INDEFINITE TRANS-SASAKIAN MANIFOLD ADMITTING AN ASCREEN LIGHTLIKE HYPERSURFACE

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Abstract. We study indefinite trans-Sasakian manifold $\bar{M}$ admitting an ascreen lightlike hypersurface M. Our main results are several classification theorems of such an indefinite trans-Sasakian manifold.

1. Introduction

Oubina [14] introduced the notion of an indefinite trans-Sasakian manifold, of type (α, β). Indefinite Sasakian manifold is an important kind of indefinite trans-Sasakian manifold with $\alpha = 1$ and $\beta = 0$. Indefinite cosymplectic manifold is another kind of indefinite trans-Sasakian manifold such that $\alpha = \beta = 0$. Indefinite Kenmotsu manifold is also an example with $\alpha = 0$ and $\beta = 1$.

Alegre, Blair and Carriazo [1] introduced generalized Sasakian space form $\bar{M}(f_1, f_2, f_3)$. Indefinite Sasakian space form, indefinite Kenmotsu space form and indefinite cosymplectic space form are important kinds of indefinite generalized Sasakian space forms such that

$$f_1 = \tfrac{c+3}{4}, f_2 = f_3 = \tfrac{c-1}{4}; \ f_1 = \tfrac{c-3}{4}, f_2 = f_3 = \tfrac{c+1}{4}; \ f_1 = f_2 = f_3 = \tfrac{c}{4},$$

where c is a constant J-sectional curvature of respective space forms.

The theory of lightlike submanifolds is one of the most important topics of differential geometry. The study of such notion was initiated by Duggal and Bejancu [2] and later studied by many authors (see recent results in two books [3, 6]). Recently many authors have studied lightlike submanifolds M of indefinite almost contact metric manifolds $\bar{M}$ ([4]~[13]). The authors in above papers principally assumed that the

structure vector fields of $\bar{M}$ is tangent to M, which are called *tangential lightlike submanifolds*. There are several different types of non-tangential lightlike hypersurface of an indefinite trans-Sasakian manifold $\bar{M}$ according to the form of the structure vector field of $\bar{M}$. We study a type of them here, named by *ascreen lightlike hypersurfaces*.

In this paper, we study indefinite trans-Sasakian manifold $\bar{M}$ admitting an ascreen lightlike hypersurface M. The main results are several classification theorems of such an indefinite trans-Sasakian manifold.

2. Lightlike hypersurfaces

Let M be a lightlike hypersurface of a semi-Riemannian manifold $\bar{M}$. Then the normal bundle $TM^{\perp}$ of M is a vector subbundle of the tangent bundle TM, of rank 1. Therefore there exists a non-degenerate complementary vector bundle $S(TM)$ of $TM^{\perp}$ in TM, which is called a *screen distribution* on M, such that

$$(2.1) \qquad TM = TM^{\perp} \oplus_{orth} S(TM),$$

where $\oplus_{orth}$ denotes the orthogonal direct sum. We denote such a lightlike hypersurface by $M = (M, g, S(TM))$. Denote by $F(M)$ the algebra of smooth functions on M and by $\Gamma(E)$ the $F(M)$ module of smooth sections of a vector bundle E over M. For any null section ξ of $TM^{\perp}$ on a coordinate neighborhood $\mathcal{U} \subset M$, there exists a unique null section N of a unique vector bundle $tr(TM)$ of rank 1 in $S(TM)^{\perp}$ satisfying

$$\bar{g}(\xi, N) = 1, \quad \bar{g}(N, N) = \bar{g}(N, X) = 0, \ \forall X \in \Gamma(S(TM)).$$

In this case, the tangent bundle $T\bar{M}$ of $\bar{M}$ is decomposed as follow:

$$(2.2) \qquad T\bar{M} = TM \oplus tr(TM) = \{TM^{\perp} \oplus tr(TM)\} \oplus_{orth} S(TM).$$

We call $tr(TM)$ and N the *transversal vector bundle* and the *null transversal vector field* of M with respect to $S(TM)$ respectively.

