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점진적 충격파모형의 함축적 의미와 검산

Implications and numerical application of the asymptotical shock wave model

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요 약

Lighthill과 Whitham의 충격파모형에 따르면 동일한 속도를 유지하는 교통류 흐름상태에서도 충격파가 존재하며, 이는 라디오 전파처럼 보이지도 않고 관측할 수도 없다고 하였다. 최근의 한 논문은 이 문제에 대해 새로운 접근방법을 통해 위와 같은 모순이 어떻게 발생하였는지를 보여주었고, 이를 개선하기 위해 점근적 충격파모형 (asymptotical shock wave model) 을 제시하였다. 점근적 충격파모형은 동일한 속도로 이동하는 균일한 교통류에서 라디오 전파와 같은 관측 불가능한 충격파가 존재하지 않는 것을 증명하였다. 그러나 상기 논문은 모형의 유도와 증명에 치중하였고 모형으로서의 해석이나 구체적인 수치를 적용한 모형의 검증은 아직 실행된 적이 없다.

본 논문은 점근적 충격파모형의 내포된 의미를 해석하고, 구체적인 수치를 바탕으로 한 시나리오를 통해 모형의 성능을 시험하였다. 그 결과 점근적 충격파모형은 기존 모형에 비해 수식상의 큰 차이는 없었지만, 유일한 차이인 등식의 세 번째 항목이 모형 결과에 결정적인 차이를 나타냄을 확인하였다. 새 모형에 도입된 파라메터는 적용된 수치의 대소에 따라 그 결과가 다르게 나타났다. 이는 기존의 충격파모형에는 없는 특징으로서, 적절한 수치를 선정한다면 다양한 교통흐름에 신축적으로 모형을 적용할 수 있을 것으로 판단된다. 또한 구체적인 수치를 적용한 점근적모형의 시나리오별 시험 결과 동일한 조건에서 새로운 모형은 기존 모형에 비해 충격파가 교통류의 하류 측으로 더 진행됨을 확인하였다. 양 모형간의 이러한 차이는 통계적 유의성 검토에서도 확인되었으며, 향후 현장 자료를 적용한 추가적 비교연구가 필요한 것으로 사료된다.

Abstract

According to the Lighthill and Whitham's shock wave model, a shock wave exists even in a homogeneous speed condition. They referred this wave as unobservable-- analogous to a radio wave that cannot be seen. Recent research has attempted to identify how such a counterintuitive conclusion results from the Lighthill and Whitham's shock wave model, and derive a new asymptotical shock wave model. The asymptotical model showed that the shock wave in a homogenous speed traffic stream is identical to the ambient vehicle speed. Thus, no radio wave-like shock wave exists. However, performance tests of the asymptotical model using numerical values have not yet been performed. We investigated the new asymptotical model by examining the implications of the new model, and tested it using numerical values based on a test scenario. Our investigation showed that the only difference between both models is in the third term of the equations, and that this difference has a crucial role in the model output. Incorporation of model parameter α is another distinctive feature of the asymptotical model. This parameter makes the asymptotical model more flexible. In addition, due to various choices of α values, model calibration to accommodate various traffic flow situations is achievable. In Lighthill and Whitham's model, this is not possible. Our numerical test results showed that the new model yields significantly different outputs: the predicted shock wave speeds of the asymptotical model tend to lean toward the downstream direction in most cases compared to the shock wave speeds of Lighthill and Whitham's model for the same test environment. Statistical tests of significance also indicate that the outputs of the new model are significantly different than the corresponding outputs of Lighthill and Whitham's model.

Key words : Shock wave, traffic flow theory, model investigation, asymptotical model, mathematical modeling

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Ⅰ. Introduction

1. Background Information

Shock waves are defined as boundary conditions in the time-space domain that denote a discontinuity in flow-density conditions[1]. Many traffic problems have been analyzed by deploying this model, and it is useful to analyze such traffic phenomena as backups and queuing on a highway, or at an urban signalized intersection[1, 2, and 3]. Many researchers have suggested more complex transformed models including Daganzo[3, 4], Newell[5, 6], Zhang[7], and Michalopoulos et al. [8]. However, the basic form of the shock wave model (SWM) has remained unchanged for more than five decades.

Lighthill and Whitham first pointed out that there are some traffic situations in which shock waves are not observable in the field, whereas the model predicts the existence of waves [2]. An example is the shock wave in a homogeneous speed condition. Lighthill and Whitham referred to this wave as unobservable; that is, analogous to a radio wave that cannot be seen. Gerlough and Huber[9] also described this wave as imaginary, but useful as an analytical tool. This contradictory example demonstrates the paradox of Lighthill and Whitham's model. Cho[10] suggested that there is no logical reason why this particular wave is unobservable or imaginary while all other waves are observable in the field. He denoted this specific case as the SWM paradox, and attempted to resolve the paradox by deriving a new asymptotical shock wave model. By evaluating the development of Lighthill and Whitham's model, Cho shows that oversimplified assumptions regarding the relationships between speed, density, and flow are the direct causes of the model distortion. Although the simplified assumptions in Lighthill and Whitham's model allow a very simple derivation procedure, its outputs for certain conditions are severely distorted. The asymptotical model requires a more complex derivation, but it resolves the contradictory output of the existing model.

