} (x_{ij},\alpha'_{ij},\beta'_{ij},\gamma'_{ij})_{JMD} = \sum_{k:(k,1)\in A} (x_{ki},\alpha'_{ki},\beta'_{ki},\gamma'_{ki})_{JMD} \oplus (1,0,0,0)$$

$$\sum_{j:(i,j)\in A} (x_{ij},\alpha'_{ij},\beta'_{ij},\gamma'_{ij})_{JMD} = \sum_{k:(k,i)\in A} (x_{ki},\alpha'_{ki},\beta'_{ki},\gamma'_{ki})_{JMD}, \qquad i \neq 1,n$$

$$\sum_{j:(n,j)\in A} (x_{ij},\alpha'_{ij},\beta'_{ij},\gamma'_{ij})_{JMD} \oplus (1,0,0,0) = \sum_{k:(k,n)\in A} (x_{ki},\alpha'_{ki},\beta'_{ki},\gamma'_{ki})_{JMD}$$

$(x_{ij},\alpha'_{ij},\beta'_{ij},\gamma'_{ij})_{JMD}$ is a non-negative trapezoidal fuzzy number $\forall\ (i,j) \in A$

Step 2 Using ranking formula, presented in Section 5.2, the LP formulation of FFCP problems can be written as:

Minimize $\Re[\sum_{(i,j)\in A} (c_{ij},\delta_{ij},\zeta_{ij},\eta_{ij})_{JMD} \otimes (x_{ij},\alpha'_{ij},\beta'_{ij},\gamma'_{ij})_{JMD}]$

subject to

$$\sum_{j:(1,j)\in A} (x_{ij},\alpha'_{ij},\beta'_{ij},\gamma'_{ij})_{JMD} = \sum_{k:(k,1)\in A} (x_{ki},\alpha'_{ki},\beta'_{ki},\gamma'_{ki})_{JMD} \oplus (1,0,0,0)$$

$$\sum_{j:(i,j)\in A} (x_{ij},\alpha'_{ij},\beta'_{ij},\gamma'_{ij})_{JMD} = \sum_{k:(k,i)\in A} (x_{ki},\alpha'_{ki},\beta'_{ki},\gamma'_{ki})_{JMD}, \qquad i \neq 1,n$$

$$\sum_{j:(n,j)\in A} (x_{ij},\alpha'_{ij},\beta'_{ij},\gamma'_{ij})_{JMD} \oplus (1,0,0,0) = \sum_{k:(k,n)\in A} (x_{ki},\alpha'_{ki},\beta'_{ki},\gamma'_{ki})_{JMD}$$

$(x_{ij},\alpha'_{ij},\beta'_{ij},\gamma'_{ij})_{JMD}$ is a non-negative trapezoidal fuzzy number $\forall\ (i,j) \in A$

Step 3 Using the arithmetic operations, described in Section 5.1, and Definitions 10, 11, the FLP problem, obtained in Step 2, is converted into the following CLP problem:

Minimize $\Re[\sum_{(i,j)\in A} (c_{ij},\delta_{ij},\zeta_{ij},\eta_{ij})_{JMD} \otimes (x_{ij},\alpha'_{ij},\beta'_{ij},\gamma'_{ij})_{JMD}]$

subject to

$$\sum_{j:(1,j)\in A} x_{ij} = \sum_{k:(k,1)\in A} x_{ki} + 1, \quad \sum_{j:(1,j)\in A} \alpha'_{ij} = \sum_{k:(k,1)\in A} \alpha'_{ki} + 1$$

$$\sum_{j:(1,j)\in A} \beta'_{ij} = \sum_{k:(k,1)\in A} \beta'_{ki} + 1, \quad \sum_{j:(1,j)\in A} \gamma'_{ij} = \sum_{k:(k,1)\in A} \gamma'_{ki} + 1$$

$$\sum_{j:(i,j)\in A} x_{ij} = \sum_{k:(k,i)\in A} x_{ki}, \quad \sum_{j:(i,j)\in A} \alpha'_{ij} = \sum_{k:(k,i)\in A} \alpha'_{ki}, \qquad i \neq 1,n$$

$$\sum_{j:(i,j)\in A} \beta'_{ij} = \sum_{k:(k,i)\in A} \beta'_{ki}, \quad \sum_{j:(i,j)\in A} \gamma'_{ij} = \sum_{k:(k,i)\in A} \gamma'_{ki}, \qquad i \neq 1, n$$

$$\sum_{j:(n,j)\in A} x_{ij} + 1 = \sum_{k:(k,n)\in A} x_{ki}, \quad \sum_{j:(n,j)\in A} \alpha'_{ij} + 1 = \sum_{k:(k,n)\in A} \alpha'_{ki}$$

$$\sum_{j:(n,j)\in A} \beta'_{ij} + 1 = \sum_{k:(k,n)\in A} \beta'_{ki}, \quad \sum_{j:(n,j)\in A} \gamma'_{ij} + 1 = \sum_{k:(k,n)\in A} \gamma'_{ki}$$

x_{ij}, α'_{ij}, β'_{ij} $\gamma'_{ij} \geq 0 \; \forall (i,j) \in A$

Step 4 Find the optimal solution x_{ij}, α'_{ij}, β'_{ij}, γ'_{ij} by solving the CLP problem, obtained in Step 3.