Let $\bar{\nabla}$ be the Levi-Civita connection of $\bar{M}$ and P the projection morphism of TM on $S(TM)$ with respect to the decomposition (2.1). Then the local Gauss and Weingartan formulas M and $S(TM)$ are given by

$$(2.3) \qquad \bar{\nabla}_X Y = \nabla_X Y + B(X, Y)N,$$
$$(2.4) \qquad \bar{\nabla}_X N = -A_N X + \tau(X)N,$$
$$(2.5) \qquad \nabla_X PY = \nabla_X^* PY + C(X, PY)\xi,$$
$$(2.6) \qquad \nabla_X \xi = -A_\xi^* X - \tau(X)\xi,$$

for all $X, Y \in \Gamma(TM)$, where ∇ and ∇^* are the liner connections on M and $S(TM)$ respectively, B and C are the local second fundamental forms on M and $S(TM)$ respectively, A_N and A_ξ^* are the shape operators on M and $S(TM)$ respectively and τ is a 1-form on TM.

Since $\bar{\nabla}$ is torsion-free, ∇ is also torsion-free and B is symmetric on TM. From the fact that $B(X, Y) = \bar{g}(\bar{\nabla}_X Y, \xi)$, we show that B is independent of the choice of $S(TM)$ and satisfies

$$(2.7) \qquad B(X, \xi) = 0, \quad \forall X \in \Gamma(TM).$$

The above two local second fundamental forms B and C are related to their shape operators by

$$(2.8) \qquad B(X, Y) = g(A_\xi^* X, Y), \qquad \bar{g}(A_\xi^* X, N) = 0,$$

$$(2.9) \qquad C(X, PY) = g(A_N X, PY), \quad \bar{g}(A_N X, N) = 0.$$

From (2.8), A_ξ^* is $S(TM)$-valued self-adjoint on TM such that

$$(2.10) \qquad\qquad A_\xi^* \xi = 0.$$

The induced connection ∇ of M is not a metric one and satisfies

$$(2.11) \qquad (\nabla_X g)(Y, Z) = B(X, Y)\, \eta(Z) + B(X, Z)\, \eta(Y),$$

for any $X, Y, Z \in \Gamma(TM)$, where η is a 1-form such that

$$\eta(X) = \bar{g}(X, N), \quad \forall X \in \Gamma(TM).$$

But the connection ∇^* on $S(TM)$ is a metric one.

3. Indefinite trans-Sasakian manifolds

An odd-dimensional semi-Riemannian manifold $(\bar{M}, \bar{g})$ is called an *indefinite almost contact metric manifold* ([4] ∼ [14]) if there exist a $(1, 1)$-type tensor field J, a vector field ζ which is called the *structure vector field* of $\bar{M}$ and a 1-form θ such that

$$(3.1) \qquad J^2 X = -X + \theta(X)\zeta, \qquad \theta(\zeta) = 1,$$
$$\bar{g}(JX,\, JY) = \bar{g}(X,\, Y) - \epsilon\theta(X)\theta(Y),$$

for any vector fields X and Y on $\bar{M}$, where $\epsilon = 1$ or -1 according as ζ is spacelike or timelike respectively. The set $\{J, \zeta, \theta, \bar{g}\}$ is called an *indefinite almost contact metric structure* of $\bar{M}$.

In an indefinite almost contact metric manifold, we show that $J\zeta = 0$ and $\theta \circ J = 0$. Such a manifold is said to be an *indefinite contact metric manifold* if $d\theta = \Phi$, where $\Phi(X, Y) = \bar{g}(X, JY)$ is called the *fundamental 2-form* of $\bar{M}$.

The indefinite almost contact metric structure of $\bar{M}$ is said to be *normal* if $[J, J](X, Y) = -2d\theta(X, Y)\zeta$ for any vector fields X and Y on $\bar{M}$, where $[J, J]$ denotes the Nijenhuis tensor field of J given by

$$[J, J](X, Y) = J^2[X, Y] + [JX, JY] - J[JX, Y] - J[X, JY].$$

An indefinite normal contact metric manifold is called an *indefinite Sasakian manifold*. It is well known [6] that an indefinite almost contact metric manifold $(\bar{M}, \bar{g}, J, \zeta, \theta)$ is indefinite Sasakian if and only if

$$(\bar{\nabla}_X J)Y = \bar{g}(X, Y)\zeta - \epsilon\theta(Y)X.$$

Definition 1. An indefinite almost contact metric manifold $(\bar{M}, \bar{g})$ is called an *indefinite trans-Sasakian manifold* [14] if there exist two smooth functions α and β such that

