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DEA의 가중치 제약의 특성 연구

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Characterization of Weight Restrictions in DEA

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본 논문에서는 Cone-ratio 가중치 제약이 DEA 단일 입력, 다중 출력(또는 다중 입력, 단일 출력)에 맞게 constant return을 하는 문제에 적용될 때 효율성을 측정하는 방법에 필요한 특성을 제안하였다. 또한 제안된 특성들을 기반으로 DEA에서의 단일 입력, 이중 출력에 대한 다중 해결 문제의 예시를 제시한 후 Cone-ratio 및 Wong과 Beasley 가중치 제약의 특성들을 도식적으로 비교 설명하였다.

Keywords : Data Envelopment Analysis, Weight restriction, Cone-ratio, Wong and Beasley

1. Introduction

Since the CCR model from Charnes et al. [2] does not have any weight restrictions (except for positivity), it can have the flexibility in choosing weights for each DMU (Decision Making Unit) j. And this flexibility is considered to be one of the major advantages of DEA to measure efficiency in many DEA literatures, since using CCR model has the practical advantage that a user need not identify prior relative values of inputs and outputs. Unfortunately, the imputed input and output values of CCR model may be problematic when a user has certain value judgments that should be taken into account in the assessment and those values do not coincide with the imputed values of CCR model. Thanassoulis [9] mentioned some of the circumstances when we would wish to incorporate value judgment in a DEA assessment as follows. 1) Imputed values may not accord well with prior views on the marginal rates of substitution, 2) Certain inputs and outputs may have a special relation within the production process, 3) We may wish to know some kind of ‘overall efficiency,’ 4) We may wish to discriminate between Pareto-efficient DMUs. The comprehensive range of weight restrictions that can be used with CCR model to incorporate value judgments in DEA under constant returns to scale are well represented in Allen et al. [1], Among the weight restriction methods in DEA, we focused on the cone-ratio (hereafter, we call C/R) and Wong and Beasley (hereafter, we call W/B) weight restrictions in this paper. C/R DEA model was initiated by Charnes et al. [4], in which assurance regions are defined by bounds on weights reflecting the relative importance of inputs or outputs. On the other hand, Wong and Beasley [10] suggested a method that restricts virtual inputs or outputs rather than restricting actual inputs or outputs.

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The goal of this paper is to show some characteristics on the C/R weight restriction (more specifically, the following (1) in the category of assurance region type I by Thompson et al. [9] and W/B weight restriction, (the following (2)) using suggested property. Also we proposed graphical illustration of multiple solution problem with one-input, two-outputs in DEA. We think that explanation using graph, even it is limited in 2-dimensional case, can be useful which provide intuitional knowledge for further analysis in many cases. Finally, we showed an alternative way of C/R weight restriction using weights obtained by AHP.

$$Max\ h_{j_o} = \sum_{r=1}^{s} \mu_r y_{rj_o}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} \nu_i x_{ij_o} = 1$$

$$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \nu_i x_{ij} \le 0, \ j = 1, \cdots, n$$

$$\alpha_i \le \frac{\nu_i}{\nu_{i+1}} \le \beta_i, \ \alpha_r \le \frac{\mu_r}{\mu_{r+1}} \le \beta_r \qquad (1)$$

$$\alpha_i \le \frac{\nu_i x_{ij}}{\sum_{i=1}^{m} \nu_i x_{ij}} \le \beta_i, \ \alpha_r \le \frac{\mu_r y_{rj}}{\sum_{r=1}^{s} \mu_r y_{rj}} \le \beta_r \qquad (2)$$

$$(0 \le \alpha \le \beta \le 1)$$

where,
y_{rj} = amount of output r from unit j
x_{ij} = amount of input i to unit j
μ_r = the weight given to output r
ν_i = the weight given to input i
n = the number of units
s = the number of outputs
m = the number of inputs

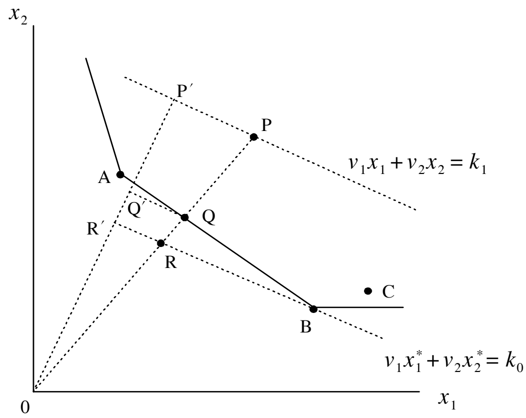
2. The Characteristics of Cone-ratio Weight Restrictions in DEA

