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노트 : 점진적 복수할인이 있는 뉴스벤더모델의 상시 이점에 대한 추측 증명

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Note : Proof of the Conjecture on the Consistent Advantage of the Newsvendor Model under Progressive Multiple Discounts

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■ Abstract ■

In this note, a recent work in Won (2011) which investigates properties of the newsvendor model under progressive multiple discounts is revisited and a complete proof is provided for the conjecture on the consistent advantage of progressive multiple discounts over no-discounts in terms of the expected profit. The proof considers the generalized newsvendor model under progressive multiple discounts extended with positive shortage cost and salvage value which have not been considered in the previous newsvendor models under progressive multiple discounts. Without relying on derivatives, we prove that the expected profit under progressive multiple discounts are consistently greater than or equal to the one under no-discounts for every order quantity as far as her multiple discounts do not decrease customer demand, and therefore, the optimal expected profit under progressive multiple discounts is always greater than or equal to the one under no-discounts. As by-products from the proof, some interesting features of the generalized newsvendor model under progressive multiple discounts are revealed.

Keyword : Newsvendor Problem, Progressive Multiple Discounts

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1. Introduction

Khouja [3, 4] formulated an extended newsvendor problem under progressive multiple discounts and used a numerical example to show that multiple discounts provide larger optimal order quantity and higher expected profit than using no-discounts. Recently, Won [11] has investigated properties of the newsvendor model under progressive multiple discounts and presented a proof of the consistent advantage of the newsvendor model under progressive multiple discounts over no-discounts in terms of the riskless profit. However, the consistent advantage of the newsvendor model under progressive multiple discounts over no-discounts in terms of the expected profit is presented as a robust conjecture with experimental results based on a variety of probability distribution of demand.

In this note, an easy alternative proof of the consistent advantage of progressive multiple discounts over no-discounts in terms of the riskless profit is presented by adopting derivative-free approach. Unlike previous newsvendor models under progressive multiple discounts assuming zero shortage cost and salvage value, our model considers positive shortage cost and salvage value. As by-products from the proof, some interesting features of the generalized newsvendor model under progressive multiple discounts are revealed.

As can be seen in the next section, complicated mathematical expressions for the expected profit and cost of the newsvendor facing progressive multiple discounts prevent from adopting derivative-based approach for the purpose of proving the consistent advantage of progressive multiple discounts over no-discounts in terms of the expected profit. Encouraged by the benefit from a derivative-free approach for proving the consistent advantage of progressive multiple discounts over no-discounts in terms of the riskless profit, we provide a complete proof of the consistent advantage of progressive multiple discounts over no-discounts in terms of the expected profit without using derivatives.

Our proof is motivated by a recent trend in which the optimal order quantity is derived without using derivatives, such as in determining economic order quantity [2, 5, 7, 9, 10]. Because our proof shows that using multiple discounts results in constantly higher or equal profit than using no-discounts for *every* order quantity, the retailer can safely implement multiple discounts for customers even if she may not order the optimal quantity due to the realistic restriction such as budget or warehouse capacity limit as far as her multiple discounts do not decrease customer demand. Higher optimal expected profit of progressive multiple discounts over no-discounts follows immediately as a by-product of our proof. In addition, since our proof considers with the generalized newsvendor model under progressive multiple discounts extended with shortage cost and salvage value which have not been considered in the previous newsvendor models under progressive multiple discounts, it can increase practical usefulness of the decision based on the result obtained from the proof.

The note is organized as follows. In section 2, notation and definitions are introduced and previous related work is described briefly. Section 3 provides rigorous proofs for the consistent advantage of progressive multiple discounts over no-discounts in terms of the riskless profit and expected profit. The last section gives a summary of the note and a suggestion for future research.