Step 5 Find the fuzzy optimal solution $\tilde{x}_{ij}$ by putting the values of x_{ij}, α'_{ij}, β'_{ij} and γ'_{ij} in $\tilde{x}_{ij} = (x_{ij}, \alpha'_{ij}, \beta'_{ij}, \gamma'_{ij})$

Step 6 Find the fuzzy shortest distance by putting the values of $\tilde{x}_{ij}$ in $\sum_{(i,j)\in A} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$.

Step 7 Find the FSP by combining all the (i, j) arcs such that $\tilde{x}_{ij}$ = (1, 0, 0, 0)

Remark 2. The JMD representation of trapezoidal fuzzy number may be called as "JAI MATA DI" or "JAI MEHAR DI" Mehar is a lovely daughter of Parmpreet Kaur (Research scholar under the supervision of Dr. Amit Kumar).

6. Advantages of JMD representation over the existing representation of trapezoidal fuzzy numbers

In this section, it is shown that it is better to use the JMD representation of trapezoidal fuzzy numbers instead of existing representations of trapezoidal fuzzy numbers, for finding the fuzzy optimal solution of FFSP problems.
It is obvious from Section 4.1, 4.2 and 5.2 that

- **(i)** If all the parameters of FFSP problems are represented by (a, b, c, d) type trapezoidal fuzzy numbers and FLP problem is converted into the corresponding CLP problem by using the proposed method, presented in Section 4.1, then number of constraints in CLP problem = 4 × number of constraints in FLP problem + 3 × number of fuzzy variables.
- **(ii)** If all the parameters of FFSP problems are represented by (m, n, α, β) type trapezoidal fuzzy numbers and FLP problem is converted into the corresponding CLP problem by using the proposed method, presented in Section 4.2, then number of constraints in CLP problem = 4 × number of constraints in FLP problem + 2 × number of fuzzy variables.
- **(iii)** If all the parameters of FFSP problems are represented by $(x, \alpha', \beta', \gamma')_{JMD}$ type trapezoidal fuzzy numbers and FLP problem is converted into the corresponding CLP problem by using the proposed method, presented in

Section 5.2, then number of constraints in CLP problem = 4 × number of constraints in FLP problem.

On the basis of above results it can be concluded that if all the parameters are represented by existing type trapezoidal fuzzy numbers then total number of constraints, in the obtained CLP problem, will be more than as compared to number of constraints in CLP probelm, obtained by representing all the parameters as JMD type trapezoidal fuzzy numbers. So it is better to use JMD representation of trapezoidal fuzzy numbers, for finding the fuzzy optimal solution of FFSP problems, as compared to the existing representations of trapezoidal fuzzy numbers.

7. Numerical example

To show the advantages of JMD representation over existing representations of trapezoidal fuzzy numbers, the same numerical example is solved by using all three representations of trapezoidal fuzzy numbers.
The problem is to find the FSP and fuzzy shortest distance of a network with 5 nodes and 7 arcs as shown in Fig. 1. The fuzzy path of each arc is represented by the following (a, b, c, d) type trapezoidal fuzzy numbers:
$\tilde{c}_{12}$ = (3,4,5,6), $\tilde{c}_{13}$ = (2,4,6,8), $\tilde{c}_{24}$ = (7,8,9,11), $\tilde{c}_{25}$ = (5,10,15,20), $\tilde{c}_{34}$ = (8,9,12,15), $\tilde{c}_{35}$=(10,13,17,20), $\tilde{c}_{45}$=(10,20,30,40).

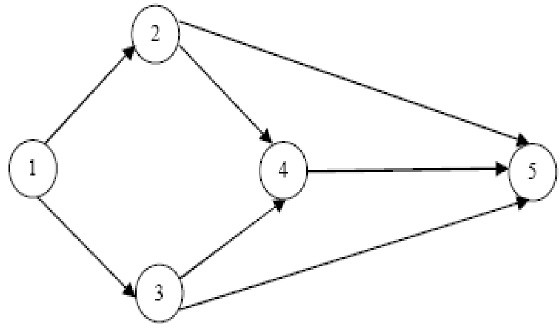

Fig. 1. A network with 5 nodes and 7 arcs

7.1. Fuzzy optimal solution using (a, b, c, d) representation of trapezoidal fuzzy numbers.

Step 1 Using Subsection 4.1, the given problem can be formulated as follows:
Minimize $((3,4,5,6)\otimes(x_{12}, y_{12}, z_{12}, w_{12})\oplus(2,4,6,8)\otimes(x_{13}, y_{13}, z_{13}, w_{13})\oplus(7,8,9,11)\otimes(x_{24}, y_{24}, z_{24}, w_{24})\oplus(5,10,15,20)\otimes(x_{25}, y_{25}, z_{25}, w_{25})\oplus(8,9,12,15)\otimes$

$(x_{34}, y_{34}, z_{34}, w_{34}) \oplus (10, 13, 17, 20) \otimes (x_{35}, y_{35}, z_{35}, w_{35}) \oplus (10, 20, 30, 40) \otimes (x_{45}, y_{45}, z_{45}, w_{45}))$
subject to