2.1 The View Focused on Weight Vectors in CCR Model

When we analyze the result of CCR model, most of previous DEA literatures have focused mainly on the value of each input and output multipliers rather than the ratio scales of them. In this paper, we slightly changed our view in analyzing the result of CCR model like follows. That is, when we have the result of each input, output multipliers in CCR model, we consider that the corresponding cone-ratio weight vectors are assigned to each DMU j since they are most favorable. For example in one input, two outputs case, if the CCR result for DMU j are $\mu_1 = 0.111$, $\mu_2 = 0.111$, then we consider that DMU j took the output weight vector μ_1/μ_2 = 1.00 since it is most favorable rather than focusing each value itself. And we believe that this view enables us to have more clear interpretation on the characteristics of some DEA models when combined with the suggested property.

2.2 The Way of Measuring Efficiency with Cone-ratio Weight Restrictions in DEA

We suggest the property that shows the way to measure the efficiency when C/R weight restrictions are applied under constant returns to scale with one- input, multiple-outputs (or multiple-inputs, one-output) in DEA. Let us assume the case that 1) each DMU uses two inputs (x_1, x_2) in order to yield a single output (y) under the condition of constant returns to scale 2) two inputs and one output are assumed to be all positive 3) the decision maker's weight(preference) for two inputs x_1, x_2 is given to be $\nu_1/\nu_2 = k$.



<Figure 1> Iso-weight (preference) Lines

Then, in <Figure 1>, $\nu_1 x_1 + \nu_2 x_2 = k_1$ and $\nu_1 x_1^* + \nu_2 x_2^* = k_0$ are each of iso-weight (preference) lines for DMU B and DMU P that are parallel to each other. Also OP' represents an orthogonal vector to the iso-weight lines, which passes through the origin. It is clear from above figure that we can find the unique vector, which is perpendicular to the iso-weight lines and passes through the origin. And Q', R' are the projection points which are projected perpendicular to the vector

OP' from Q and B(or R) respectively. Since Q and Q', R and R', P and P' lie on each iso-weight (preference) line, these points have the same weights respectively. Therefore, the following relation (3) should be hold that is also obvious by the property of right-angled triangle in $\triangle OPP'$.

$$\text{Efficiency score of (DMU P)} : \frac{OQ}{OP} = \frac{OQ'}{OP'}, \quad (3)$$

$$\text{Similarly, in case of (DMU B)} : \frac{OB}{OB} = \frac{OR'}{OR'}.$$

Now we define the vector OP' as the weight vector with the following property (P1)

(P1) The weight vector $\vec{w}$ is a vector, which is perpendicular to the iso-weight lines (planes) of DMU *j* and passes through the origin.

Generally if there is a DMU *j*, which uses m inputs and the weights are given by C/R weight restrictions among inputs, then the iso-weight plane of DMU *j* can be expressed as $\nu_1 x_1 + \cdots + \nu_m x_m = k$. The equation of $\nu_1 x_1 + \cdots + \nu_m x_m = k$ is the general form of the plane equation, which intersects each of m axes with the following points ,i.e. $(k/\nu_1, 0, 0, \cdots, 0)$, $(0, k/\nu_2, 0, \cdots, 0)$, $\cdots$, $(0, 0, 0, \cdots, k/\nu_m)$ and $(\nu_1, \cdots, \nu_m)$ represents orthogonal vector to this plane that is passing through the origin. Therefore we have the following property on weight vector (P2).

(P2) The weight vector of DMU *j*, which uses m inputs and the weights are given by C/R weight restrictions among inputs, can be represented such as input weight vector, $\overrightarrow{w_i} = (\nu_1, \cdots, \nu_m)$ and the output weight vector, $\overrightarrow{w_r} = (\mu_1, \cdots, \mu_m)$.

For simplicity, we assumed that the C/R weight for two inputs are the same for all DMUs in the case of <Figure 1>. Therefore we have all different iso-weight lines but parallel to each other to all DMUs and a unique weight vector. However in case that each DMU's C/R weight for each input is different, each DMU will have its own weight vector.