2. Notation and Related Work

2.1 Notation and Definition

To prove the consistent advantage of progressive multiple discounts over no-discounts, we define the following notation :

n = number of discounts offered by the retailer

Q = retailer order quantity

a_k = unit selling price during the kth discount period, $a_0 > a_1 > \cdots > a_k > \cdots > a_n$, $k = 0, 1, \cdots, n$

b = purchasing cost per unit

s = shortage cost per unsatisfied unit

g = salvage value per unsold unit, $g < b$

t_k = fraction of realized demand at the regular price that can be additionally sold by discounting the product to price a_k offered by the retailer in the kth discount stage, $k = 0, 1, \cdots, n$

$r-1$ = number of periods for which the first multiple discounts are profitable

$\alpha_k = a_k - b$ = nonnegative cost of underestimating demand in the kth discount stage, $k = 0, 1, \cdots, r-1$

$\beta_k = b - a_k$ = nonnegative cost of overestimating demand in the kth discount stage, $k = r, r+1, \cdots, n$

X = random variable denoting the demand,

x = realized value of random variable X, $0 \le x \le \infty$

$f(x)$ = probability density function of x

$F(x)$ = cumulative distribution function of x

$C^I(x, Q)$ = retailer cost of underestimating and overestimating demand for order quantity Q given demand x under no-discounts

$R^I(x, Q)$ = retailer profit for demand x and order quantity Q under no-discounts

$CU_k^{II}(x, Q)$ = retailer cost of underestimating demand for order quantity Q under demand x in the kth discount stage, $k = 0, 1, \cdots, r-1$

$CO_k^{II}(x, Q)$ = retailer cost of overestimating demand for order quantity Q under demand x in the kth discount stage, $k = r, r+1, \cdots, n$

$R_k^{II}(x, Q)$ = retailer profit for demand x and order quantity Q in the kth discount stage, $k = 0, 1, \cdots, n$

$EC^I(Q)$ = expected retailer cost for order quantity Q under no-discounts

$ER^I(Q)$ = expected retailer profit for order quantity Q under no-discounts

$EC^{II}(Q)$ = expected retailer cost for order quantity Q under multiple discounts

$ER^{II}(Q)$ = expected retailer profit for order quantity Q under multiple discounts

$ES^I(Q) = EC^I(Q) + ER^I(Q)$ = riskless profit under no-discounts

$ES^{II}(Q) = EC^{II}(Q) + ER^{II}(Q)$ = riskless profit under multiple discounts

The previous newsvendor model under multiple discounts did not consider shortage cost and salvage value, while our model considers those factors. Since the first $(r-1)$ multiple discounts are profitable, $a_k \ge 0$, $k = 0, 1, \cdots, r-1$, and $\beta_k \ge 0$, $k = r, r+1, \cdots, n$. We assume that $t_0 = 1$, $t_k \ge 0$, $k = 1, 2, \cdots, n-1$, and $t_n = \infty$ [3, 4]. Let $V_k = \sum_{i=0}^{k} t_i$, $h_k = Q/V_k$, $W_k = \sum_{i=0}^{k} \alpha_i t_i$, $S_k = \sum_{i=r}^{k} \beta_i t_i$, $V_{-1} = W_{-1} = S_{-1} = 0$, and $E_q(X) = \int_0^Q x f(x) dx$.

The retailer cost function for order quantity Q under demand x in the classical newsvendor problem under no-discounts is given by

$$C^{I}(x,\ Q)=\begin{cases}(\alpha_0+s)(x-Q), & x\geq Q\\ (b-g)(Q-x), & x<Q\end{cases}. \qquad (1)$$

The retailer profit function for order quantity Q under demand x in the classical newsvendor problem under no-discounts is given by

$$R^{I}(x,\ Q)=\begin{cases}\alpha_0 Q-s(x-Q), & x\geq Q\\ a_0 x-bQ+g(Q-x), & x<Q\end{cases}. \qquad (2)$$