$(x_{12}, y_{12}, z_{12}, w_{12}) \oplus (x_{13}, y_{13}, z_{13}, w_{13}) = (1,\ 1,\ 1,\ 1)$,
$(x_{24}, y_{24}, z_{24}, w_{24}) \oplus (x_{25}, y_{25}, z_{25}, w_{25}) = (x_{12}, y_{12}, z_{12}, w_{12})$,
$(x_{34}, y_{34}, z_{34}, w_{34}) \oplus (x_{35}, y_{35}, z_{35}, w_{35}) = (x_{13}, y_{13}, z_{13}, w_{13})$,
$(x_{34}, y_{34}, z_{34}, w_{34}) \oplus (x_{24}, y_{24}, z_{24}, w_{24}) = (x_{45}, y_{45}, z_{45}, w_{45})$,
$(x_{25}, y_{25}, z_{25}, w_{25}) \oplus (x_{35}, y_{35}, z_{35}, w_{35}) \oplus (x_{45}, y_{45}, z_{45}, w_{45}) = (1,\ 1,\ 1,\ 1)$
$(x_{12}, y_{12}, z_{12}, w_{12})$, $(x_{13}, y_{13}, z_{13}, w_{13})$, $(x_{24}, y_{24}, z_{24}, w_{24})$, $(x_{25}, y_{25}, z_{25}, w_{25})$, $(x_{34}, y_{34}, z_{34}, w_{34})$, $(x_{35}, y_{35}, z_{35}, w_{35})$,$(x_{45}, y_{45}, z_{45}, w_{45})$ are non-negative trapezoidal fuzzy numbers.

Step 2 Using ranking formula the FLP problem, formulated in Step 1, can be written as:
Minimize $(\Re[(3, 4, 5, 6) \otimes (x_{12}, y_{12}, z_{12}, w_{12}) \oplus (2, 4, 6, 8) \otimes (x_{13}, y_{13}, z_{13}, w_{13}) \oplus (7, 8, 9, 11) \otimes (x_{24}, y_{24}, z_{24}, w_{24}) \oplus (5, 10, 15, 20) \otimes (x_{25}, y_{25}, z_{25}, w_{25}) \oplus (8, 9, 12, 15) \otimes (x_{34}, y_{34}, z_{34}, w_{34}) \oplus (10, 13, 17, 20) \otimes (x_{35}, y_{35}, z_{35}, w_{35}) \oplus (10, 20, 30, 40) \otimes (x_{45}, y_{45}, z_{45}, w_{45})])$
subject to

$(x_{12}, y_{12}, z_{12}, w_{12}) \oplus (x_{13}, y_{13}, z_{13}, w_{13}) = (1,\ 1,\ 1,\ 1)$,
$(x_{24}, y_{24}, z_{24}, w_{24}) \oplus (x_{25}, y_{25}, z_{25}, w_{25}) = (x_{12}, y_{12}, z_{12}, w_{12})$,
$(x_{34}, y_{34}, z_{34}, w_{34}) \oplus (x_{35}, y_{35}, z_{35}, w_{35}) = (x_{13}, y_{13}, z_{13}, w_{13})$,
$(x_{34}, y_{34}, z_{34}, w_{34}) \oplus (x_{24}, y_{24}, z_{24}, w_{24}) = (x_{45}, y_{45}, z_{45}, w_{45})$,
$(x_{25}, y_{25}, z_{25}, w_{25}) \oplus (x_{35}, y_{35}, z_{35}, w_{35}) \oplus (x_{45}, y_{45}, z_{45}, w_{45}) = (1,\ 1,\ 1,\ 1)$
$(x_{12}, y_{12}, z_{12}, w_{12})$, $(x_{13}, y_{13}, z_{13}, w_{13})$, $(x_{24}, y_{24}, z_{24}, w_{24})$, $(x_{25}, y_{25}, z_{25}, w_{25})$, $(x_{34}, y_{34}, z_{34}, w_{34})$, $(x_{35}, y_{35}, z_{35}, w_{35})$, $(x_{45}, y_{45}, z_{45}, w_{45})$ are non-negative trapezoidal fuzzy numbers.

Step 3 Using the arithmetic operations, described in Section 2.3.1, the FLP problem, obtained in Step 2, is converted into the following CLP problem:
Minimize $(.75\ x_{12} + y_{12} + 1.25\ z_{12} + 1.5\ w_{12} + .5\ x_{13} + y_{13} + 1.5\ z_{13} + 2\ w_{13} + 1.75\ x_{24} + 2\ y_{24} + 2.25\ z_{24} + 2.75\ w_{24} + 1.25\ x_{25} + 2.5 y_{25} + 3.75\ z_{25} + 5 w_{12} + 2 x_{34} + 2.25\ y_{34} + 3 z_{34} + 3.75\ w_{34} + 2.5 x_{35} + 3.25 y_{35} + 4.25\ z_{35} + 5\ w_{35} + 2.5 x_{45} + 5\ y_{45} + 7.5\ z_{45} + 10\ w_{45})$
subject to
$x_{12} + x_{13} = 1$, $y_{12} + y_{13} = 1$, $z_{12} + z_{13} = 1$, $w_{12} + w_{13} = 1$, $x_{24} + x_{25} = x_{12}$, $y_{24} + y_{25} = y_{12}$, $z_{24} + z_{25} = z_{12}$, $w_{24} + w_{25} = w_{12}$, $x_{34} + x_{35} = x_{13}$, $y_{34} + y_{35} = y_{13}$, $z_{34} + z_{35} = z_{13}$, $w_{34} + w_{35} = w_{13}$, $x_{34} + x_{24} = x_{45}$, $y_{34} + y_{24} = y_{45}$, $z_{34} + z_{24} = z_{45}$, $w_{34} + w_{24} = w_{45}$, $x_{25} + x_{35} + x_{45} = 1$, $y_{25} + y_{35} + y_{45} = 1$, $z_{25} + z_{35} + z_{45} = 1$, $w_{25} + w_{35} + w_{45} = 1$,
$y_{12} - x_{12} \geq 0$, $z_{12} - y_{12} \geq 0$, $w_{12} - z_{12} \geq 0$, $y_{13} - x_{13} \geq 0$, $z_{13} - y_{13} \geq 0$, $w_{13} - z_{13} \geq 0$, $y_{24} - x_{24} \geq 0$, $z_{24} - y_{24} \geq 0$, $w_{24} - z_{24} \geq 0$, $y_{25} - x_{25} \geq 0$, $z_{25} - y_{25} \geq 0$, $w_{25} - z_{25} \geq 0$, $y_{34} - x_{34} \geq 0$, $z_{34} - y_{34} \geq 0$, $w_{34} - z_{34} \geq 0$, $y_{35} - x_{35} \geq 0$, $z_{35} - y_{35} \geq 0$, $w_{35} - z_{35} \geq 0$, $y_{45} - x_{45} \geq 0$, $z_{45} - y_{45} \geq 0$, $w_{45} - z_{45} \geq 0$,
x_{12}, x_{13}, x_{24}, x_{25}, x_{34}, x_{35}, x_{45}, y_{12}, y_{13}, y_{24}, y_{25}, y_{34}, y_{35}, y_{45}, z_{12}, z_{13}, z_{24}, z_{25},

