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Naïve Decode-and-Forward Relay Achieves Optimal DMT for Cooperative Underwater Communication

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Abstract

Diversity-multiplexing tradeoff (DMT) characterizes the fundamental relationship between the diversity gain in terms of outage probability and the multiplexing gain as the normalized rate parameter r, where the limiting transmission rate is given by $r\log\mathrm{SNR}$ (here, SNR denote the received signal-to-noise ratio). In this paper, we analyze the DMT and outage performance of an underwater network with a cooperative relay. Since over an acoustic channel, the propagation delay is commonly considerably higher than the processing delay, the existing transmission protocols need to be explained accordingly. For this underwater network, we briefly describe two well-known relay transmissions: decode-and-forward (DF) and amplify-and-forward (AF). As our main result, we then show that an instantaneous DF relay scheme achieves the same DMT curve as that of multiple-input single-output channels and thus guarantees the DMT optimality, while using an instantaneous AF relay leads at most only to the DMT for the direct transmission with no cooperation. To validate our analysis, computer simulations are performed in terms of outage probability.

Index Terms: Acoustic channel, Amplify-and-forward (AF), Decode-and-forward (DF), Diversity-multiplexing tradeoff (DMT), Underwater network

I. INTRODUCTION

Underwater networks have attracted considerable attention due to recent advances in acoustic communications technology [1, 2]. However, underwater acoustic channels normally have limited bandwidth and severe signal attenuation as well as very low propagation speed, which are the main features that distinguish underwater systems from wireless radio links.

In underwater networks, a natural way to partially overcome such difficulties and to further improve the performance is the use of cooperation between terminals. Cooperative relay techniques have the advantages of extending the coverage and enhancing the end-to-end quality in terms of capacity and reliability (e.g., [3-5] for terrestrial radio networks). In the case of underwater networks, it was shown that cooperation gains could be achieved via simple maximum ratio combining [6] or distributed spacetime block coding [7]. To support the practical implementation of such a cooperative framework, a sparse channel estimation method [8] and a receiver structure including various detectors [9] were introduced.

In a quasi-static channel environment in which the transmitters do not have perfect channel state information (CSI), a fundamental performance measure to evaluate

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various cooperative strategies is the diversity-multiplexing tradeoff (DMT), originally introduced by Zheng and Tse [10] for point-to-point multiple antenna systems. In the high signal-to-noise ratio (SNR) regime, they defined the diversity gain as the rate of decay of the error probability (or outage probability) and the multiplexing gain as the rate of increase in the transmission rate, with increasing SNR. This work has stimulated a number of research efforts to extend the optimal DMT for wireless radio networks with cooperation [5, 11]. For underwater systems, since only a small amount of CSI via delayed limited feedback may be available at the transmitters due to the low speed of sound in water (i.e., the slow propagation velocity), thus causing outages, characterizing the DMT is crucial in practice.

In this paper, we analyze the DMT and outage behavior for a three-terminal underwater network using an acoustic signal, where a single relay helps a source to better transmit its message to a destination. In the network, the construction of an optimal cooperative strategy in terms of DMT remains a challenge. Since the processing time, due to a variety of operations, at the relay node does not cause significant changes in the overall delay along the source-relay-destination path owing to the long propagation delay over an acoustic channel, the existing transmission protocols may operate in a fundamentally different manner from those in wireless radio channels and thus, need to be explained accordingly. For an underwater system, two relay transmissions, called decode-and-forward (CD) and amplify-and-forward (AF), are briefly described. Our results then indicate that a naïve instantaneous DF relay scheme achieves the same DMT curve as that of 2×1 multiple-input single-output (MISO) channels, thereby guaranteeing the DMT optimality. Meanwhile, the DMT achieved by an instantaneous AF relay is upper-bounded by that of a direct transmission with no cooperation. To validate our analysis, computer simulations are performed with respect to the outage probability for a fixed target rate.

The rest of this paper is organized as follows: Section II describes the system and channel models. The DMT curves for underwater systems are derived in Section III, and the numerical evaluation is discussed in Section IV. Finally, we summarize the paper with some concluding remarks in Section V.

Throughout this paper, the superscript H denotes the conjugate transpose of a vector. $E\{\cdot\}$ represents the expectation. $CN(m, \sigma^2)$ indicates the complex circular Gaussian with mean m and variance σ^2 per complex dimension. Unless otherwise stated, all logarithms are assumed to be to the base 2.