2. Study purpose and approach

The asymptotical model is more flexible than Lighthill and Whitham's model since it incorporates the speed-space relationship during the speed or spacing transition procedure. However, Cho's previous work[10] concentrated on the derivation of the asymptotical model and on the resolution of the SWM paradox. The asymptotical model is not self-explanatory, and implications of the new model have not been explored. Investigation of the features and the implications of the new model, especially in association with numerical speed values, were all passed over to the further study. Further, the asymptotical model incorporated a new parameter α, but no numerical values were used to reveal the relation of the parameter to the model outputs.

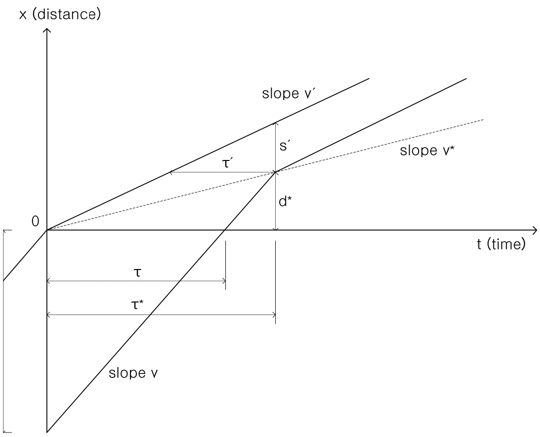
We explored the implications and applicability issues of the asymptotical SWM, and performed rigorous numerical tests to demonstrate the performance of the new model. We reviewed Cho's model, including the comparison between the new and the classical models in terms of derivation procedure, assumptions, and features. To assess the applicability of the new asymptotical model, a set of traffic data is cited from a textbook, and the data are applied to the new model based on several scenarios.

Ⅱ. Review of Shock Wave Models

We reviewed the derivation procedures of the shock wave models of Lighthill-Whitham, and Cho, both of which are shown in Cho's article[10]. We assumed that the highway has a one-directional lane. The lane maintains a geometric condition in the time and space domains. Vehicle conservation induces a relationship between the traffic flow rate change with respect to the space between two consecutive vehicles, and the associated density.

1. Lighthill and Whitham's shock wave model

Cho[10] cited the geographic derivation procedure of Lighthill and Whitham's SWM. It was assumed that a driver is traveling along a homogeneous highway at a constant speed **v**, and then suddenly changes the speed to **v′** and maintains this speed for an arbitrary length of time. A following driver may accelerate or decelerate in some manner. If unable to pass, the following driver will also adjust to the new speed **v′**. Regardless of the details of the trajectory, we extrapolated the trajectory at speeds **v** and **v′** until the two asymptotes intersected, and we imagined that the following car had such a piecewise linear trajectory. By drawing the trajectories on a sufficiently coarse scale of distance, the details of the transition would not be seen, as shown in Fig. 1.



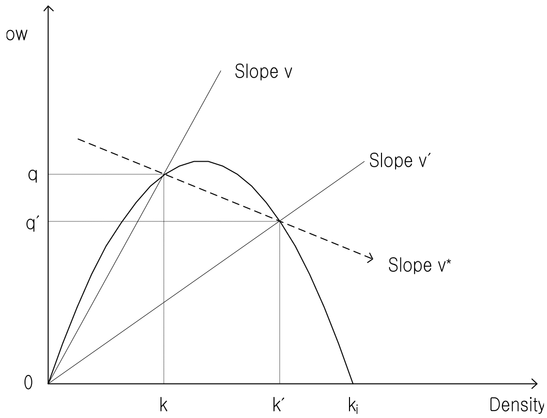
〈Fig. 1〉 Trajectories of Lighthill and Whitham's model.

If a sequence of cars is traveling at speed **v** have spacing s, flow **q**, etc., and when traveling at speed **v′** have spacing **s′**, flow **q′**, etc., (both determined from the same curves of **v** vs. **s**, and **q** vs. **k**, etc.), then the trajectory corners will all lie on a straight line. The path upon which the speed change propagates is called a shock wave. From the geographic conditions and associated mathematical computation, Lighthill-Whitham's SWM was formulated as follows:

$$v^* = \frac{d^*}{\tau^*} = -\frac{(\tau'-\tau)(v'-v)}{\left(\frac{1}{v'}-\frac{1}{v}\right)(s'-s)} = \frac{(q-q')v'vk'k}{qq'(k-k')} = \frac{q'-q}{k'-k} \tag{1}$$

where **v*** is the speed of the shock wave, **d*** is the distance the wave travels from one car to the next, and τ^* is the time for the wave to propagate from one car to the next.