z_{34}, z_{35}, z_{45}, w_{12}, w_{13}, w_{24}, w_{25}, w_{34}, w_{35}, $w_{45} \geq 0$

Step 4 On solving CLP problem, obtained in Step 3, an optimal solution is $x_{12} = x_{25} = y_{12} = y_{25} = z_{12} = z_{25} = w_{12} = w_{25} = 1$ and $x_{13} = x_{24} = x_{34} = x_{35} = x_{45} = y_{13} = y_{24} = y_{34} = y_{35} = y_{45} = z_{13} = z_{24} = z_{34} = z_{35} = z_{45} = w_{13} = w_{24} = w_{34} = w_{35} = w_{45} = 0$

Step 5 Putting the values of x_{ij}, y_{ij}, z_{ij} and w_{ij} in $\tilde{x}_{ij} = (x_{ij}, y_{ij}, z_{ij}, w_{ij})$, the fuzzy optimal solution is $\tilde{x}_{12}$ = (1,1,1,1), $\tilde{x}_{25}$ = (1,1,1,1)

Step 6 Using the fuzzy optimal solution, the FSP is $1 \to 2 \to 5$. Putting the values of $(x_{ij}, y_{ij}, z_{ij}, w_{ij})$ in objective function the fuzzy shortest distance is (8,14,20,26)

7.2. Fuzzy optimal solution using (m, n, α, β) representation of trapezoidal fuzzy numbers. Using Section 2.2.2, the (m, n, α, β) representation of $\tilde{c}_{12}$ = (3,4,5,6), $\tilde{c}_{13}$ = (2,4,6,8), $\tilde{c}_{24}$ = (7,8,9,11), $\tilde{c}_{25}$ = (5,10,15,20), $\tilde{c}_{34}$ = (8,9,12,15), $\tilde{c}_{35}$=(10,13,17,20), and $\tilde{c}_{45}$=(10,20,30,40) is $\tilde{c}_{12}$ = (4,5,1,1), $\tilde{c}_{13}$ = (4,6,2,2), $\tilde{c}_{24}$ = (8,9,1,2), $\tilde{c}_{25}$ = (10,15,5,5), $\tilde{c}_{34}$ = (9,12,1,3), $\tilde{c}_{35}$=(13,17,3,3) and $\tilde{c}_{45}$=(20,30,10,10) respectively.

Step 1 Using Section 4.2, the given problem can be formulated as follows:
Minimize $((4,5,1,1)\otimes(y_{12}, z_{12}, \alpha_{12}, \beta_{12})\oplus(4,6,2,2)\otimes(y_{13}, z_{13}, \alpha_{13}, \beta_{13})\oplus(8,9,1,2)\otimes(y_{24}, z_{24}, \alpha_{24}, \beta_{24})\oplus(10,15,5,5)\otimes(y_{25}, z_{25}, \alpha_{25}, \beta_{25})\oplus(9,12,1,3)\otimes(y_{34}, z_{34}, \alpha_{34}, \beta_{34})\oplus(13,17,3,3)\otimes(y_{35}, z_{35}, \alpha_{35}, \beta_{35})\oplus(20,30,10,10)\otimes(y_{45}, z_{45}, \alpha_{45}, \beta_{45}))$
subject to

$(y_{12}, z_{12}, \alpha_{12}, \beta_{12}) \oplus (y_{13}, z_{13}, \alpha_{13}, \beta_{13}) = (1, 1, 0, 0)$,
$(y_{24}, z_{24}, \alpha_{24}, \beta_{24}) \oplus (y_{25}, z_{25}, \alpha_{25}, \beta_{25}) = (y_{12}, z_{12}, \alpha_{12}, \beta_{12})$,
$(y_{34}, z_{34}, \alpha_{34}, \beta_{34}) \oplus (y_{35}, z_{35}, \alpha_{35}, \beta_{35}) = (y_{13}, z_{13}, \alpha_{13}, \beta_{13})$,
$(y_{34}, z_{34}, \alpha_{34}, \beta_{34}) \oplus (y_{24}, z_{24}, \alpha_{24}, \beta_{24}) = (y_{45}, z_{45}, \alpha_{45}, \beta_{45})$,
$(y_{25}, z_{25}, \alpha_{25}, \beta_{25}) \oplus (y_{35}, z_{35}, \alpha_{35}, \beta_{35}) \oplus (y_{45}, z_{45}, \alpha_{45}, \beta_{45}) = (1, 1, 0, 0)$
$(y_{12}, z_{12}, \alpha_{12}, \beta_{12})$, $(y_{13}, z_{13}, \alpha_{13}, \beta_{13})$, $(y_{24}, z_{24}, \alpha_{24}, \beta_{24})$, $(y_{25}, z_{25}, \alpha_{25}, \beta_{25})$, $(y_{34}, z_{34}, \alpha_{34}, \beta_{34})$, $(y_{35}, z_{35}, \alpha_{35}, \beta_{35})$, $(y_{45}, z_{45}, \alpha_{45}, \beta_{45})$ are non-negative trapezoidal fuzzy numbers.