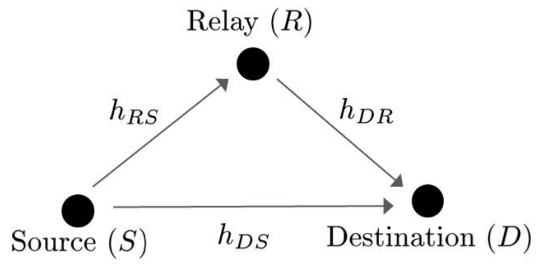

Fig. 1. System model of three-node network, including S, R, and D.

II. SYSTEM AND CHANNEL MODELS

Consider a three-terminal relay system [3], in which a source S aims to transmit its message to the corresponding destination D with the help of an intermediate relay R, as illustrated in Fig. 1.

Thus, there exists a direct link from S to D. It is assumed that R is located close to the direct transmission path. Each node has an average transmit power constraint P (constant). The relay node R is assumed to operate in the full duplex mode [3] and either to amplify what it receives (i.e., AF protocol) or to fully decode, re-encode, and retransmit the source message (i.e., DF protocol). As in [12], we consider slotted transmission protocols, where a cooperative block is composed of multiple time slots, each having a large number of symbols.

Now, let us turn to channel modeling. Due to the highly frequency-selective nature of underwater channels, multicarrier modulation (e.g., orthogonal frequency-division multiplexing) is an attractive choice for reduction in receiver complexity. For analytical convenience, coding is assumed to be performed over a subchannel in a slot experiencing relatively flat fading (through channel coding across all the subchannels, full frequency diversity can be utilized, resulting in a better outage performance, which remains for further work). In this work, we focus on a subcarrier under the assumption that the same relay technique is applied to every subcarrier.

As stated earlier, suppose that the processing delay, taking place due to a variety of operations (e.g., receiving and reading a packet), at the relay is negligible as compared to the propagation delay in water (the propagation speed of an acoustic signal in water is around 1,500 m/s [13], which is five orders of magnitude lower than that of a radiowave). This is because the processing delay is at most on the order of a few milliseconds, while the propagation delay can be of several seconds according to the distance between nodes. Such an assumption was similarly made in [14] only when the AF relay was used in the underwater system even if the AF protocol could not utilize the full spatial diversity, which will be specified in Section III-A. In this model, the symbol generated at R is immediately forwarded to D, instead of waiting until the next time slot. That is, no idle time is assumed at R. Then, when the relative propagation delay between the direct and the relay paths is only a multiple of the basic symbol duration (far less than the length of each slot) under our network topology, the signal sent from S and the signal forwarded by R can be regarded as two paths in the frequency domain at a certain time by allowing a sufficiently long guard interval between the symbols. That is, synchronous cooperative communications can be possible owing to the use of multi-carrier modulation (refer to [15] for the detailed description). Thus, unlike in the case of a wireless radio [5, 16], no additional time slot is required for cooperative transmission.

When the two instantaneous full-duplex relay schemes are used at a certain subcarrier (symbol), the output signals at the relay R and the destination D are given by

$$y_R = h_{RS}x_S + z_R, \tag{1}$$

and

$$y_D = h_{DR}x_R + h_{DS}x_S + z_D, \tag{2}$$

where y_R and y_D denote the signals received at R and D, respectively, x_S and x_R represent the transmitted symbols from S and R, respectively, and z_R and z_D refer to the independent and the identically distributed (i.i.d.) additive white Gaussian noises with variance N_0. Here, h_{RS}, h_{RD}, and h_{DS} denote the i.i.d. channel coefficients of the S-R, R-D, and S-D links, respectively, where all of them follow $CN(0, 1)$, i.e., Rayleigh fading (Note that Rician fading provides a good match for underwater acoustic channels [17]. However, since the high SNR outage behaviors of Rayleigh and Rician channels are shown to be identical [18], we simply consider Rayleigh fading in this work). Moreover, we assume the quasi-static channel model, in which the channel coefficients are constant over time during one block transmission and change to a new independent value for the next block. The CSI is assumed to be available at the receivers, but not at the transmitters.

For the AF transmission, the transmitted symbol at R is given by

$$x_R = gy_R, \tag{3}$$

where g represents the amplification factor and is given by [5]

$$g = \sqrt{\frac{P}{|h_{RS}|^2 P + N_0}}.$$

For DF transmission, the relay processes y_R by decoding an estimate of the symbol transmitted from S. The relay codebook is assumed to be independent of the source codebook. The relay R transmits the encoded symbol if it decodes the received signal successfully, i.e., the effective SNR $|h_{RS}|^2/N_0$ at R exceeds a predetermined threshold. Otherwise, x_R is set to 0, i.e., no transmission at R.