According to Eq. (1), the shock wave speed is the ratio of the flow difference (**q′** - **q**) and the density difference (**k′**- **k**). The shock wave speed is shown graphically as the slope of the line passing the points (**k**, **q**)and(**k′**, **q′**) on a flow-density diagram (Fig. 2).



〈Fig. 2〉 Lighthill and Whitham's shock wave speed on a flow-density diagram.

2. The shock wave model paradox

Cho argued that since the derivation of Eq. (1) relies on assumptions of relationships among **q**, **v**, and **k**, the shock wave equation is valid for **v** < **v′**, **v** > **v′**, and **v** = **v′**. That is, the equation should accommodate any situation, as follows:

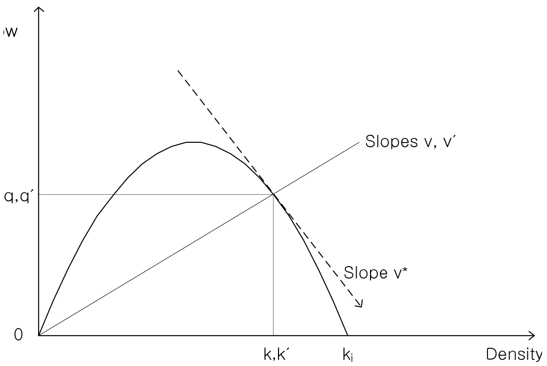
$\triangle > 0$,

$\triangle < 0$, or

$\triangle = 0$

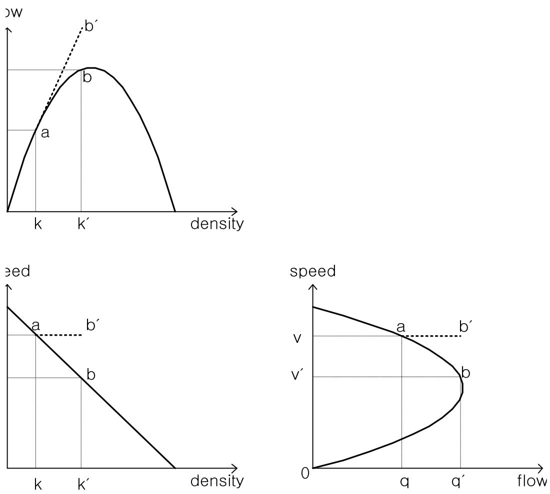
where $\triangle = \mathbf{v} - \mathbf{v}'$.

For the case in which $\triangle = 0$ or $\mathbf{v} = \mathbf{v}'$ (homogeneous speed traffic), all drivers maintain an identical speed **v**. Therefore, no waves propagate down ward or upward. The meaning of “no wave existence” is not that **v***=0, but that the wave speed is “identical” to **v**(the speed of the vehicles). If **v*** is a value other than **v**, then a shock wave should be observable. However, the solution wave speed for this case in the Lighthill-Whitham’s model is **v***(i.e., not **v**), as shown in Fig. 3. This means that in a traffic stream where all the vehicles are cruising at the same speed, the model predicts a wave that propagates forward or backward. This is a clear contradiction, since we cannot detect a shock wave in traffic traveling at a homogeneous speed.



〈Fig. 3〉 Lighthill and Whitham's shock wave speed in a homogeneous speed flow.

Cho showed that the contradictory outcome of Lighthill and Whitham’s model stemmed from ignorance of the speed-space relationship between consecutive vehicles. Cho demonstrated that the speed-density trajectory of the second vehicle against the first follows **ab**′ rather than **ab**, as shown in Fig. 4. Similar violations of initial assumptions are illustrated in both the flow-density and flow-speed diagrams. Trajectories follow **ab**′ rather than **ab**. In both cases, the speed did not change, whereas the flow and density changed from **q** to **q**′ and **k** to **k**′, respectively.

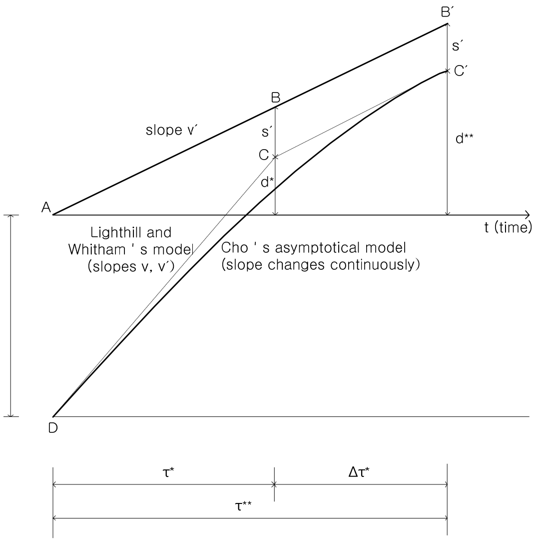


〈Fig. 4〉 Distorted diagrams of the Lighthill and Whitham's model.