When the positive shortage cost and salvage value are considered, the retailer cost for order quantity Q under demand x in the kth discount stage $(k=0,\ 1,\ \cdots,\ r-1)$ is

$$\begin{aligned}CU_k^{II}(x,\ Q)&=(\alpha_k+s)\left[\left(\sum_{i=0}^{k}t_i\right)x-Q\right]+\left[\sum_{i=k+1}^{r-1}(\alpha_i+s)t_i\right]x\\ &=\left(\alpha_k V_k+\sum_{i=k+1}^{r-1}\alpha_i t_i+sV_{r-1}\right)x-(\alpha_k+s)Q\\ &=\left[W_{r-1}+sV_{r-1}+\sum_{i=0}^{k}(\alpha_k-\alpha_i)t_i\right]x-(\alpha_k+s)Q. \qquad (3)\end{aligned}$$

In equation (3), the term $(\alpha_k+s)\left[\left(\sum_{i=0}^{k}t_i\right)x-Q\right]$ indicates the cost of underestimating demand increased during the first k discount stages and the term $\left[\sum_{i=k+1}^{r-1}(\alpha_i+s)t_i\right]x$ indicates the cost of underestimating demand increased during the next profitable discount stages.

The retailer cost of overestimating demand for order quantity Q under demand x in the kth discount stage $(k=r,\ r+1,\ \cdots,\ n)$ is

$$\begin{aligned}CO_k^{II}(x,\ Q)&=(\beta_k-g)(Q-V_{k-1}x)+\left[\sum_{i=r}^{k-1}(\beta_i-g)t_i\right]x\\ &=(S_{k-1}-\beta_k V_{k-1}+gV_{r-1})x+(\beta_k-g)Q. \qquad (4)\end{aligned}$$

In equation (4), the term $(\beta_k-g)(Q-V_{k-1}x)$ indicates the cost of overestimating demand during the first $(k-1)$ discount stages and the term $\left[\sum_{i=r}^{k-1}(\beta_i-g)t_i\right]x$ indicates the cost of overestimating demand during the unprofitable discount stages up to $(k-1)$ discount stages.

The retailer profit function for order quantity Q given a demand x in the kth discount stage is given by

$$R_k^{II}(x,\ Q)=\begin{cases}\alpha_0 Q-s(x-Q), & k=0\\ \left(\sum_{i=0}^{k-1}\alpha_i t_i\right)x+\alpha_k(Q-V_{k-1}X), & k=1,\ \cdots,\ r-1\\ \left(\sum_{i=0}^{r-1}\alpha_i t_i\right)x-(\beta_k-g)(Q-V_{k-1}x) & \\ -\left[\sum_{i=r}^{k-1}(\beta_i-g)t_i\right]x, & k=r,\ r+1,\ \cdots,\ n\end{cases}$$

(5)

When $x\geq Q$, i.e., $k=0$, $R^{I}(x,\ Q)=R^{II}(x,\ Q)$. As compared with the profit function under the zero shortage cost and salvage value, the profit function during the profitable discount stages, i.e., $k=1,\ \cdots,\ r-1$, does not change even if positive shortage cost and salvage value are assumed, whereas the profit function during the unprofitable discount stages, i.e., $k=r,\ r+1,\ \cdots,\ n$, changes due to positive salvage value. For $k=1,\ 2,\ \cdots,\ n$, simplifying the equation for $R_k^{II}(x,\ Q)$ leads to

$$R_k^{II}(x,\ Q)=\begin{cases}\alpha_k Q+(W_{k-1}-\alpha_k V_{k-1})x, & k=1,\ \cdots,\ r-1\\ (W_{r-1}-S_{k-1}+\beta_k V_{k-1}-gV_{r-1})x & \\ -(\beta_k-g)Q, & k=r,\ r+1,\ \cdots,\ n\end{cases}$$

(6)