After all, when the C/R weight restrictions (weight vectors) are applied to the general CCR model, all DMUs are projected to the weight vector along with the iso-weight lines (planes), and the efficiency score is measured by the following ratio (P3).

(P3) When the C/R weight restrictions (weight vectors) are applied to the CCR model, efficiency score of DMU *j* can be measured by the following ratio.

(one-input, multiple output or multiple-input, one output case)
The efficiency score of DMU j =

$$\frac{\textit{(Norms of orthogonal projection of DMU j to the weight vector)}}{\textit{(Norms of orthogonal projection of DMU j* to the weight vector)}}$$

where, DMU *j** has the largest norm (output maximization case) or smallest norm (input minimization case) when projected to the weight vector.

(multiple-input, multiple-output case)
The efficiency score of DMU j =

$$\frac{\textit{virtual output of DMU j / virtual input of DMU j}}{\textit{virtual output of DMU j* / virtual input of DMU j*}}$$

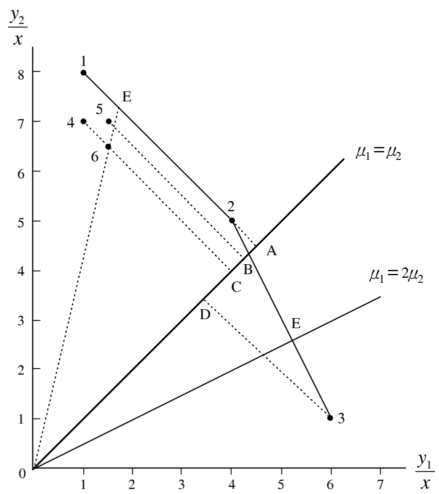
where, DMU *j** has the largest efficiency score with the same input and output weight vectors with DMU *j* respectively.

We explain above descriptions using the following example 1. <Table 1> shows the data set (left table) and the CCR results of each DMU (right table). The final column in <Table 1> shows the ratio of output multipliers of each DMU, in which we can see that only DMU 3 chose the ratio of output multipliers μ_1/μ_2 as 2.00 and all the other five DMUs chose the ratio of 1.00. <Figure 2> is drawn with overlapping two planes, one is input-output variable plane $(y_1/x, y_2/x) = (y_1, y_2)$ and the other is corresponding multiplier plane $(\mu_1/\nu, \mu_2/\nu) = (\mu_1, \mu_2)$ in this case. And we showed the weight vectors $\mu_1 = \mu_2$, $\mu_1 = 2\mu_2$ in this overlapped plane. First, when we assume that the decision maker's weight (preference) for two outputs are given to equal and apply this preference as C/R weight restriction $\mu_1/\mu_2 = 1.00$, then the efficiency score of DMU *j* can be measured by the suggested property.

That is, 1) $\overline{1E2A}$, $\overline{5B}$, $\overline{46C}$, $\overline{3D}$ represent the iso-weight (preference) lines for each DMU, where A, B, C and D represent the projection points of DMUs 1, 5, 4 (6) and 3 respectively 2) Therefore, the efficiency score of each DMU can be measured as follows.

<Table 1> Data and CCR Results of Example 1

Data				CCR Multipliers-Input Oriented				
DMU	x	y_1	y_2	CCR	v_1	μ_1	μ_2	μ_1/μ_2
1	1	1.0	8.0	1	1	0.1111	0.1111	1.00
2	1	4.0	5.0	1	1	0.1111	0.1111	1.00
3	1	6.0	1.0	1	1	0.1538	0.0769	2.00
4	1	1.0	7.0	0.8889	1	0.1111	0.1111	1.00
5	1	1.5	7.0	0.9444	1	0.1111	0.1111	1.00
6	1	1.5	6.5	0.8889	1	0.1111	0.1111	1.00



<Figure 2> One-input, Two-outputs Case of Example 1

$$\frac{OA}{OA} = 1(\text{DMUs 1 and 2}),\ \frac{OD}{OA} = 0.777(\text{DMU3}),$$
$$\frac{OC}{OA} = 0.888(\text{DMUs 4 and 6}),\ \frac{OB}{OA} = 0.944(\text{DMU5}).$$