Step 2 Using ranking formula the FLP problem, formulated in Step 1, can be written as:
Minimize $(\Re[(4,5,1,1) \otimes (y_{12}, z_{12}, \alpha_{12}, \beta_{12}) \oplus (4,6,2,2) \otimes (y_{13}, z_{13}, \alpha_{13}, \beta_{13}) \oplus (8,9,1,2) \otimes (y_{24}, z_{24}, \alpha_{24}, \beta_{24}) \oplus (10,15,5,5) \otimes (y_{25}, z_{25}, \alpha_{25}, \beta_{25}) \oplus (9,12,1,3) \otimes (y_{34}, z_{34}, \alpha_{34}, \beta_{34}) \oplus (13,17,3,3) \otimes (y_{35}, z_{35}, \alpha_{35}, \beta_{35}) \oplus (20,30,10,10) \otimes (y_{45}, z_{45}, \alpha_{45}, \beta_{45})])$
subject to

$(y_{12}, z_{12}, \alpha_{12}, \beta_{12}) \oplus (y_{13}, z_{13}, \alpha_{13}, \beta_{13}) = (1, 1, 0, 0)$,
$(y_{24}, z_{24}, \alpha_{24}, \beta_{24}) \oplus (y_{25}, z_{25}, \alpha_{25}, \beta_{25}) = (y_{12}, z_{12}, \alpha_{12}, \beta_{12})$,
$(y_{34}, z_{34}, \alpha_{34}, \beta_{34}) \oplus (y_{35}, z_{35}, \alpha_{35}, \beta_{35}) = (y_{13}, z_{13}, \alpha_{13}, \beta_{13})$,
$(y_{34}, z_{34}, \alpha_{34}, \beta_{34}) \oplus (y_{24}, z_{24}, \alpha_{24}, \beta_{24}) = (y_{45}, z_{45}, \alpha_{45}, \beta_{45})$,
$(y_{25}, z_{25}, \alpha_{25}, \beta_{25}) \oplus (y_{35}, z_{35}, \alpha_{35}, \beta_{35}) \oplus (y_{45}, z_{45}, \alpha_{45}, \beta_{45}) = (1, 1, 0, 0)$

$(y_{12}, z_{12}, \alpha_{12}, \beta_{12})$, $(y_{13}, z_{13}, \alpha_{13}, \beta_{13})$, $(y_{24}, z_{24}, \alpha_{24}, \beta_{24})$, $(y_{25}, z_{25}, \alpha_{25}, \beta_{25})$, $(y_{34}, z_{34}, \alpha_{34}, \beta_{34})$, $(y_{35}, z_{35}, \alpha_{35}, \beta_{35})$, $(y_{45}, z_{45}, \alpha_{45}, w_{45})$ are non-negative trapezoidal fuzzy numbers.

Step 3 Using the arithmetic operations, described in Section 2.3.2, the FLP problem, obtained in Step 2, is converted into the following CLP problem:
Minimize $(1.75y_{12}+2.75z_{12}-0.75\alpha_{12}+1.5\beta_{12}+1.5y_{13}+3.5z_{13}-0.5\alpha_{13}+2\beta_{13}+3.75y_{24}+5z_{24}-1.75\alpha_{24}+2.75\beta_{24}+3.75y_{25}+8.75z_{25}-1.25\alpha_{25}+5\beta_{12}+4.25y_{34}+6.75z_{34}-2\alpha_{34}+3.75\beta_{34}+5.75y_{35}+9.25z_{35}-2.5\alpha_{35}+5\beta_{35}+7.5y_{45}+17.5z_{45}-2.5\alpha_{45}+10\beta_{45})$
subject to
$y_{12}+y_{13}=1$, $z_{12}+z_{13}=1$, $\alpha_{12}+\alpha_{13}=0$, $\beta_{12}+\beta_{13}=0$, $y_{24}+y_{25}=y_{12}$, $z_{24}+z_{25}=z_{12}$, $\alpha_{24}+\alpha_{25}=\alpha_{12}$, $\beta_{24}+\beta_{25}=\beta_{12}$, $y_{34}+y_{35}=y_{13}$, $z_{34}+z_{35}=z_{13}$, $\alpha_{34}+\alpha_{35}=\alpha_{13}$, $\beta_{34}+\beta_{35}=\beta_{13}$, $y_{34}+y_{24}=y_{45}$, $z_{34}+z_{24}=z_{45}$, $\alpha_{34}+\alpha_{24}=\alpha_{45}$, $\beta_{34}+\beta_{24}=\beta_{45}$, $y_{25}+y_{35}+y_{45}=1$, $z_{25}+z_{35}+z_{45}=1$, $\alpha_{25}+\alpha_{35}+\alpha_{45}=0$, $\beta_{25}+\beta_{35}+\beta_{45}=0$,
$y_{12}-\alpha_{12}\geq 0$, $z_{12}-y_{12}\geq 0$, $y_{13}-\alpha_{13}\geq 0$, $z_{13}-y_{13}\geq 0$, $y_{24}-\alpha_{24}\geq 0$, $z_{24}-y_{24}\geq 0$, $y_{25}-\alpha_{25}\geq 0$, $z_{25}-y_{25}\geq 0$, $y_{34}-\alpha_{34}\geq 0$, $z_{34}-y_{34}\geq 0$, $y_{35}-\alpha_{35}\geq 0$, $z_{35}-y_{35}\geq 0$, $y_{45}-\alpha_{45}\geq 0$, $z_{45}-y_{45}\geq 0$, y_{12}, y_{13}, y_{24}, y_{25}, y_{34}, y_{35}, y_{45}, z_{12}, z_{13}, z_{24}, z_{25}, z_{34}, z_{35}, z_{45}, α_{12}, α_{13}, α_{24}, α_{25}, α_{34}, α_{35}, α_{45}, β_{12}, β_{13}, β_{24}, β_{25}, β_{34}, β_{35}, $\beta_{45}\geq 0$