From the above settings, the expected costs and profits for a specific order quantity Q under no-discounts and multiple discounts over all discount periods are expressed as follows :

$$EC^{I}(Q) = (\alpha_0 + s)\int_{Q}^{\infty}(x-Q)f(x)dx \tag{7}$$
$$+(b-g)\int_{0}^{Q}(Q-x)f(x)dx$$
$$= \alpha_0 E(X) - (a_0 - g)E_Q(X) + s\left[E(X) - E_Q(X)\right]$$
$$-(\alpha_0 + s)Q + (a_0 - g + s)QF(Q).$$

$$ER^{I}(Q) = (\alpha_0 + s)\int_{Q}^{\infty}Qf(x)dx - s\int_{Q}^{\infty}xf(x)dx \tag{8}$$
$$+(a_0 - g)\int_{0}^{Q}xf(x)dx - (b-g)\int_{0}^{Q}Qf(x)dx$$
$$= (a_0 - g)E_Q(X) - s\left[E(X) - E_Q(X)\right]$$
$$+(\alpha_0 + s)Q - (a_0 - g + s)QF(Q).$$

$$EC^{II}(Q) = \sum_{k=0}^{r-1}\left[W_{r-1} + sV_{r-1} + \sum_{i=0}^{k}(\alpha_k - \alpha_i)t_i\right] \tag{9}$$
$$\times\left[E_{h_{k-1}}(X) - E_{h_k}(X)\right]$$
$$-Q\left(\sum_{k=0}^{r-1}(\alpha_k + s)[F(h_{k-1}) - F(h_k)]\right)$$
$$+\sum_{k=r}^{n}(S_{k-1} - \beta_k V_{k-1} + gV_{r-1})[E_{h_{k-1}}(X) - E_{h_k}(X)]$$
$$+Q\left(\sum_{k=r}^{n}(\beta_k - g)[F(h_{k-1}) - F(h_k)]\right).$$

$$ER^{II}(Q) = (\alpha_0 + s)Q[1 - F(Q)] - s[E(X) - E_Q(X)] \tag{10}$$
$$+\sum_{k=1}^{r-1}(W_{k-1} - \alpha_k V_{k-1})[E_{h_{k-1}}(X) - E_{h_k}(X)]$$
$$+Q\left(\sum_{k=1}^{r-1}\alpha_k\left[F(h_{k-1}) - F(h_k)\right]\right)$$
$$+\sum_{k=r}^{n}(W_{r-1} - S_{k-1} + \beta_k V_{k-1} - gV_{r-1})[E_{h_{k-1}}(X)$$
$$-E_{h_k}(X)] - Q\left(\sum_{k=r}^{n}(\beta_k - g)[F(h_{k-1}) - F(h_k)]\right).$$

2.2 Won's Conjecture

The conjecture raised in Won [11] is that the expected profit under progressive multiple discounts are consistently greater than or equal to the one under no-discounts for *every* order quantity under the assumption of zero shortage cost and salvage value as far as her multiple discounts do not decrease customer demand, that is, the following inequality holds for every order quantity Q :

$$ER^{II}(Q) \geq ER^{I}(Q).$$

The above conjecture justifies the newsvendor's implementing progressive multiple discounts to customers in the sense that the newsvendor can improve the expected profit under multiple discounts even if she maintains her non-optimal order quantity under no-discounts. In this note, however, in addition to providing a rigorous proof of the above conjecture under the assumption of zero shortage cost and salvage value, we show that the conjecture still holds even if positive shortage cost and salvage value are assumed.

2.3 Derivative-Free Approach for the Proof

Khouja [3] found that $ER^{II}(Q_{II}^{*}) \geq ER^{I}(Q_{I}^{*})$ for optimal order quantities Q_{I}^{*} and Q_{II}^{*} of the newsvendor problems under no-discounts and multiple discounts, respectively. Therefore, Won's statement is much stronger than Khouja's statement. However, as can be seen in equations (7) through (10), direct approach attempting to prove the conjecture by comparing $ER^{I}(Q)$ with $ER^{II}(Q)$ for every order quantity Q seems to be not so promising due to their complicated formulas. The most popular measure for the optimal order quantity of the newsvendor model is the critical fractile, which is the ratio based on the unit underestimating cost of demand and the unit overestimating cost of demand, and the critical fractile can be found by using derivatives. However, in this note we do not adopt the critical fractile in the course of proof because we attempt to prove the consistent advantage of progressive multiple discounts over no-discounts for *every* order quantity rather than the optimal order quantity alone. These are main reasons why we do not adopt derivative-based approach in order to prove Won's conjecture.