When we think the case of DMU 6, it is clear that $\left(\frac{O6}{OE} = \frac{OC}{OA} = 0.8889\right)$ from the iso-weight lines and also the property of right-angled triangle (in this case $\triangle AOE$). Actually we can see that DMU 3 takes the weight vector $\mu_1 = 2\mu_2$ from the CCR result but here we assumed that all DMUs take the same weight vector$\mu_1 = \mu_2$, therefore the measure efficiency score for DMU 3 is $\frac{OD}{OA} = 0.777 < 1$. This means that DMU 3 can be evaluated as technically efficient when the DMU's weight (preference) ratio on the output variables is $\mu_1 = 2\mu_2$. Therefore if the DMU's weight (preference) ratio is $\mu_1 = \mu_2$, the overall efficiency score of DMU 3 is less than technical efficiency score of 1. Until now, we showed the way of measuring efficiency when we have C/R weight restrictions in DEA. Generally in DEA, it is admitted that C/R weight restrictions represent the decision maker's decision of relative importance. Also this relative importance has been explained as the marginal rates of substitution between inputs or between outputs [2, 8].

However, the fact that C/R weight restriction in DEA implies the relation of perfect substitution among inputs (or outputs) has not been emphasized in previous DEA literature. Since the C/R weight restriction implies the relation of perfect substitution among inputs (or outputs), it can have possible drawback when the decision maker's weight (preference) on input (output) variables doesn't allow the substitution among inputs or outputs at all or allow the substitution only in certain ranges, i.e. not allow the relation of perfect substitution. That is, there can be a case that even though (decision maker's) revealed relative importance of the two outputs is $(\mu_1 : \mu_2) = (1:1)$, which means the marginal rate of substitution between two outputs is -1, i.e. a 1 unit increase of output 1 would be compensated for by a 1 unit decrease of output 2, but it doesn't mean that it is also acceptable (no difference) in the following case 2 or case 3. 1) Case 1 : (output 1: output 2) = (3 units: 1 unit) and (1 unit : 3 units) 2) Case 2 : (output 1: output 2) = (100 units : 1 unit) and (1 unit : 100 units) 3) Case 3 : (output 1 : output 2) = (1000 units : 1 unit) and (1 unit : 1000 units).

3. Graphical Illustration on the Multiple Solution Problems in DEA

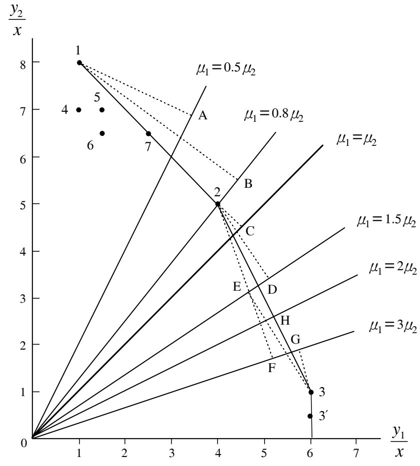
One of the benefits of using the view focused on weight vectors and properties shown in previous section is that we can make graphical interpretation on the characteristics of some DEA models although it is limited in a 2-dimensional case. However, even in a 2-dimensional case, we think that the graphical explanation may be useful which provide intuitional knowledge for further analysis in many cases. One of possible explanations is on the multiple solution problems with one-input, two-outputs case in DEA. The detailed statements or mathematical proofs on the multiple solution problems in DEA can be found in Charnes et al. [3, 4]. In their papers, they suggested the classification and characterization of DMUs as follows after dividing DMUs into 6 classes, E, E', F, NE, NE', NF. 1) DMUs E, E', F are scale efficient but only DMUs E, E' are Pareto-Koopmans efficient, 2) DMU E is efficient and is characterized by the property that their sets of optimizing multipliers are all of the maximal dimension $s+m$, 3) DMU E' are also efficient and have at least one optimizing multiplier with all component positive, however they differ from E in that their set of optimizing multipliers have dimension less than $s+m$, 4) DMU F is also on the frontier but is associated with DMU that is not efficient. This has no optimizing multiplier in which all components are strictly positive. On the other hand, the multiple solution problems in DEA has been explained more generally using the dual form of CCR model in several studies [3, 5, 7] that certain DMUs can have multiple λ values in their optimal solutions. But we cannot get graphical intuitions on the possible multiple solutions using the dual form of CCR model, which makes it rather difficult to understand. Here, we show the above characteristics of 1) ~ 4) using the data of example 1 and <Figure 3> which shows the projections of DMU 1, 2 and 3 to various weight vectors.