3. Cho’s asymptotical shock wave model

Cho’s new approach attempted to eliminate the distortion in the relationships among flow, density, and speed from the derivation of the Lighthill-Whitham’s model by modifying the vehicle trajectories in a time-space diagram to accommodate the changed speed at every instant as the spacing changes from **s** to **s**′ (Fig. 5). That is, when a preceding vehicle changes speed from **v**to **v**′, the following vehicle continuously changes speed in a way such that the relationship between the spacing of the preceding vehicle and the speed of the following vehicle strictly follows the presumed relationships; i.e., Greenshield’s model. Figure 5 shows the trajectories of the two different models.

In Fig. 5, the trajectory of the preceding vehicle is the same in both models. The trajectories of the following vehicle in the existing and the revised models differ; one exhibits piecewise linear lines whereas the other is a monotonic curve. The dashed line in Fig. 5 represents the trajectory of the following vehicle in the existing model, and the solid curve below it represents the trajectory in the revised model. Thus, the time required for the following vehicle to change its spacing from **s** to **s′** is different in each model: one is τ^* and the other is τ^{**}. Cho noted that in the case of the revised approach, the speed changes continuously as the spacing changes from **s** to **s′**. Therefore, the relationships among flow, density, and speed satisfy the presumed relationships to eliminate the modeling distortion of the existing approach.



〈Fig. 5〉 Comparison of time-space trajectories.

From this revised trajectory, Cho's asymptotical shock wave model is written as follows:

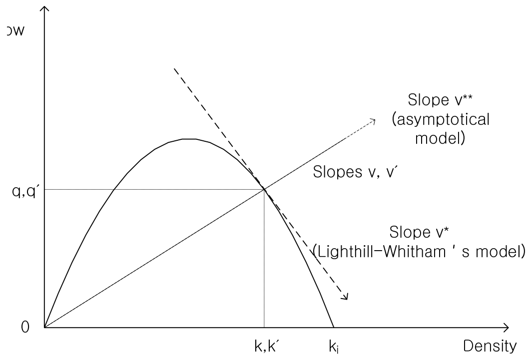
$$v^{**} = \frac{v_f}{k_j}\left\{k_j - k' - \frac{k'}{\frac{k'-k}{k} + \ln\frac{k'-k}{(\alpha-1)k}}\right\} \quad (2)$$

Equation (2) represents the shock wave speed expressed by **k** and **k′** in the new asymptotical model. In the equation, k_j and $\frac{v_f}{k_j}$ are constant under given highway conditions, and α is a model parameter.

In a homogeneous traffic stream, **v** and **v′**, and **s** and **s′** are identical and constant, respectively. In this case, the wave velocity **v****can be represented by letting **k′** approach **k** in Eq.(2):

$$\begin{aligned}\lim_{k'\to k}(v^{**}) &= \lim_{k'\to k}\left(\frac{v_f}{k_j}\left\{k_j - k' - \frac{k'}{\frac{k'-k}{k} + \ln\frac{k'-k}{(\alpha-1)k}}\right\}\right)\\ &= \frac{v_f}{k_j}\left\{k_j - k - \frac{k}{-\infty}\right\}\\ &= v_f\left\{1 - \frac{k}{k_j}\right\}\\ &= v\end{aligned} \quad (3)$$

Thus, in the asymptotical model, the shock wave speed in a homogeneous traffic stream is always identical to the ambient vehicle speed. Graphically, the slopes of **v**, **v′**, and **v**** of the revised model shown in Fig. 6 are all the same, whereas the shock wave speed **v*** of Light hill and Whitham's model differs from the traffic speed(**v** or **v′**).



〈Fig. 6〉 Comparison of wave speeds in homogeneous traffic stream.

Ⅲ. Implications and Test of the Asymptotical Shock Wave Model

We investigated the features of the asymptotical model in comparison with the Lighthill-Whitham's model. Due to the different approaches of the models, the final mathematical equations and their numerical outputs for any given traffic situation are distinguishable. The only known functional difference is that the asymptotical model yields a shock wave speed identical to the ambient traffic speed in a homogeneous flow condition, whereas the Lighthill-Whitham's model produces a shock wave speed that is quite different from the traffic speed.

1. Comparison of the models

(1) Trajectory of the following vehicle in the model

A distinctive aspect of the asymptotical shock wave model in comparison with the classic model is that it deploys a stringent speed-space relationship between two consecutive vehicles in traffic while the model was derived. We note that in the classic shock wave model, the following vehicle maintains speed **v** until the space reach to **s**′, which satisfies the given speed-space relationship. Then, the speed is reduced to **v**′ abruptly. Thus, the trajectory of the following vehicle in the Lighthill-Whitham's model is represented by the two linear lines shown in Fig. 5. Cho argued that this assumption significantly distorts the flow-density-speed relationships. He suggested that as the following vehicle approaches the front vehicle, the assumed speed-density-flow relationships follow the **a b**′ lines instead of the **ab** curves(the lines in Fig. 4).