3. Proof of the Consistent Advantage of Progressive Multiple Discounts

3.1 Riskless Profit

First, we provide an easy alternative proof of the consistent advantage of progressive multiple discounts over no-discounts in terms of the riskless profit. From equations (1) and (2), the sum of retailer cost and profit for order quantity Q under demand x in the classical newsvendor problem under no-discounts is given by

$$C^I(x,\ Q)+R^I(x,\ Q)=\alpha_0 x=W_0 x. \tag{11}$$

The expected value of the sum of retailer cost and profit for order quantity Q , called the riskless profit [6] or the maximum profit [1], for the newsvendor problem under no-discounts is given by

$$\begin{aligned} ES^I(Q) &= EC^I(Q)+ER^I(Q) \\ &= \alpha_0 E(X)=W_0 E(X). \end{aligned} \tag{12}$$

From equations (3) and (5), for $k=0$, we have

$$CU_0^{II}(x,\ Q)+R_0^{II}(x,\ Q)=\alpha_0 x=W_0 x, \tag{13}$$

for $k=1, \cdots, r-1$,

$$CU_k^{II}(x,\ Q)+R_k^{II}(x,\ Q)=W_{r-1}x+s(V_{r-1}x-Q), \tag{14}$$

and for $k=r, r+1, \cdots, n$,

$$CU_k^{II}(x,\ Q)+R_k^{II}(x,\ Q)=W_{r-1}x. \tag{15}$$

Instead of attempting to prove the consistent advantage of progressive multiple discounts over no-discounts by directly dealing the expected values of both problems in terms of the riskless profit, we attempt to prove it by closer examination of the profits and costs of both problems for any combination of x and Q.

Obviously, for $k=0$, $C^I(x,\ Q)+R^I(x,\ Q)=C^{II}(x,\ Q)+R^{II}(x,\ Q)$. For $k=1, \cdots, r-1$, we have

$$\begin{aligned} CO_0^{II}(x,\ Q)+R_0^{II}(x,\ Q) &= W_{r-1}x+s(V_{r-1}x-Q) \\ &\geq W_{r-1}x \geq W_0 x. \end{aligned} \tag{16}$$

The first and second inequalities immediately follow from the condition that $Q/V_{r-1} \leq x < Q/V_{r-2}$ for $k=r-1$ and the definition of W_{r-1}.

For $k=r, r+1, \cdots, n$, we have

$$CO_k^{II}(x,\ Q)+R_k^{II}(x,\ Q)=W_{r-1}x \geq W_0 x.$$

Thus, we have proved that $ES^{II}(Q) \geq ES^I(Q)$, that is, the riskless profit under progressive multiple discounts is consistently greater than or equal to the one under no-discounts for every order quantity Q as far as $t_k \geq 0$, $k=0, 1, \cdots, n$. ■

From equation (16), an interesting finding can be addressed with regard to the equivalence of the expected cost minimization approach and the expected profit maximization approach for the newsvendor problem under progressive multiple discounts. It is well known that in the standard newsvendor problem the expected cost minimization approach yields the same optimal order quantity as the expected profit maximization approach. From equation (16), this statement also applies to the newsvendor problem under progressive multiple discounts with zero shortage cost and salvage value because the riskless profit given as $W_{r-1}x$ under zero shortage cost and salvage value is constant for every order quantity Q. But the statement does not apply to the newsvendor problem under progressive multiple discounts with positive shortage cost and salvage value because the riskless profit varies as the order quantity Q varies. This implies that if positive shortage cost and salvage value are considered the newsvendor seeking to maximize the expected profit under progressive multiple discounts may find the optimal order quantity which is different from the optimal order quantity that the newsvendor seeking to minimize the expected cost finds.