(1) At first, DMU 1 and 2 have the same projection point at C(4.5, 4.5) when projected to the weight vector $\mu_1 = \mu_2$ which results in efficiency score 1. However if projected to any weight vector in relation of $\mu_1 < \mu_2$ (we showed in <Figure 3> the two cases of $\mu_1 = 0.5\mu_2$ and $\mu_1 = 0.8\mu_2$), DMU 1 is the only one which has the efficiency score of 1. This implies that DMU 1 has multiple optimal solutions.

(2) DMU 2 and 3 have the same projection point at H(5.2, 2.6) when projected to the weight vector $\mu_1 = 2\mu_2$ which results in efficiency score 1.

However if projected to any weight vectors $\mu_2 < \mu_1 < 2\mu_2$ (we showed in <Figure 3> the one case of $\mu_1 = 1.5\mu_2$), DMU 2 is the only one which has the efficiency score of 1. This also implies that DMU 2 also has multiple optimal solutions.

(3) For DMU 3, if projected to any weight vectors $\mu_1 > 2\mu_2$ (we showed in <Figure 3> the one case of $\mu_1 = 3\mu_2$), DMU 3 is the only one which has the efficiency score of 1. This also implies that DMU 3 has multiple optimal solutions.



<Figure 3> The Range of Optimal Multipliers for DMUs

(4) If we assume that there is another DMU 3' which corresponds to F in DMU classification of [3, 4], the only possible weight vector with which the efficiency score of DMU 3' is 1 is $\mu_2 = 0$. With any other weight vector, DMU 3' cannot be the only one which has the efficiency score of 1. This coincides with the above explanation 4) by [3, 4].

(5) DMUs 1, 2 and 3 are belong to E and all of corresponding optimal multipliers are linearly independent. Therefore their sets of optimizing multipliers have the maximal dimension $s+m=3$, which coincides with the above explanation 2) by [3, 4].

(6) If we assume that there is another DMU 7 shown in <Figure 3>, which belongs to E', it can have efficiency score 1 only when projected to the weight vector $\mu_1 = \mu_2$. Therefore DMU 7 has the unique solution with relation of $\mu_1 = \mu_2$, and the dimension is $2 < s+m = 3$, which coincides with the above explanation 3) by [3, 4].

4. The Characteristics of Wong and Beasley Weight Restrictions in DEA

Rather than restricting the actual weights, Wong and Beasley [10] suggested a method which restricts virtual inputs or outputs. (Here, we used the expression of 'restricting actual weights' to follow the classification of [1, 8]. They proposed that 1) The proportion of output r devoted to the total outputs for DMU j can be represented as $\frac{\mu_r y_{rj}}{\sum_{r=1}^{s}\mu_r y_{rj}}$ and it means the 'importance' attached to output measure r by DMU j, 2) It is because the larger this value, the more DMU j depends on output measure r in determining its efficiency.

Based on the decision maker's value judgments, we can set the lower and upper limits for the importance of output r in DMU j and this can be expressed shown in (2).

Shang and Sueyoshi [6] showed an application using this W/B weight restrictions with weights obtained by AHP. However they didn't mention why W/B weight restrictions are most appropriate in that case (weights obtained by AHP) and the differences between using some direct weight restrictions. Also on the W/B weight restrictions [1] indicated that 1) Even though restrictions on virtual input or output weights have received relatively little attention in DEA literature, more research is necessary to explore the pros and cons of setting restrictions on the virtual inputs and outputs. 2) Heretofore, there has been no attempt to compare methods for setting restrictions on the actual DEA weights with those restricting virtual inputs and/or outputs. Therefore, we will examine the characteristics of W/B weight restriction.

Based on the decision maker's value judgments, W/B weight restriction sets the lower and upper limits for the importance of output r of DMU j like (2). When we think the case of one input, two outputs (y_1, y_2 with the W/B weight restriction (4), each DMU take the projection vector by the relationship of (5).

$$\frac{\mu_1 y_1}{\mu_1 y_1+\mu_2 y_2}=\alpha,\ \frac{\mu_2 y_2}{\mu_1 y_1+\mu_2 y_2}=\beta \tag{4}$$

then $\mu_1 y_1 : \mu_2 y_2 = \alpha : \beta \Rightarrow \alpha\mu_2 y_2 = \beta\mu_1 y_1$

$$\text{therefore } \frac{\mu_1}{\mu_2}=\frac{\alpha y_2}{\beta y_1} \tag{5}$$

Similarly, in the case of one input, multiple outputs, we can get all pairwise projection vectors, which make the plane equation using the same manner with (5).