Step 4 On solving CLP problem, obtained in Step 3, an optimal solution is $y_{12}=y_{25}=z_{12}=z_{25}=1$ and $y_{13}=y_{24}=y_{34}=y_{35}=y_{45}=z_{13}=z_{24}=z_{34}=z_{35}=z_{45}=\alpha_{12}=\alpha_{13}=\alpha_{24}=\alpha_{25}=\alpha_{34}=\alpha_{35}=\alpha_{45}=\beta_{12}=\beta_{13}=\beta_{24}=\beta_{34}=\beta_{35}=\beta_{45}=0$

Step 5 Putting the values of y_{ij}, z_{ij}, α_{ij} and β_{ij} in $\tilde{x}_{ij}=(y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})$, the fuzzy optimal solution is $\tilde{x}_{12}=(1,1,0,0)$, $\tilde{x}_{25}=(1,1,0,0)$

Step 6 Using the fuzzy optimal solution, the FSP is $1\rightarrow 2\rightarrow 5$. Putting the values of $(y_{ij}, z_{ij}, \alpha_{ij}, \beta_{ij})$ in objective function the fuzzy shortest distance is (14,20,6,6)

7.3. Fuzzy optimal solution using JMD representation of trapezoidal fuzzy numbers. Using Definition 12, the $(x, \alpha', \beta', \gamma')_{JMD}$ representation of $\tilde{c}_{12}=(3,4,5,6)$, $\tilde{c}_{13}=(2,4,6,8)$, $\tilde{c}_{24}=(7,8,9,11)$, $\tilde{c}_{25}=(5,10,15,20)$, $\tilde{c}_{34}=(8,9,12,15)$, $\tilde{c}_{35}=(10,13,17,20)$, and $\tilde{c}_{45}=(10,20,30,40)$ is $\tilde{c}_{12}=(3,1,1,1)_{JMD}$, $\tilde{c}_{13}=(2,2,2,2)_{JMD}$, $\tilde{c}_{24}=(7,1,1,2)_{JMD}$, $\tilde{c}_{25}=(5,5,5,5)_{JMD}$, $\tilde{c}_{34}=(8,1,3,3)_{JMD}$, $\tilde{c}_{35}=(10,3,4,3)_{JMD}$, and $\tilde{c}_{45}=(10,10,10,10)_{JMD}$ respectively.

Step 1 Using Section 5.2, the given problem can be formulated as follows:
Minimize $((3,1,1,1)_{JMD}\otimes(x_{12}, \alpha'_{12}, \beta'_{12}, \gamma'_{12})_{JMD}\oplus(2,2,2,2)_{JMD}\otimes(x_{13}, \alpha'_{13}, \beta'_{13}, \gamma'_{13})_{JMD}\oplus(7,1,1,2)_{JMD}\otimes(x_{24}, \alpha'_{24}, \beta'_{24}, \gamma'_{24})_{JMD}\oplus(5,5,5,5)_{JMD}\otimes(x_{25}, \alpha'_{25}, \beta'_{25}, \gamma'_{25})_{JMD}\oplus(8,1,3,3)_{JMD}\otimes(x_{34}, \alpha'_{34}, \beta'_{34}, \gamma'_{34})_{JMD}\oplus(10,3,4,3)_{JMD}\otimes(x_{35}, \alpha'_{35}, \beta'_{35}, \gamma'_{35})_{JMD}\oplus(10,10,10,10)_{JMD}\otimes(x_{45}, \alpha'_{45}, \beta'_{45}, \gamma'_{45})_{JMD})$
subject to

$(x_{12},\alpha'_{12},\beta'_{12},\gamma'_{12})_{JMD} \oplus (x_{13},\alpha'_{13},\beta'_{13},\gamma'_{13})_{JMD} = (1,0,0,0)_{JMD}$,
$(x_{24},\alpha'_{24},\beta'_{24},\gamma'_{24})_{JMD} \oplus (x_{25},\alpha'_{25},\beta'_{25},\gamma'_{25})_{JMD} = (x_{12},\alpha'_{12},\beta'_{12},\gamma'_{12})_{JMD}$,
$(x_{34},\alpha'_{34},\beta'_{34},\gamma'_{34})_{JMD} \oplus (x_{35},\alpha'_{35},\beta'_{35},\gamma'_{35})_{JMD} = (x_{13},\alpha'_{13},\beta'_{13},\gamma'_{13})_{JMD}$
$(x_{34},\alpha'_{34},\beta'_{34},\gamma'_{34})_{JMD} \oplus (x_{24},\alpha'_{24},\beta'_{24},\gamma'_{24})_{JMD} = (x_{45},\alpha'_{45},\beta'_{45},\gamma'_{45})_{JMD}$
$(x_{25},\alpha'_{25},\beta'_{25},\gamma'_{25})_{JMD} \oplus (x_{35},\alpha'_{35},\beta'_{35},\gamma'_{35})_{JMD} \oplus (x_{45},\alpha'_{45},\beta'_{45},\gamma'_{45})_{JMD} = (1,0,0,0)_{JMD}$