(3.2) $(\bar{\nabla}_X J)Y = \alpha\{\bar{g}(X, Y)\zeta - \epsilon\,\theta(Y)X\} + \beta\{\bar{g}(JX, Y)\zeta - \epsilon\theta(Y)JX\},$

for any vector fields X and Y on $\bar{M}$, where $\bar{\nabla}$ is the Levi-Civita connection of $\bar{M}$ with respect to the semi-Riemannian metric $\bar{g}$. We say that $\{J, \zeta, \theta, \bar{g}\}$ is an *indefinite trans-Sasakian structure of type* (α, β).

By replacing Y by ζ in (3.2), we get

(3.3) $\bar{\nabla}_X\zeta = -\epsilon\alpha JX + \epsilon\beta(X - \theta(X)\zeta).$

Remark 1. If $\beta = 0$, then $\bar{M}$ is called an *indefinite α-Sasakian manifold*. Indefinite Sasakian manifolds [4, 5, 7, 10] appear as examples of indefinite α-Sasakian manifolds, with $\alpha = 1$. Another important kind of indefinite trans-Sasakian manifold is that of indefinite cosymplectic manifolds [9, 13] obtained for $\alpha = \beta = 0$. If $\alpha = 0$, then $\bar{M}$ is called an *indefinite β-Kenmotsu manifold*. Indefinite Kenmotsu manifolds [8, 11] are particular examples of indefinite β-Kenmotsu manifold, with $\beta = 1$.

It is well-known ([7] $\sim$ [10]) that, for any lightlike hypersurface M of an indefinite almost contact metric manifold $\bar{M}$, $J(TM^\perp)$ and $J(tr(TM))$ are subbundles of $S(TM)$, of rank 1. In the sequel, we shall assume that ζ is a unit spacelike vector field without loss of generality, *i.e.*, $\epsilon = 1$. Let a and b be the smooth functions given by $a = \theta(N)$ and $b = \theta(\xi)$.

Definition 2. A lightlike hypersurface M of an indefinite almost contact metric manifold $\bar{M}$ is called an *ascreen lightlike hypersurface* [9, 10, 12] if the structure vector field ζ belongs to $S(TM)^\perp = TM^\perp \oplus tr(TM)$.

Example 3.1. Define a hypersurface M of $\bar{M} = (R_2^3, J, \zeta, \theta, \bar{g})$ by

$$X(x,\, y) = (x,\, y,\, \frac{1}{\sqrt{2}}(x + y)).$$

By direct calculations we easily check that

$$TM = \text{Span}\{\xi = \partial x + \partial y + \sqrt{2}\partial z,\ U = \partial x - \partial y\},$$
$$Rad(TM) = \text{Span}\{\xi\}, \qquad S(TM) = \text{Span}\{U\},$$
$$tr(TM) = \text{Span}\{N = \frac{1}{4}(-\partial x - \partial y + \sqrt{2}\partial z)\}.$$

From these equations, we show that $J\xi = U$, $Rad(TM) \cap J(Rad(TM)) = \{0\}$, $JN = -\frac{1}{4}U$, $JN = -\frac{1}{4}J\xi$, $J(Rad(TM)) = J(tr(TM))$ and $J\zeta = 0$. As $\zeta = \frac{1}{2\sqrt{2}}\xi + \sqrt{2}N$, M is an ascreen lightlike hypersurface of an indefinite trans-Sasakian manifold $\bar{M}$.