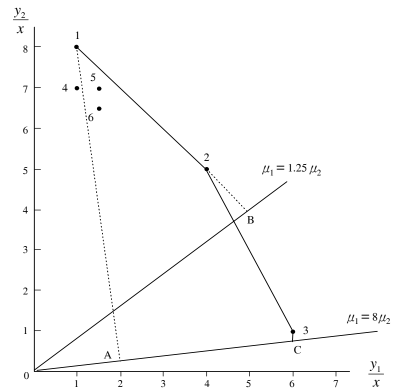
For example, when $(y_1, y_2)=(1,8)$ and if

1) $\alpha:\beta=1:1 \Rightarrow \frac{\mu_1}{\mu_2}=\frac{\alpha y_2}{\beta y_1}=\frac{y_2}{y_1}=8,\ \mu_1=8\mu_2$

2) $\alpha:\beta=3:7 \Rightarrow \frac{\mu_1}{\mu_2}=\frac{3y_2}{7y_1}=\frac{24}{7},\ \mu_1=3.43\mu_2$

To explain the characteristics of W/B weight restriction, we use the example 1 data again and also assume the equal W/B weights (importance) on two outputs such that $\alpha:\beta=0.5:0.5$ in (4). <Table 2> shows the results comparison between C/R (with restriction of μ_1=μ_2) and W/B weight restriction ($\alpha:\beta=0.5:0.5$ in (4)). The columns under C/R and W/B represent the efficiency scores when each weight restriction is applied. Only DMU 2 has the same efficiency score and all the other DMUs have much lower scores when applied W/B restriction. The way for measuring efficiency score can be explained by using weight vectors in <Figure 4>, in a similar way as in C/R weight restriction. For example, DMU 1 takes the weight vector μ_1 = $8\mu_2$, which can be seen from the last column in <Table 2>, and the efficiency score is measured by $\frac{OA}{OC}$.

The coordinates of A and C are A(1.9692, 0.2462), C (6.0307, 0.7538). Therefore $\frac{OA}{OC}=\frac{\sqrt{1.9692^2+0.24615^2}}{\sqrt{6.030769^2+0.753846^2}}=\frac{1.9845}{6.0777}=0.3265$, which is exactly same with the results in <Table 2>. On the other hand, DMU 2 takes the weight vector μ_1 = $1.25\mu_2$ and the efficiency score is $OB/OB=1$, where the coordinate of B is B(4.878, 3.9024).



<Figure 4> Illustration of W/B Weight Restriction

Here we can indicate the following characteristics on W/B weight restriction compared with C/R weight restriction.

1) When the C/R weight restriction is applied, all DMUs take the same weight vector μ_1 = μ_2 and the efficiency score is measured by the ratio of norms of projections to the same weight vector. However when W/B weight restriction is applied, each DMU takes a different weight vector and the efficiency score is measured by the ratio of projection norms on that vector.

<Table 2> Results Comparison between C/R and W/B Weight Restriction

DMU	C/R	Multipliers(C/R)				W/B	Multipliers(W/B)			
		ν_1	μ_1	μ_2	μ_1/μ_2		ν_1	μ_1	μ_2	μ_1/μ_2
1	1	1.0	0.111	0.111	1.00	0.327	1.0	0.163	0.020	8.00
2	1	1.0	0.111	0.111	1.00	1	1.0	0.125	0.100	1.25
3	0.778	1.0	0.111	0.111	1.00	0.245	1.0	0.020	0.122	0.17
4	0.889	1.0	0.111	0.111	1.00	0.326	1.0	0.163	0.023	7.00
5	0.944	1.0	0.111	0.111	1.00	0.483	1.0	0.161	0.034	4.67
6	0.889	1.0	0.111	0.111	1.00	0.481	1.0	0.160	0.037	4.33

2) W/B weight restriction makes pretty low efficiency scores for some DMUs In this case, all DMUs except DMU 2 got much lower score than those under C/R weight restriction. Actually, as long as there is any DMU which keeps its production of $y_1 = 1$, no matter how many units of y_2 are produced it can not beat DMU 5 in its efficiency score (0.483) under W/B weight restriction ($\alpha : \beta = 0.5 : 0.5$ in (4)). For example, if we imagine DMU 7, which produces two outputs $(y_1, y_2) = (1, \infty)$, efficiency score under W/B weight restriction will be 0.333, which is still lower than that of DMU 5.