III. DMT ANALYSIS

In this section, the DMT curves for three-node underwater acoustic systems using the AF and DF protocols are analyzed after briefly reviewing DMT [10].

A. Overview of DMT

Let r and d denote the multiplexing and diversity gains, respectively. Then,

$$r = \lim_{\rho\to\infty} \frac{R_0(\rho)}{\log\rho}, \tag{4a}$$

and

$$d = \lim_{\rho\to\infty} \frac{\log P_e(\rho)}{\log\rho}, \tag{4b}$$

where $R_0(\rho)$ represents the target rate (b/s/Hz) for a given SNR $\rho = \frac{P}{N_0 W}$ and $P_e(\rho)$ denotes the error probability assuming the maximum likelihood decoding (To simplify notation, R_0 (ρ) will be written as R_0 if dropping ρ does not cause any confusion). Here, W represents the bandwidth. For the sake of simplicity, the notation $\doteq$ is used for representing the relation in (4b): particularly,

$$P_e(\rho) \doteq \rho^{-d}$$

is identical to (4b), and $\doteq$ is referred to as the exponential equality.

The optimal DMT curve represents the maximum diversity gain for a given multiplexing gain r and is given by $d^*(r)$. It was shown in [10] that the outage probability

$$P_{\text{out}}(R_0, \rho) = \Pr\{I < R_0\} \tag{5}$$

satisfies

$$P_{\text{out}}(R_0, \rho) \doteq \rho^{-d^*(r)},$$

where I denotes the maximum average mutual information between the input and the output, and the error probability $P_e(\rho)$ of an optimal DMT-achieving scheme also satisfies $P_e(\rho) \doteq \rho^{-d^*(r)}$ if the block length is sufficiently large.

B. Achievability

In this subsection, we show that the simple instantaneous DF relay scheme achieves an optimal DMT curve. An upper bound on the DMT based on an instantaneous AF relay is also derived for the sake of comparison. We start from the following lemma:

Lemma 1. *Let* $F(x;k)$ *denote the cumulative distribution function of a chi-squared random variable* x *with* k *degrees of freedom*. Then, it follows that

$$F(x;2) = 1 - e^{-x/2}, \tag{6}$$

and

$$F(x;4) = 1 - \tfrac{1}{2}(x+2)e^{-x/2}. \tag{7}$$

The proof of this lemma is presented in [19]. From Lemma 1, it can be easily concluded that $F(x;2) = O(x)$ and $F(x;4) = O(x^2)$ for small x— $f(x) = O(g(x))$ means that positive constants M and m exist such that $f(x) \le Mg(x)$ for all $x > m$. Now, we are ready to derive the achievable DMT curve for underwater acoustic systems by using the DF relay protocol.

Theorem 1. *Suppose that the instantaneous DF relay scheme is used in three-node underwater systems.* Then,

$$d^*(r) = 2(1-r) \tag{8}$$

is achievable.

Proof. If the relay R fully decodes the source message, i.e., $\log(1+\rho|h_{RS}|^2) \ge R_0$, the maximum average mutual information I of the DF protocol is given by

$$I = \log(1+\rho|h_{DS}|^2 + \rho|h_{DR}|^2),$$

which is the same as that of a 2×1 MISO system with the input covariance matrix

$$E = \left\{\begin{bmatrix} x_S \\ x_R \end{bmatrix}\begin{bmatrix} x_S \\ x_R \end{bmatrix}^H\right\} = P\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

under the quasi-static channel assumption [20]. If R fails to decode the symbol transmitted from S, i.e., there is an outage at R, then we have

$$I = \log(1+\rho|h_{DS}|^2),$$

which leads to the same performance as the direct transmission case with no cooperation. Since the two aforementioned events are mutually exclusive, the outage probability $P_{\text{out}}(R_0,\rho)$ in (5) becomes a sum

$$\Pr\{|h_{RS}|^2 \ge g(\rho)\}\Pr\{|h_{DS}|^2 + |h_{DR}|^2 < g(\rho)\}$$
$$+\Pr\{|h_{RS}|^2 < g(\rho)\}\Pr\{|h_{DS}|^2 \ge g(\rho)\},$$

where $g(\rho) = (2^{R_0}-1)/\rho$. Since $|h_{DS}|^2 + |h_{DR}|^2$ follows the chi-squared distribution with 4 degrees of freedom, the use of (6) and (7) yields

$$P_{\text{out}}(R_0,\rho) = e^{-\frac{g(\rho)}{2}}\left(1 - \frac{1}{2}(g(\rho)+2)e^{-\frac{g(\rho)}{2}}\right) + \left(1 - e^{-\frac{g(\rho)}{2}}\right)^2,$$

whose high SNR behavior is readily shown to be

$$\begin{aligned} P_{\text{out}}(R_0,\rho) &= O\left(e^{-g(\rho)}g^2(\rho) + g^2(\rho)\right) \\ &= O\left(\tfrac{(2^{R_0}-1)^2}{\rho^2}\right) \\ &\doteq \tfrac{\rho^{2r}}{\rho^2}, \end{aligned}$$

due to the fact that $g(\rho) \to 0$ as $\rho \to \infty$, where the exponential equality comes from (4a), thus resulting in (8). This completes the proof. □