3.2 Expected Profit

Our derivative-free proof technique for showing the consistent advantage of the riskless profit under progressive multiple discounts over the one under no-discounts for every order quantity Q immediately motivates proof of the consistent advantage of the expected profit under progressive multiple discounts over the one under no-discounts for every order quantity Q since the information with which the newsvendor is concerned actually is the expected profit rather than the riskless profit.

Since $R_0^{II}(x, Q) = R^I(x, Q)$ for any combination of x and Q such that $x \geq Q$ and, therefore, it suffices to show that $R_k^{II}(x, Q) \geq R^I(x, Q)$ for any combination of x and Q such that $x < Q$, i.e., $k = 1, 2, \cdots, n$.

For $k = 1, 2, \cdots, r-1$, we have

$$\begin{aligned}
R_k^{II}(x, Q) - R^I(x, Q) &= (W_{k-1} - a_0 + g)x \\
&\quad + \alpha_k(Q - V_{k-1}x) + (b-g)Q \\
&\geq (W_{k-1} - a_0 + g)x + (b-g)Q \\
&\geq (W_{k-1} - a_0 + g)x + (b-g)V_{k-1}x \\
&= (W_{k-1} - a_0 + g + bV_{k-1} - gV_{k-1})x.
\end{aligned}$$

The first and second inequalities hold because the smallest Q is at $V_{k-1}x$ from the condition that $Q/V_k \leq x < Q/V_{k-1}$ for $k = 1, 2, \cdots, r-1$ and $b > g$. The conventional assumption that $b > g$ is generally accepted because the unit salvage value of leftover inventory at the end of the sales period is quite low [8]. Because $t_i \geq 0$ for every i, the term in the parenthesis of the last equality is

$$\begin{aligned}
W_{k-1} - a_0 + bV_{k-1} + g - gV_{k-1} &= \sum_{i=0}^{k-1} \alpha_i t_i - a_0 \\
&\quad + b\sum_{i=0}^{k-1} t_i - g\sum_{i=1}^{k-1} t_i = \sum_{i=0}^{k-1}(\alpha_i + b)t_i - a_0 - g\sum_{i=1}^{k-1} t_i \\
&= \sum_{i=0}^{k-1} a_i t_i - a_0 - g\sum_{i=1}^{k-1} t_i = \sum_{i=1}^{k-1} a_i t_i - g\sum_{i=1}^{k-1} t_i \\
&= \sum_{i=1}^{k-1}(a_i - g)t_i \geq 0.
\end{aligned}$$

The last inequality follows from the condition that $a_i > b > g$ for $k = 1, 2, \cdots, r-1$. Therefore, $R_k^{II}(x, Q) \geq R^I(x, Q)$ for $k = 1, 2, \cdots, r-1$.

For $k = r, r+1, \cdots, n$, we have

$$\begin{aligned}
&R_k^{II}(x, Q) - R^I(x, Q) = [W_{r-1} - S_{k-1} + \beta_k V_{k-1} \\
&\quad - gV_{r-1} - a_0 + g]x + (b - \beta_k)Q \\
&= \left[W_{r-1} - S_{k-1} + \beta_k V_{k-1} - g\sum_{i=0}^{r-1} t_i - a_0 + g\right]x + a_k Q \\
&\geq \left[W_{r-1} - S_{k-1} + \beta_k V_{k-1} - g\sum_{i=0}^{r-1} t_i - a_0 + g + a_k V_{k-1}\right]x
\end{aligned}$$

The last inequality follows since the smallest Q is at $V_{k-1}x$ from the condition that $Q/V_k < x <$