In case M is ascreen lightlike hypersurface of $\bar{M}$, ζ is decomposed by

$$(3.4) \qquad\qquad\qquad \zeta = a\xi + bN.$$

As $\bar{g}(\zeta, \zeta) = 1$, we have $2ab = 1$. Thus $a \neq 0$ and $b \neq 0$. Consider a local unit timelike vector field V on $S(TM)$ and its 1-form v defined by

$$(3.5) \qquad V = -b^{-1}J\xi, \quad v(X) = -g(X, V), \quad \forall X \in \Gamma(TM).$$

Applying J to (3.4) and using (3.5) and the fact $J\zeta = 0$, we have

$$(3.6) \qquad\qquad\qquad JN = aV.$$

Applying J to $(3.5)_1$ and using (3.1), (3.4) and the fact $2ab = 1$, we have

$$(3.7) \qquad\qquad\qquad JV = a\xi - bN.$$

As $JN = -\frac{a}{b}J\xi$, we show that $J(TM^\perp) = J(tr(TM))$. From the fact $J(TM^\perp)$ is vector subbundle of $S(TM)$, there exists a non-degenerate almost complex distribution D with respect to J such that

$$TM = TM^\perp \oplus_{orth} \{J(TM^\perp) \oplus_{orth} D\}.$$

Denote by Q the projection morphism of TM on D. Any vector field X on M is expressed as $X = QX + v(X)V + \eta(X)\xi$. Applying J to this representation of X and using $(3.5)_1$ and (3.7), we obtain

$$(3.8) \qquad JX = fX - \theta(X)V + av(X)\xi - bv(X)N,$$

where f is a tensor field of type $(1,1)$ defined on M by $f = J \circ Q$. Applying J to (3.8) and using (3.1) and (3.4) $\sim$ (3.7), we have

$$(3.9) \qquad f^2 X = -X + v(X)V + \eta(X)\xi = -QX.$$

Applying $\bar{\nabla}_X$ to (3.4) and using $(2.3) \sim (2.7)$, (3.3) and (3.8), we have

(3.10) $aA_\xi^* X + bA_N X = \alpha fX - \beta QX - \{\alpha\theta(X) + \beta v(X)\}V,$

(3.11) $Xb + b\tau(X) = b\{\alpha v(X) - \beta\theta(X)\},$

(3.12) $Xa - a\tau(X) = -a\{\alpha v(X) - \beta\theta(X)\}.$

Applying f to (3.10) and using (3.9) and the fact $fV = 0$, we have

(3.13) $af(A_\xi^* X) + bf(A_N X) + \alpha QX + \beta fX = 0.$

Applying $\bar{\nabla}_X$ to $bV = -J\xi$ and using (2.3), (2.6), (2.7), (3.1), (3.2), $(3.4) \sim (3.9)$, (3.8) and (3.13), we get

(3.14) $\nabla_X^* V = af(A_\xi^* X) - bf(A_N X), \quad \forall X \in \Gamma(TM).$

Definition 3. A lightlike hypersurface M of $\bar{M}$ is said to be

(a) *totally umbilical*[2] if there is a smooth function ρ on any coordinate neighborhood $\mathcal{U}$ in M such that $A_\xi^* X = \rho PX$, or equivalently,

$$B(X, Y) = \rho g(X, Y), \quad \forall X, Y \in \Gamma(TM).$$

In case $\rho = 0$ on $\mathcal{U}$, we say that M is *totally geodesic.*

(b) *screen totally umbilical* [2] if there exist a smooth function γ on $\mathcal{U}$ such that $A_N X = \gamma PX$, or equivalently,

$$C(X, PY) = \gamma g(X, Y), \quad \forall X, Y \in \Gamma(TM).$$

(c) *screen conformal* [3] if there exist a non-vanishing smooth function φ on $\mathcal{U}$ such that $A_N = \varphi A_\xi^*$, or equivalently,

$$C(X, PY) = \varphi B(X, Y), \quad \forall X, Y \in \Gamma(TM).$$

Proposition 3.1. *Let M be an ascreen lightlike hypersurface of an indefinite trans-Sasakian manifold $\bar{M}$. The function α satisfies $\alpha = 1$. Furthermore M is neither screen conformal nor screen totally umbilical.*

Proof. Taking the scalar product with V to (3.10), we have

(3.15) $aB(X, V) + bC(X, V) = \alpha\theta(X) + \beta v(X).$

Replacing X by ξ to this and using (2.7) and $b \neq 0$, we have $C(\xi, V) = \alpha$.
    Using $\eta(Y) = \bar{g}(Y, N)$ and (2.4), we obtain