On the other hand, Cho's asymptotical model adjusts the spacing continuously to incorporate the speed-density-flow relationships. This does not necessarily mean that one model is better than the other. Any field observations will show that the asymptotical model mimics the space adjusting behavior of the following vehicle in a more realistic manner. Several researchers have demonstrated the shortcomings of hydrodynamic or fluid models to represent traffic flow[9]. Applying to traffic those models implies greater concern in the over-all statistical behavior of the traffic stream than in the interaction between vehicles[11]. Cho showed that the prediction of radio wave-like imaginary propagation in the Lighthill-Whitham's model is an example of the shortcomings of a fluid model. The numerical comparisons of the performance of both models are discussed in the following section.

(2) Comparison of model equations

The governing mathematical equation of the asymptotical model is given by Eq. (2), which includes the free flow speed $\mathbf{v_f}$, jam density $\mathbf{k_j}$, and the density before and after the speed change(**k** and **q**′, respectively). For a side-by-side comparison, Lighthill-Whitham's model (Eq. (1)) can be rewritten with the same variables in Eq. (2) by substituting the flows (**q** and **q**′) with free the flow speed ($\mathbf{v_f}$), jam density($\mathbf{k_j}$), and the density before and after the speed change(**k** and **k**′). Thus, the variable-substituted Lighthill-Whitham's model governing equation is given by

$$v^* = \frac{v_f}{k_j}\left(k_j - k' - k\right) \quad (4)$$

A comparison of Eqs. (2) and (4) shows that the third term in the parentheses of both equations is the only difference between the two models. However, a comparison of the third term of each equation cannot be made directly. The third term of the asymptotical model includes **k**, **k**′ , andα, and the third term of the Lighthill-Whitham's model has only one variable, **k**. Although the complete implications of the combined effect of **k**, **k**′ , and α in the asymptotical model is not clear, it is certain that the third term in the parenthesis of Eq. (2) plays the crucial role in the asymptotical model by establishing that the shock wave speed of the homogeneous flow condition is identical to the ambient speed. The use of actual numerical values in the equations enables a quantitative comparison of both models, as described in Section 2.

Incorporation of model parameter α is another feature of the asymptotical model. Equation (2) clearly indicates that the value of α affects the results for any flow condition. However, since Cho focused on a verification that are no radio wave-like imaginary shock waves at a homogeneous speed condition, a specific numerical value of α was omitted in his article except that it is slightly greater than 1.0. Deployment of α into the model requires a more complex computational procedure. However, when the appropriate value is available, the model would be more flexible; thus, it can be applied to various traffic situations by accommodating specific driving behavior.

2. Test of the asymptotical model with numerical values

To demonstrate the performance of the asymptotical shock wave model, a set of numerical values is deployed in the model. For convenient comparison and data accessibility, numerical values are cited from a contemporary traffic engineering textbook written by Garber and Hoel[11]. For the same reason, the distance scale uses miles (mi) instead of kilometers (km). The numerical traffic values are as follows:

Saturation flow rate $\mathbf{q_{max}}$=2000 veh/hr/ln
Jam density $\mathbf{k_j}$=150 veh/ln/mi

From Greenshild's speed-density relationship and equation $\mathbf{q=k \bullet v}$, the free flow speed $\mathbf{v_f}$ was determined as follows:

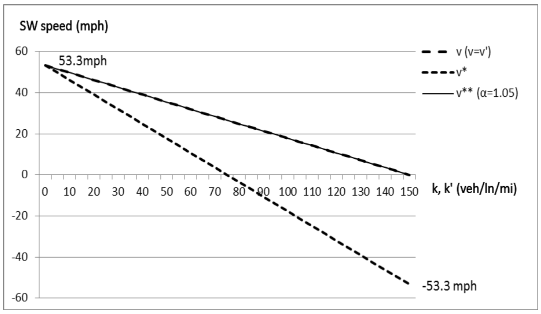
Free flow speed $\mathbf{v_f}$ = 53.3 miles per hour(mph)

The Greenshild speed-density relationship was used in Cho's paper; thus, no other advanced density-speed relationships were considered for this numerical test. Since the asymptotical model was applicable only to deceleration flow conditions (i.e., $\mathbf{v>v'}$), the numerical tests of the model were restricted accordingly. In addition to the traffic data, the value of α should be decided for the numerical test of the asymptotical model. When the model was derived, Cho mentioned that α is a number slightly greater than 1.0 but there was no further description on it. Since there has been no empirical study on the appropriate value of α, we assumed that it lies somewhere in between 1.05 and 1.0005 which means that the following driver stops his/her vehicle spacing adjustment with 0.05 to 5 percent margin.