Q/V_{k-1} for $k=r, r+1, \cdots, n$. Here, the term in the parenthesis of the last inequality is

$$\begin{aligned} &W_{r-1}-S_{k-1}+\beta_k V_{k-1}-g\sum_{i=0}^{r-1}t_i-a_0+g+a_k V_{k-1} \\ &=\sum_{i=0}^{r-1}(a_i-b)t_i-\sum_{i=r}^{k-1}(b-a_i)t_i \\ &+(b-a_k)\sum_{i=0}^{k-1}t_i-g\sum_{i=1}^{r-1}t_i-a_0+a_k\sum_{i=0}^{k-1}t_i \\ &=\sum_{i=0}^{r-1}a_i t_i-\sum_{i=0}^{r-1}bt_i-\sum_{i=r}^{k-1}bt_i+\sum_{i=r}^{k-1}a_i t_i+\sum_{i=0}^{k-1}bt_i \\ &-g\sum_{i=1}^{r-1}t_i-a_0=\sum_{i=1}^{r-1}(a_i-g)t_i+\sum_{i=r}^{k-1}a_i t_i \geq 0 \end{aligned}$$

Therefore, $R_k^{II}(x, Q) \geq R^I(x, Q)$ for $k=r, r+1, \cdots, n$. Because $R_k^{II}(x, Q) \geq R^I(x, Q)$ for all k's and any combination of x and Q, we thus have proved that $ER^{II}(Q) \geq ER^I(Q)$ for any order quantity Q as far as $t_k \geq 0, k=0, 1, \cdots, n$. ■

It follows immediately that the optimal expected profit under progressive multiple discounts is greater than or equal to the one under no-discounts. However, we cannot conclude that the consistent relationship between the expected profits also holds between the expected costs for every order quantity Q, i.e., $EC^{II}(Q) \leq EC^I(Q)$.

It is interesting to notice from the above proof that the lower bound for the gap between $R_k^{II}(x, Q)$ and $R^I(x, Q)$ when considering positive shortage cost and salvage value is less than the one for the gap between $R_k^{II}(x, Q)$ and $R^I(x, Q)$ when considering zero shortage cost and salvage value as far as $t_k \geq 0, k=0, 1, \cdots, n$.

4. Concluding Remarks

In this note, we have provided a complete proof of the consistent advantage of progressive multiple discounts over no-discounts in terms of both the riskless profit and the expected profit in the newsvendor problem. Our proof proceeds by closer inspection of the profits and costs of both problems without using derivatives. Previous newsvendor model under progressive multiple discounts is extended with positive shortage cost and salvage value. Because our proof shows the consistent advantage of progressive multiple discounts for every order quantity as well as the optimal order quantity as far as her multiple discounts do not decrease customer demand, the newsvendor can safely implement multiple discounts for customers even if she may not order the optimal quantity due to the realistic restriction such as budget or warehouse capacity limit as far as her multiple discounts do not decrease customer demand. As by-products from the proof of the conjecture, we have found :

- In the newsvendor model under multiple discounts with zero shortage cost and salvage value, the expected cost minimization approach yields the same optimal order quantity as the expected profit maximization approach since the riskless profit is constant for every order quantity.
- But in the generalized newsvendor model under multiple discounts extended with positive shortage cost and salvage value, the expected cost minimization approach may not yield the same optimal order quantity as the expected profit maximization approach since the riskless profit is not constant for every order quantity. Such a property of the newsvendor problem under progressive multiple discounts with positive shortage cost has never been addressed in

previous researches.

- The lower bound for the gap between the expected profit of the newsvendor model under progressive multiple discounts and the one of the newsvendor model under no-discounts grows less as compared with the case of zero shortage cost and salvage value if positive shortage cost and salvage value are considered.

However, one question still remains unanswered rigorously. Does using progressive multiple discounts yield always larger optimal order quantity than using no-discounts even if positive shortage cost and salvage value are considered? Providing the answer to this question is another future work to be done.

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