5. The Difference between C/R and W/B Weight Restriction

The important difference between C/R and W/B weight restriction comes from the difference of interpretation of weights. Therefore, in order to reflect the decision maker's preferences more precisely, we have to know the decision maker's preferences more clearly and then decide which restriction method is more appropriate. The following <Figure 5> shows the difference of C/R and W/B weight restriction more clearly.

There are 3 DMUs of A, B and C which produce two outputs (y_1, y_2) like A(5, 5), B(2, 7), C(7, 2) using one input, and the decision maker's preference (importance) between two outputs is given to be $\mu_1 = \mu_2$ in C/R weight restriction and ($\alpha : \beta = 0.5 : 0.5$ in (4)) in W/B weight restriction.

When we apply the above C/R weight restriction,

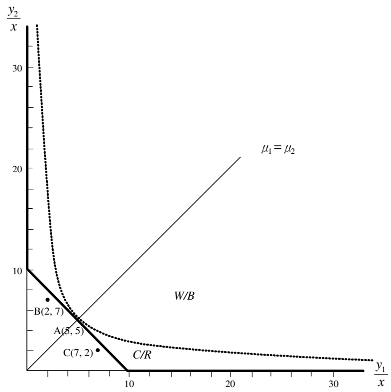
1) Every DMU located to the right side of boldline will dominate the DMU A.
2) DMUs located under the bold line don't affect to the efficiency score of the DMUs located upper side of the bold line.

On the other hand, when we apply the above W/B weight restriction

1) Every DMU located to the right side of dotted curve will dominate the DMU A.
2) DMUs located under the dotted curve affect the efficiency score of the DMUs located upper side of the bold line. That is, DMU (1, 40) takes the projection weight vector $\mu_1 = 40\mu_2$ and its W/B efficiency score 0.2836 is measured compared with DMU C(7, 2). While, DMU (40, 1) takes the projection weight vector $\mu_2 = 40\mu_1$ and its W/B efficiency score 0.2836 is measured compared with DMU B (2, 7). Therefore, if we have different DMUs (not DMU B and C) in the sample, W/B efficiency score will also become different.

*1) Results of W/B efficiency score of each DMU with comparison of DMU A, where the results are represented by [(y_1, y_2), W/Bscore], DMU A (W/B score)
[(1, 40), 0.2836] dominates DMU A (0.2439),
[(2, 17), 0.5528] dominates DMU A(0.5263),
[(3, 10), 0.7895] dominates DMU A (0.7692),
[(4, 7), 0.9874] dominates DMU A (0.9091)
but (1, 39), (2, 16), (3, 9), (4, 6) don't.

*2) Each DMU's W/B score shows the characteristics of W/B weight restrictions well.
[(1, 4), 0.2836] < [(2, 17), 0.5528] < [(3, 1), 0.7895] < [(4, 7), 0.9874]



<Figure 5> Difference between C/R and W/B Weight Restriction

After all, we can say on the characteristics of C/R and W/B weight restrictions as follows.

1) Even though we have the common W/B weight restriction to all DMUs, each DMU takes its own weight vector and its W/B efficiency is measured compared with a certain DMU in the sample. That is, each DMU's efficiency score is assigned by comparison with different DMUs and also it depends on the sample DMUs.
2) While C/R weight restriction implies perfect substitution (flexible) between inputs or outputs, W/B weight restriction would not allow sufficient substitution (inflexible). That is, if W/B weight restriction between two outputs is given such as (4) then to get the high efficiency score, each DMU should try to produce each output with the proportion of $\frac{y_1}{y_2} = \frac{\beta}{\alpha}$.

6. Conclusion

In this paper, we suggested the property that shows the way to measure the efficiency when C/R weight restrictions are applied under constant returns to scale with one-input, multiple-outputs (or multiple-inputs, one-output) in DEA. Based on the suggested property, we showed some characteristics of C/R and W/B weight restrictions with an illustrative example. Although C/R or W/B weight restriction can be a useful tool for incorporating value judgment in DEA, it is still necessary to develop more precise weighting schemes that can reflect decision maker's more complicate preferences well.

For example, when we have the preference such that C/R and W/B weight restriction should be used at the same time, it often makes infeasible solution for some DMUs. Also the linearity assumption in DEA may be problematic when the preference of decision maker cannot be represented as a linear function.

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