(1) Test 1: The existence of a radio wave-like imaginary shock wave

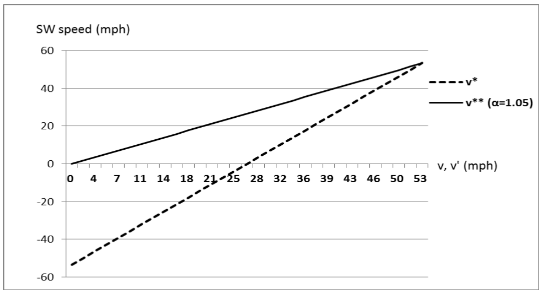
Our initial concern with the asymptotical model was the existence of a radio wave-like imaginary shock wave in the homogeneous traffic flow condition. Shock wave speeds $\mathbf{v^*}$ and $\mathbf{v^{**}}$ were computed using Eqs. (1) and (2), respectively. The input densities $\mathbf{k}$ and $\mathbf{k'}$ ($\mathbf{k = k'}$) ranged from 0 to 150 veh/l/mi with an increment of 5. The computation results are plotted in Figs. 8 and 9.

Figure. 8 is an integrated plot of the numerical test results for density for the homogeneous traffic flow condition. Throughout the entire data range, the shock wave speeds of the asymptotical model are identical to the ambient traffic speed. This result indicates that the asymptotical shock wave model is valid for homogeneous traffic conditions with numerical data. In addition, the shock wave speeds of the asymptotical model are not affected by a change in the value of α, as shown by Eq. (3) when **k** approaches **k′**.



〈Fig. 7〉 Comparison of wave speeds in homogeneous traffic stream along density.

On the contrary, the shock wave speeds of Lighthill and Whitham's model linearly decrease according to the increase in ambient density. In Figs. 7 and 8, all the shock wave speeds from Lighthill and Whitham's model are not identical to the ambient speed, except when the ambient speed is a free flow condition. These speeds are the radio wave-like imaginary shock wave speeds referred to by several researchers. When the density reached the jam density, the shock wave speed was -53.3 mph. When compared to the ambient traffic speeds v and **v′** (**v**=**v′**), except the point at which **k**=0, the shock wave speeds of Lighthill and Whitham's model are always smaller than corresponding ambient speeds, and the difference in the two speeds increase in proportion to the density.

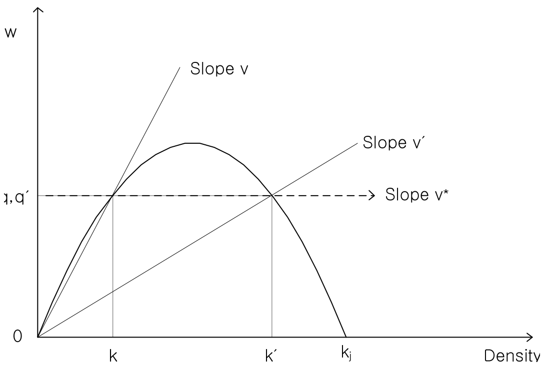


〈Fig. 8〉 Comparison of wave speeds in homogeneous traffic stream along speed.

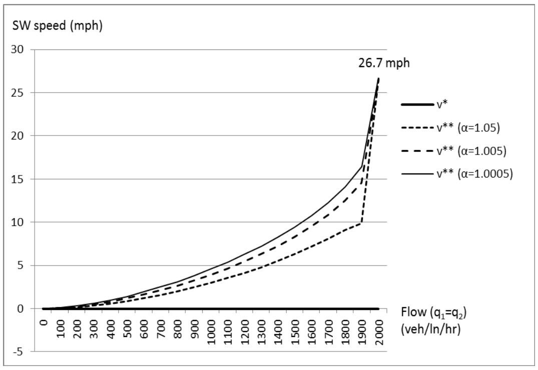
(2) Test 2: Shock waves with flow-conserved traffic conditions

We considered a particular case in which the front vehicle changes its speed from **k** to **k′** but the flows before and after the speed change remained unchanged such that **q**=**q′**. According to Lighthill and Whitham's model, the shock wave speed **v*** is stationary; thus, it is depicted as a horizontal line as shown in Fig. 9. The theoretically possible numbers of such a stationary shock wave are as large as the maximum flow. Figure 10 shows the all the numerical test results in a crude scale. The shock wave speeds of Lighthill and Whitham's model with the test data under such conditions are plotted on the horizontal axis in the figure.

In Figure 10, we plotted the shock wave speed of the asymptotical model for three different α values. The three shock waves, regardless of α value, increased gradually. When the flow was 2000 veh/ln/hr, all three speeds reached 26.7 mph, which is identical to the ambient speed. We note that the smaller α value resulted in the larger shock wave speed at a given flow. Overall, the shock wave speed of the asymptotical model is larger than that of Lighthill and Whitham's model, which is stationary regardless of the prevailing flow density. In other words, the shock waves of the asymptotical model propagate in the downstream direction, and those from



〈Fig. 9〉 A shock wave in which the flows are unchanged.



〈Fig. 10〉 Comparison of shock waves in which the flows are unchanged.