Further, a DMT upper bound based on an AF relay can be found as follows:

Theorem 2. *Suppose that the instantaneous AF relay scheme is used in three-node underwater systems*. Then, the DMT curve is upper-bounded by

$$d^*(r) = 1 - r.$$

Proof. From a genie-aided removal of the noise z_R at the relay R, resulting in an upper bound on the performance, the output signal at the destination can be written from (1)–(3) as

$$y_D = (gh_{DR}h_{RS} + h_{DS})x_S + z_D.$$

Here, it is seen that $g(\rho) = 1/|h_{RS}|$ under the condition of noise removal, and thus, gh_{DR} is modeled as a random variable with uniform phases distributed over $[0,2\pi)$. Since the characteristics of the complex circular Gaussian distribution are invariant to the phase rotation, the term $gh_{DR}h_{RS}$, independent of h_{DS}, also follows $CN(0,1)$. Hence, the performance of the DMT is bounded by the transmission case with a direct link satisfying $CN(0,2)$, which completes the proof. □

On the basis of Theorems 1 and 2, we present the following interesting discussion regarding performance comparison.

Remark 1. To verify the optimality, we consider an upper bound on the DMT in three-node underwater systems by assuming a genie-aided perfect cooperation between S and R, which leads to 2×1 MISO channels. We conclude that since the 2×1 MISO DMT curve, given by $d^*(r) = 2(1-r)$ [10], exactly matches (8), the simple instantaneous DF protocol is DMT-optimal, whereas for three-node wireless communications systems, the construction of an optimal DMT-achieving scheme is still a challenge. On the other hand, the instantaneous AF protocol does not guarantee the optimality in underwater systems because it cannot exploit the full spatial diversity unlike the case of wireless radio systems [5, 11].

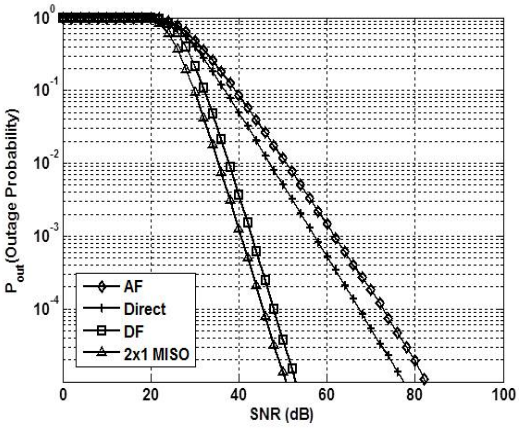

Fig. 2. Outage probabilities for the following four schemes: direct, amplify-and-forward (AF), decode-and-forward (DF), and 2 × 1 multiple-input single-output (MISO) transmissions, where $R_0 = 10$. SNR: signal-to-noise ratio.

IV. NUMERICAL EVALUATION

In this section, computer simulations are described to confirm our achievability results with respect to the outage performance. We compare the following four schemes: direct transmission with no relay, instantaneous

AF protocol, instantaneous DF protocol, and 2×1 MISO transmission. For $R_0 = 10$, that is, a fixed target rate, the simulated channels are generated 10^7 times for each scheme, and the outage probability $P_{\text{out}}(R_0, \rho)$ is evaluated. The results are shown in Fig. 2. As expected, in the case of a high SNR, the slopes, representing the maximum diversity gain, of the outage curves for DF and 2×1 MISO look identical, whereas there exists a certain SNR gap. It is also observed that the outage performance of the AF protocol is rather worse than that of direct transmission, in sharp contrast to the case of wireless radio systems.

V. CONCLUSION

The DMT and the outage probability for cooperative underwater acoustic systems have been analyzed in this study. It was shown that the use of the simple instantaneous DF protocol was indeed DMT-optimal. Meanwhile, an instantaneous AF relay was shown not to provide a better DMT performance than the direct transmission with no cooperation. As a result, vital information on how to design optimal cooperative strategies in underwater systems was provided in terms of the outage performance.

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