(3.16) $2d\eta(X, Y) = g(X, A_N Y) - g(A_N X, Y) + \tau(X)\eta(Y) - \tau(Y)\eta(X),$

for all $X, Y \in \Gamma(TM)$. Also, using $\theta(Y) = b\eta(Y)$, we have

$$2d\theta(X, Y) = 2bd\eta(X, Y) + (Xb)\eta(Y) - (Yb)\eta(X),$$

for all $X, Y \in \Gamma(TM)$. Substituting (3.16) and the fact $d\theta(X, Y) = \bar{g}(X, JY)$ into the last equation and using (3.11), we get

$$(3.17) \quad 2\bar{g}(X, JY) = b\{g(X, A_N Y) - g(A_N X, Y)\}$$
$$+ b\{\alpha v(X) - \beta\theta(X)\}\eta(Y) - b\{\alpha v(Y) - \beta\theta(Y)\}\eta(X),$$

for all $X, Y \in \Gamma(TM)$. Taking $X = V$ and $Y = \xi$ to this and using the fact $g(A_N \xi, V) = C(\xi, V) = \alpha$, we obtain $b = b\alpha$. Taking the product with $2a$ to this result and using the fact that $2ab = 1$, we obtain $\alpha = 1$.

Assume that M is either screen conformal or screen totally umbilical. Then we have the following impossible results:

$$\alpha = C(\xi, V) = \varphi B(\xi, V) = 0 \quad \text{or} \quad \alpha = C(\xi, V) = \gamma g(\xi, V) = 0$$

Thus there exist no screen conformal or screen totally umbilical ascreen lightlike hypersurface of an indefinite trans-Sasakian manifold $\bar{M}$.

Corollary 1. *Any indefinite trans-Sasakian manifold $\bar{M}$ admitting an ascreen lightlike hypersurface is neither indefinite β-Kenmotsu manifold nor indefinite cosymplectic manifold.*

4. Indefinite generalized Sasakian space form

Definition 4. An indefinite almost contact metric manifold $\bar{M}$ is called an *indefinite generalized Sasakian space form* [1] and denote it by $\bar{M}(f_1, f_2, f_3)$ if there exist smooth functions f_1, f_2 and f_3 such that

$$(4.1) \quad \bar{R}(X, Y)Z = f_1\{\bar{g}(Y, Z)X - \bar{g}(X, Z)Y\}$$
$$+ f_2\{\bar{g}(X, JZ)JY - \bar{g}(Y, JZ)JX + 2\bar{g}(X, JY)JZ\}$$
$$+ f_3\{\theta(X)\theta(Z)Y - \theta(Y)\theta(Z)X$$
$$+ \bar{g}(X, Z)\theta(Y)\zeta - \bar{g}(Y, Z)\theta(X)\zeta\},$$

for any vector fields X, Y and Z on $\bar{M}$, where $\bar{R}$ is the curvature tensor of the Levi-Civita connection $\bar{\nabla}$ on $\bar{M}$.

Example 4.1. An *indefinite Sasakian space form*, *i.e.*, an indefinite Sasakian manifold with constant J-sectional curvature c, such that ζ is spacelike, is an indefinite generalized Sasakian space form with

$$f_1 = \frac{c+3}{4}, \qquad f_2 = f_3 = \frac{c-1}{4}.$$

Example 4.2. An *indefinite Kenmotsu space form*, *i.e.*, an indefinite Kenmotsu manifold with constant J-sectional curvature c, such that ζ is spacelike, is an indefinite generalized Sasakian space form with

$$f_1 = \frac{c-3}{4}, \qquad f_2 = f_3 = \frac{c+1}{4}.$$

Example 4.3. An *indefinite cosymplectic space form*, *i.e.*, an indefinite cosymplectic manifold with constant J-sectional curvature c, such that ζ is spacelike, is an indefinite generalized Sasakian space form with

$$f_1 = f_2 = f_3 = \tfrac{c}{4}.$$

Denote by R and R^* the curvature tensors of the connections ∇ and ∇^* respectively. Using the Gauss-Weingarten formulas $(2.3)\sim(2.6)$, we obtain the Gauss-Codazzi equations for M and $S(TM)$ such that

$$(4.2) \qquad \bar{g}(\bar{R}(X,Y)Z,\ PW) = g(R(X,Y)Z,\ PW)$$
$$+ B(X,Z)C(Y,PW) - B(Y,Z)C(X,PW),$$