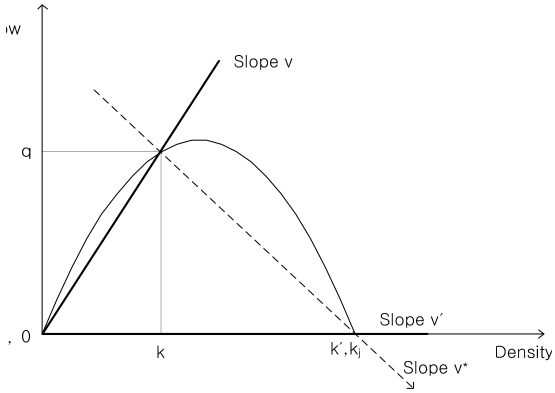
Lighthill and Whitham's model are stationary. Since an empirical test of both models is beyond the scope of this study, it is not yet clear which model replicates the real-world traffic phenomenon more correctly. Further research should include such a comparison, based on field observation.

The three curves of the asymptotical model show that there is an articulated point at which each line changes sharply between flows of 1900 and 2000 veh/ln/hr. At the data region around the crown point of the parabolic curve shown in Fig. 9, the difference between **k** and **k′** is relatively small, which means that the spacing difference between the two flow regions is very small. This is described further in Section (3).

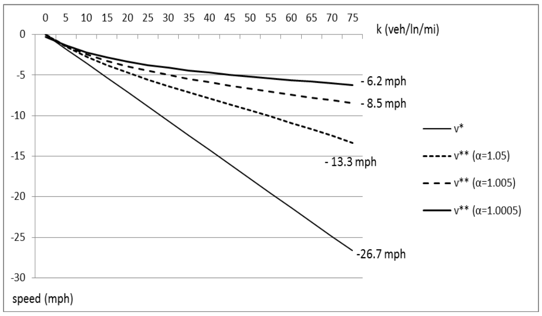
(3) Test 3: Stopping at an urban signalized intersection

One of the most frequent observations of shock waves occurs at a congested urban intersection when the signal changes to a red light. Regardless of the prevailing approaching speed **v**, the final speed **v′** of the flow is zero. Figure 11 describes such an event wherein the shock wave propagates backward at a relatively high speed.

For a detailed illustration of the shock wave comparison graph, two identical numerical tests were conducted with different density ranges. Figure 12 compares the shock waves formed when the flow condition changed from stable to no flow; thus, the initial density **k** ranged from 0(free flow condition) to 75 veh/ln/mi. Figure13 is plotted for the initial density **k** from75 to 150 veh/ln/mi.



〈Fig. 11〉 Backward shock wave at signalized intersection.

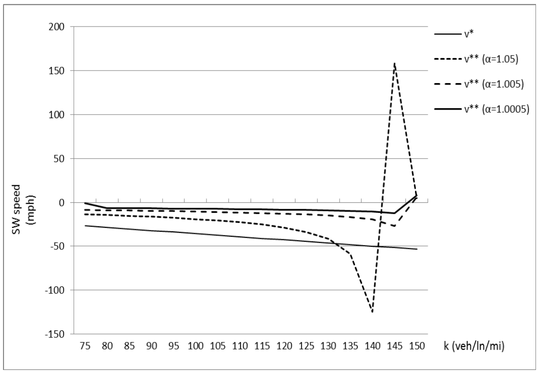


〈Fig. 12〉 Comparison of shock waves at signalized intersection: stable flow to no flow.

In Figure 12, the shock waves of the Lighthill and Whitham's model change linearly toward the upstream direction as the initial density **k** increases. The line is steeper than the other three curves of the asymptotical model. Thus, for the given flow condition, the shock wave speed of the asymptotical model is smaller than that given by the Lighthill and Whitham's model. Or, equivalently, the shock wave speed of the asymptotical model tends to lean toward the downstream direction without exception, including all situations described in Section(2). The effect of α is similar to that in the aforementioned tests: the smaller the α value, the larger the shock wave speed at a given flow.

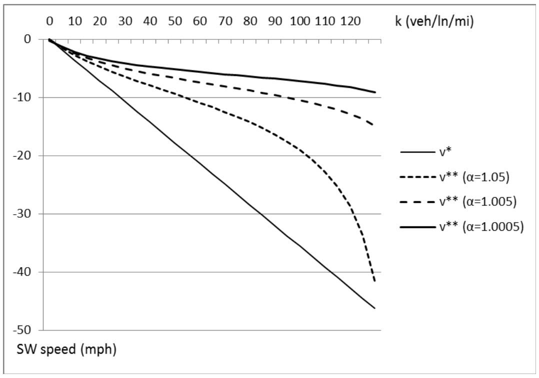
Since the output difference between the two models is distinct and systematic, we expect that tests of both models with a set of field observation data will clearly reveal which model is more applicable to the given empirical site. We note that Lighthill and Whitham's model is much stiffer since it yields only one output for a given situation in which a shock wave occurs. In contrast, in the case of the asymptotical model, the output may vary depending on the α value deployed. Thus, the flexibility of the model is apparent.