$(x_{12},\alpha'_{12},\beta'_{12},\gamma'_{12})_{JMD}$, $(x_{13},\alpha'_{13},\beta'_{13},\gamma'_{13})_{JMD}$, $(x_{24},\alpha'_{24},\beta'_{24},\gamma'_{24})_{JMD}$, $(x_{25},\alpha'_{25},\beta'_{25},\gamma'_{25})_{JMD}$, $(x_{34},\alpha'_{34},\beta'_{34},\gamma'_{34})_{JMD}$, $(x_{35},\alpha'_{35},\beta'_{35},\gamma'_{35})_{JMD}$, $(x_{45}, \alpha'_{45}, \beta'_{45}, \gamma'_{45})_{JMD}$ are non-negative trapezoidal fuzzy numbers.

Step 2 Using ranking formula the FLP problem, formulated in Step 1, may be written as:

Minimize$(\Re[(3,1,1,1)_{JMD}\otimes(x_{12},\alpha'_{12},\beta'_{12},\gamma'_{12})_{JMD}\oplus(2,2,2,2)_{JMD}\otimes(x_{13},\alpha'_{13},\beta'_{13},\gamma'_{13})_{JMD}\oplus(7,1,1,2)_{JMD}\otimes(x_{24},\alpha'_{24},\beta'_{24},\gamma'_{24})_{JMD}\oplus(5,5,5,5)_{JMD}\otimes(x_{25},\alpha'_{25},\beta'_{25},\gamma'_{25})_{JMD}\oplus(8,1,3,3)_{JMD}\otimes(x_{34},\alpha'_{34},\beta'_{34},\gamma'_{34})_{JMD}\oplus(10,3,4,3)_{JMD}\otimes(x_{35},\alpha'_{35},\beta'_{35},\gamma'_{35})_{JMD} \oplus (10,10,10,10)_{JMD} \otimes (x_{45},\alpha'_{45},\beta'_{45},\gamma'_{45})_{JMD}])$

subject to

$(x_{12},\alpha'_{12},\beta'_{12},\gamma'_{12})_{JMD} \oplus (x_{13},\alpha'_{13},\beta'_{13},\gamma'_{13})_{JMD} = (1,0,0,0)_{JMD}$,
$(x_{24},\alpha'_{24},\beta'_{24},\gamma'_{24})_{JMD} \oplus (x_{25},\alpha'_{25},\beta'_{25},\gamma'_{25})_{JMD} = (x_{12},\alpha'_{12},\beta'_{12},\gamma'_{12})_{JMD}$,
$(x_{34},\alpha'_{34},\beta'_{34},\gamma'_{34})_{JMD} \oplus (x_{35},\alpha'_{35},\beta'_{35},\gamma'_{35})_{JMD} = (x_{13},\alpha'_{13},\beta'_{13},\gamma'_{13})_{JMD}$
$(x_{34},\alpha'_{34},\beta'_{34},\gamma'_{34})_{JMD} \oplus (x_{24},\alpha'_{24},\beta'_{24},\gamma'_{24})_{JMD} = (x_{45},\alpha'_{45},\beta'_{45},\gamma'_{45})_{JMD}$
$(x_{25},\alpha'_{25},\beta'_{25},\gamma'_{25})_{JMD} \oplus (x_{35},\alpha'_{35},\beta'_{35},\gamma'_{35})_{JMD} \oplus (x_{45},\alpha'_{45},\beta'_{45},\gamma'_{45})_{JMD} = (1,0,0,0)_{JMD}$

$(x_{12},\alpha'_{12},\beta'_{12},\gamma'_{12})_{JMD}$, $(x_{13},\alpha'_{13},\beta'_{13},\gamma'_{13})_{JMD}$, $(x_{24},\alpha'_{24},\beta'_{24},\gamma'_{24})_{JMD}$, $(x_{25},\alpha'_{25},\beta'_{25},\gamma'_{25})_{JMD}$, $(x_{34},\alpha'_{34},\beta'_{34},\gamma'_{34})_{JMD}$, $(x_{35},\alpha'_{35},\beta'_{35},\gamma'_{35})_{JMD}$, $(x_{45},\alpha'_{45},\beta'_{45},\gamma'_{45})_{JMD}$ are non-negative trapezoidal fuzzy numbers.