$$(4.3) \qquad \bar{g}(\bar{R}(X,Y)Z,\ \xi) = (\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z)$$
$$+ B(Y,Z)\tau(X) - B(X,Z)\tau(Y),$$

$$(4.4) \qquad \bar{g}(\bar{R}(X,Y)Z,\ N) = g(R(X,Y)Z,\ N),$$

$$(4.5) \qquad g(R(X,Y)PZ,\ PW) = g(R^*(X,Y)PZ,\ PW)$$
$$+ C(X,PZ)B(Y,PW) - C(Y,PZ)B(X,PW),$$

$$(4.6) \qquad g(R(X,Y)PZ,\ N) = (\nabla_X C)(Y,PZ) - (\nabla_Y C)(X,PZ)$$
$$+ C(X,PZ)\tau(Y) - C(Y,PZ)\tau(X),$$

for any $X,\ Y,\ Z,\ W \in \Gamma(TM)$.

Proposition 4.1. *Any indefinite generalized Sasakian space form, with indefinite trans-Sasakian sutucture of type (α, β), admitting a totally geodesic ascreen lightlike hypersurface M satisfies*

$$\alpha = 1, \quad 2f_1 + 3f_2 - f_3 + 2\beta^2 - 2 + 4a(\xi\beta) = 0.$$

Proof. Assume that M is totally geodesic. Then, from (3.9), (3.13), (3.14), (3.15) and the fact $\alpha = 1$, for all $X \in \Gamma(TM)$, we have

$$(4.7) \qquad\qquad C(X,V) = 2a\{\theta(X) + \beta v(X)\},$$
$$(4.8) \qquad\qquad \nabla_X^* V = X - v(X)V - \eta(X)\xi + \beta f X.$$

Taking $X = \xi$ and $Y = V$ to (3.17) and using (3.5), we get

$$(4.9) \qquad\qquad g(V, A_N \xi) = 1.$$

for all $X \in \Gamma(TM)$. Substituting (4.1) and (4.6) into (4.4), we have

$$
\begin{aligned}
&f_1\{g(Y, PZ)\eta(X) - g(X, PZ)\eta(Y)\} \\
&+ af_2\{v(Y)\bar{g}(X, JPZ) - v(X)\bar{g}(Y, JPZ) + 2v(PZ)\bar{g}(X, JY)\} \\
&+ f_3\{\theta(X)\theta(PZ)\eta(Y) - \theta(Y)\theta(PZ)\eta(X) \\
&\qquad\quad + ag(X, PZ)\theta(Y) - ag(Y, PZ)\theta(X)\} \\
&= (\nabla_X C)(Y, PZ) - (\nabla_Y C)(X, PZ) \\
&\qquad\quad + C(X, PZ)\tau(Y) - C(Y, PZ)\tau(X).
\end{aligned}
$$

Replacing PZ by V to the last equation and using (4.7), we have

$$
\begin{aligned}
(4.10)\quad &(f_1 + abf_2 - abf_3)\{v(X)\eta(Y) - v(Y)\eta(X)\} + 2af_2 g(X, JY) \\
&= (\nabla_X C)(Y, V) - (\nabla_Y C)(X, V) + 2a\{\theta(X) + \beta v(X)\}\tau(Y) \\
&\quad - 2a\{\theta(Y) + \beta v(Y)\}\tau(X), \quad \forall X, Y \in \Gamma(TM).
\end{aligned}
$$

Applying $\bar{\nabla}_X$ to $v(Y) = -g(Y, V)$ and using (4.8), we have

$$
(4.11)\qquad (\nabla_X v)(Y) = -g(X, Y) - v(X)v(Y) - \beta g(fX, Y),
$$

for all $X, Y \in \Gamma(TM)$. Applying ∇_Y to (4.7) and using (4.11), we have

$$
\begin{aligned}
(\nabla_X C)(Y, V) &= 2\{Xa + av(X)\}\{\theta(Y) + \beta v(Y)\} \\
&\quad - g(A_N Y, X) - \beta C(Y, fX) \\
&\quad + 2a\{X(\theta(Y)) - \theta(\nabla_X Y) + (X\beta)v(Y) \\
&\qquad\quad - \beta[g(X, Y) + v(X)v(Y) + \beta g(fX, Y)]\}.
\end{aligned}
$$