Figure 13 shows the remaining half of the initial density **k** spectrum: 75 to 150 veh/ln/mi. The shock wave speeds of the asymptotical model with an α value of 1.05 near the jam density are noticeable larger than the neighboring speed. Figure 13 also shows two outlying data points around the jam density where the initial density **k** approaches the final density **k′**. This is similar to the case described in Section (2) in which the shock wave plots were bent sharply. Thus, the asymptotical model should be used with caution when the difference between **k** and **k′** is not significant. Both shock waves with α values of 1.005 and 1.0005 were much smaller than those given by Lighthill and Whitham's model.



〈Fig. 13〉 Comparison of shock waves at signalized intersection: forced flow to stop.

Consider the shock waves of both models excluding the outlying data region. Figure 14 combines Figs. 12 and 13, excluding the near-jam density region. The figure demonstrates the wide range of possible outcomes of the asymptotical model with respect to the choice of α values. The model appears to be more flexible and, due to the choice of various α values, model calibration to accommodate various traffic flow situations is achievable. This is not the case with Lighthill and Whitham's model.



〈Fig. 14〉 Comparison of shock waves at signalized intersection: all flow to stop.

3. Test of significance

The performance tests of the asymptotical shock wave model described in Section 2 showed that its outputs are different than the corresponding outputs of Lighthill and Whitham's model. We assessed whether the outputs of both models are different significantly in a statistical sense. Since the same numerical input data were deployed to each model, a matched difference t-test was repeatedly applied for each numerical test(Test 1, Test 2, and Test 3 as described in Section 2). The formulated null hypothesis for the significance test was defined as follows:

H_0: The observed average of the differences of both models is not significantly greater than the expected average of the difference (0).

〈Table 1〉 Tests of significance of model performance

	Sample size	Output difference average	Standard Deviation	*t-statistic*	Degree of freedom	Reference *p-value*
Test 1	31	24.3	19.3	7.006	30	$p < 0.0005$
Test 2	21	4.6	5.9	3.587	20	$0.0005 < p < 0.005$
Test 3	27	9.3	6.2	7.659	26	$p < 0.0005$

The model output differences of each numerical deployment, the standard deviations, the *t-statistics*, and the *p-values* were computed, and these results are summarized in Table 1.

In Tests 1 and 3, both *p-values* are smaller than 0.0005; thus, the null hypothesis can be rejected at the 0.05% level of significance. In Test 2, the *p-value* is smaller than 0.005 and greater than 0.0005. The null hypothesis is also rejected at the 0.5% level of significance. The three tests of significance indicate that the asymptotical model yielded significantly different outputs compared to Lighthill and Whitham's model.

Ⅳ. Concluding Remarks

One distinctive aspect of the asymptotical shock wave model in comparison with Lighthill and Whitham's classic model is that it uses a stringent speed-space relationship between two consecutive vehicles in traffic while the model was derived. Due to the different methods used in the models, the final mathematical equations and their numerical outputs for any traffic situation are distinguishable. The only known functional difference was that the asymptotical shock wave model yields a shock wave speed identical to the ambient traffic speed in a homogeneous flow condition, whereas the Lighthill and Whitham's classic model produces a shock wave that is quite different from the traffic speed. We investigated the implications of the new model, and tested it by deploying numerical values based on a set of test scenarios.

A comparison of the equations of the asymptotical and the Lighthill and Whitham's models showed that the third term (out of three terms) of the equations in each model represented the only difference between the two models. Considering the significant approaching method difference of the two models, the similarity between the models was least expected. However, we are certain that the third term in the asymptotical model ensures that the shock wave speed for a homogeneous flow condition is always identical to the ambient speed. Numerical test results support this conclusion.

The incorporation of parameter α is another distinct feature of the asymptotical model. The value of α significantly affected the results for all flow conditions considered. The deployment of α required a more complex computational procedure. However, if the appropriate value is available, the model can be more flexible; thus, it can be applied to various traffic situations by accommodating specific driving behavior. Thus the incorporation of parameter α made the asymptotical model more flexible. Due to the various choices of α, model calibration to accommodate various traffic flow situations is achievable, which is not the case with Lighthill and Whitham's model.

Numerical tests of the asymptotical model showed that the predicted shock wave speed was smaller than that predicted by Lighthill and Whitham's model in most cases. The shock wave speed of the asymptotical model tends to lean toward the downstream direction to a greater degree than the shock wave speed of

Lighthill and Whitham's model for the same test environment. When a larger α value is used, the output difference between the models tends to be mitigated. However, three tests of significance show that the asymptotical model yields significantly different outputs compared with Lighthill and Whitham's model. Since the overall output difference between the two models is distinctive and systematic, tests and comparisons with both models using sets of empirical field data should be performed in future research.

Real world wave speeds of the deceleration and acceleration of flow speed may not be identical in general. Newell [5] and Zhang [7] considered such cases. The asymptotical model addresses only the deceleration flow condition. Further study should include the acceleration case using the asymptotical model.

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