Step 3 Using the arithmetic, described in Section 5.1, the FLP problem, obtained in Step 2, is converted into following CLP problem:

Maximize $(4.5x_{12} + 3.75\alpha'_{12} + 2.75\beta'_{12} + 1.5\gamma'_{12} + 5x_{13} + 4.5\alpha'_{13} + 3.5\beta'_{13} + 2\gamma'_{13} + 8.75x_{24} + 7\alpha'_{24} + 5\beta'_{24} + 2.75\gamma'_{24} + 12.5x_{25} + 11.25\alpha'_{25} + 8.75\beta'_{25} + 5\gamma'_{25} + 11x_{34} + 9\alpha'_{34} + 6.75\beta'_{34} + 3.75\gamma'_{34} + 15x_{35} + 12.5\alpha'_{35} + 9.25\beta'_{35} + 5\gamma'_{35} + 25x_{45} + 22.5\alpha'_{45} + 17.5\beta'_{45} + 10\gamma'_{45})$

subject to

$x_{12} + x_{13} = 1$, $\alpha'_{12} + \alpha'_{13} = 0$, $\beta'_{12} + \beta'_{13} = 0$, $\gamma'_{12} + \gamma'_{13} = 0$, $x_{24} + x_{25} = x_{12}$, $\alpha'_{24} + \alpha'_{25} = \alpha'_{12}$, $\beta'_{24} + \beta'_{25} = \beta'_{12}$, $\gamma'_{24} + \gamma'_{25} = \gamma'_{12}$, $x_{34} + x_{35} = x_{13}$, $\alpha'_{34} + \alpha'_{35} = \alpha'_{13}$, $\beta'_{34}+\beta'_{35} = \beta'_{13}$, $\gamma'_{34}+\gamma'_{35} = \gamma'_{13}$, $x_{34}+x_{24} = x_{45}$, $\alpha'_{34}+\alpha'_{24} = \alpha'_{45}$, $\beta'_{34}+\beta'_{24} = \beta'_{45}$, $\gamma'_{34} + \gamma'_{24} = \gamma'_{45}$, $x_{25} + x_{35} + y_{45} = 1$, $\alpha'_{25} + \alpha'_{35} + \alpha'_{45} = 0$, $\beta'_{25} + \beta'_{35} + \beta'_{45} = 0$, $\gamma'_{25} + \gamma'_{35} + \gamma'_{45} = 0$,
x_{12}, x_{13}, x_{24}, x_{25}, x_{34}, x_{35}, x_{45}, α'_{12}, α'_{13}, α'_{24}, α'_{25}, α'_{34}, α'_{35}, α'_{45}, β'_{12}, β'_{13}, β'_{24}, β'_{25}, β'_{34}, β'_{35}, β'_{45}, γ'_{12}, γ'_{13}, γ'_{24}, γ'_{25}, γ'_{34}, γ'_{35}, $\gamma'_{45} \geq 0$

Step 4 On solving CLP problem, obtained in Step 3, an optimal solution is $x_{12} = x_{25} = 1$ and $x_{13} = x_{24} = x_{34} = x_{35} = x_{45} = \alpha'_{12} = \alpha'_{13} == \alpha'_{24} = \alpha'_{25} = \alpha'_{34} = \alpha'_{35} = \alpha'_{45} = \beta'_{12} = \beta'_{13} = \beta'_{24} = \beta'_{25} = \beta'_{34} = \beta'_{35} = \beta'_{45} = \gamma'_{12} = \gamma'_{13} =$

$\gamma'_{24} = \gamma'_{25}\gamma'_{34} = \gamma'_{35} = \gamma'_{45} = 0$

Step 5 Putting the values of x_{ij}, α'_{ij}, β'_{ij} and γ'_{ij} in $\tilde{x}_{ij} = (x_{ij}, \alpha'_{ij}, \beta'_{ij}, \gamma'_{ij})_{JMD}$, the fuzzy optimal solution is $\tilde{x}_{12} = (1, 0, 0, 0)_{JMD}$, $\tilde{x}_{25} = (1, 0, 0, 0)_{JMD}$

Step 6 Using the fuzzy optimal solution, the FSP is $1 \to 2 \to 5$. Putting the values of $(x_{ij}, \alpha'_{ij}, \beta'_{ij}, \gamma'_{ij})_{JMD}$ in objective function the fuzzy shortest distance is $(8, 6, 6, 6)_{JMD}$

7.4. Results and discussion. The results of the numerical example, obtained from section 7.1, 7.2 and 7.3, are shown in Table 1. On the basis of these results it can be easily seen that if all the parameters are represented by JMD representation of trapezoidal fuzzy numbers, instead of existing representations of trapezoidal fuzzy numbers, and proposed method is applied to find the fuzzy optimal solution of FFSP problems then the fuzzy optimal solution is same but the total number of constraints, in converted CLP problem, are less than the number of constraints, obtained by using the existing representations of trapezoidal fuzzy numbers. Hence, it is better to use JMD representation instead of existing representations of trapezoidal fuzzy numbers to find the fuzzy optimal solution of FFSP problems.

Table 1. Results using existing and proposed representation of trapezoidal fuzzy numbers

Type of trapezoidal fuzzy number	Number of constraints in FLP problem	Number of constraints in CLP problem	FSP	Fuzzy shortest distance
(a, b, c, d)	12	$(4 \times 12) + (3 \times 7) = 69$	$1 \Rightarrow 2 \Rightarrow 5$	(8,14,20,26)
(y, z, α, β)	12	$(4 \times 12) + (2 \times 7) = 62$	$1 \Rightarrow 2 \Rightarrow 5$	$(14, 20, 6, 6)$
$(x, \alpha', \beta', \gamma')_{JMD}$	12	$(4 \times 12) = 48$	$1 \Rightarrow 2 \Rightarrow 5$	$(8, 6, 6, 6)_{JMD}$

8. Conclusions

A new method is proposed to find the fuzzy optimal solution of FFSP problems and it is shown that it is better to use the proposed representation of trapezoidal fuzzy numbers instead of existing representations, to find the fuzzy optimal solution of FFSP problems.

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