Substituting this equation into (4.10) and using (3.12), we get

$$
\begin{aligned}
(4.12)\quad &(f_1 + abf_2 - abf_3)\{v(X)\eta(Y) - v(Y)\eta(X)\} + 2af_2 g(X, JY) \\
&= 2a\beta^2\{\theta(X)v(Y) - \theta(Y)v(X)\} + g(A_N X, Y) - g(X, A_N Y) \\
&\quad + \beta\{C(X, fY) - C(Y, fX)\} + 2a\{2\bar{g}(X, JY) + (X\beta)v(Y) \\
&\qquad\quad - (Y\beta)v(X) + \beta^2[g(X, fY) - g(Y, fX)]\}.
\end{aligned}
$$

Taking $X = V$ and $Y = \xi$ and using (4.9) and $2ab = 1$, we have

$$
(4.13)\qquad 2f_1 + 3f_2 - f_3 + 2\beta^2 - 2 + 4a(\xi\beta) = 0.
$$

Thus we have our Proposition.

Corollary 2. *Any indefinite Sasakian space form $\bar{M}(f_1, f_2, f_3)$ admitting a totally geodesic ascreen lightlike hypersurface M satisfies $c = 1$.*

Proof. Any indefinite Sasakian space form is an indefinite generalized Sasakian space form with $\alpha = 1$, $\beta = 0$ and $f_1 = \frac{c+3}{4}$, $f_2 = f_3 = \frac{c-1}{4}$. Substituting this result into (4.13), we obtain $c = 1$.

References

[1] P. Alegre, D.E. Blair and A. Carriazo, *Generalized Sasakian space form*, Israel J. Math. **141** (2004), 157-183.

[2] K.L. Duggal and A. Bejancu, *Lightlike Submanifolds of Semi-Riemannian Manifolds and Applications*, Kluwer Acad. Publishers, Dordrecht, 1996.

[3] K.L. Duggal and D.H. Jin, *Null curves and Hypersurfaces of Semi-Riemannian Manifolds*, World Scientific, 2007.

[4] K.L Duggal and B. Sahin, *Generalized Cauchy-Riemann lightlike Submanifolds of indefinite Sasakian manifolds*, Acta Math. Hungar. **112(1-2)** (2006), 113-136.

[5] K.L. Duggal and B. Sahin, *Lightlike Submanifolds of indefinite Sasakian manifolds*, Int. J. Math. Math. Sci. 2007, Art ID 57585, 1-21.

[6] K.L. Duggal and B. Sahin, *Differential geometry of lightlike submanifolds*, Frontiers in Mathematics, Birkhäuser, 2010.

[7] D.H. Jin, *Geometry of lightlike hypersurfaces of an indefinite Sasakian manifold*, Indian J. of Pure and Applied Math. **41(4)** (2010), 569-581.

[8] D.H. Jin, *The curvatures of lightlike hypersurfaces of an indefinite Kenmotsu manifold*, Balkan J. Geo. and Its Appl. **17(1)** (2012), 49-57.

[9] D.H. Jin, *Geometry of lightlike hypersurfaces of an indefinite cosymplectic manifold*, Commun. Korean Math. Soc. **27(1)** (2012), 185-195.

[10] D.H. Jin, *Ascreen lightlike hypersurfaces of an indefinite Sasakian manifold*, J. Korean Soc Math. Edu. Ser. B: Pure Appl. Math., **20(1)** (2013), 25-35.

[11] D.H. Jin, *Non-existence of screen homothetic half lightlike submanifolds of an indefinite Kenmotsu manifold*, Balkan J. Geo. and Its Appl. **18(1)** (2013), 22-30.

[12] D.H. Jin, *Ascreen lightlike hypersurfaces of a semi-Riemannian space form with a semi-symmetric non-metric connection*, accepted in Commun. Korean Math. Soc. 2013.

[13] D.H. Jin and J.W. Lee, *Generic lightlike submanifolds of an indefinite cosymplectic manifold*, Math. Prob. in Engineering, 2011, Art ID 610986, 1-16.

[14] J.A. Oubina, *New classes of almost contact metric structures*, Publ. Math. Debrecen **32** (1985), 187-193.

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