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Piaget's Theory in the Development of Creative Thinking[1]

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Piaget's revolutionary study on the cognitive development of children has focused on the development of logic. Logical operations and a variety of classifications based on the set of accepted rules involve convergent thinking. Children and adults have logical and creative thinking which deal with a reality of thinking. This study aims to examine a cognitive structure of students, which is closely related to the Piaget's cognitive development theories of students when creative thinking. Students were given an open mathematical problem and were expected to be able to take advantage of sensitivity, fluency, flexibility, originality, and elaboration which can be seen as clearly of their structure cognitive.

Keywords: Piaget' theory, creative thinking, convergent thinking, logical thinking
MESC Classification: C30
MSC2010 Classification: 97C30

1. INTRODUCTION

In recent years, various theories have emerged to explain and predict cognitive development in mathematics education. Authors identified two types of theories of cognitive growth are:

1) Global theory of long-term growth of the individual, such as Piaget's theory of stages (e.g., Piaget & Garcia, 1983).
2) The growth of local theories like the theory of conceptual action-process-object-schema Dubinsky (Czarnocha, Dubinsky, Prabhu & Vidakovic, 1999) or sequence-multi structural-uni structural abstract-relational model extended SOLO (Structure of the observed learning results, Biggs & Collis, 1982; 1991; Pegg, 2003).

[1] A draft version of the article was presented at the 2013 Joint International Conference on Mathematics Education held at Seoul National University, Seoul 151-742, Korea; November 1–2, 2013 (*cf.* Supratman, 2012).

Some theories (such as that of Piaget, the SOLO Model, or more broadly, the enactive-iconic-symbolic theory of Bruner (1966)) incorporate both aspects. Others such as Lakoff & Nunez (2000) and situated learning Lave & Wenger (1990) paint a broader brush-strokes showed biological and social structures involved. It has been developed for different purposes. The SOLO Model, for example, is related to the performance assessment through learning outcomes were observed. Other theories such as Davis (1984), Dubinsky (Czarnocha et al., 1999), Sfard (1991), and Gray & Tall (1994) concerned with the order in which concepts are built by an individual. But still there are some researchers who uncover cognitive structures associated with the construction of detailed knowledge about the mastery of new knowledge.

Piaget portrayed the child as a lone scientist, creating his or her own sense of the world. Then individual will interpret and act accordingly to conceptual categories or schemas that are developed in interaction with the environment. The knowledge of relationships among ideas, objects, and events is constructed by the active processes of internal assimilation, accommodation, and equilibration (Oxford, 1997, p. 39). Until children can construct a certain level of logic from the inside, they are non-conservers because they can judge on the basis of what they can see (Kamii & Ewing, 1996, p. 261).

Besides, we take into account that current learning perspectives incorporate three important assumptions as Anthony (1996) said:

(1) Learning is a process of knowledge construction, not of knowledge recording or absorption;
(2) Learning is knowledge-dependent; people use current knowledge to construct new knowledge; and
(3) The learner is aware of the processes of cognition and can control and regulate them.

Each child builds on the previous stage of cognitive development increasing the child's ability to solve more complex problems (Oxford, 1997, p. 189). The fundamental basis of learning was a discovery. Understanding is a discovering or a reconstructing by rediscovery, and such conditions must be compiled with if in the future individuals are to be formed who are capable of production, creativity and not simply repetition.

A series of activities were undertaken to identify:

1.1. Assimilation, accommodation and illustrations

Learning is an adaptation which has assimilation and accommodation in Piaget's term. To reach an understanding of basic phenomena, children have to go through the stages which Piaget presented (Bybee & Sund, 1982, p. 36). In problems solving, students construct the structure of thinking through the processes of assimilation and accommodation. Working memory capacity (that is, the capacity to hold various pieces of information simultaneously and to use them for further processing) is a critical feature of several models of human cognition, and it is widely recognized that it affects performance on many tasks (Morra, Gobbo, Marini & Sheese, 2009, p. 20). It has also been claimed that individual differences in working memory capacity account well for difference in measures of fluid intelligence (Engle, Tuholski, Laughlin & Conway, 1999; Kyllonen, 2002).

According to Fisher (1995, p. 57), thinking which is visualized and expressed can be observed and communicated. As stated by Gentner (1983) and Morrison, Doumas & Richl (2010), balancing inhibitory control in working memory and relational representation can be illustrated the process of assimilation and accommodation fundamentally. And then author adopted from assimilation and accommodation of Subanji (2007).

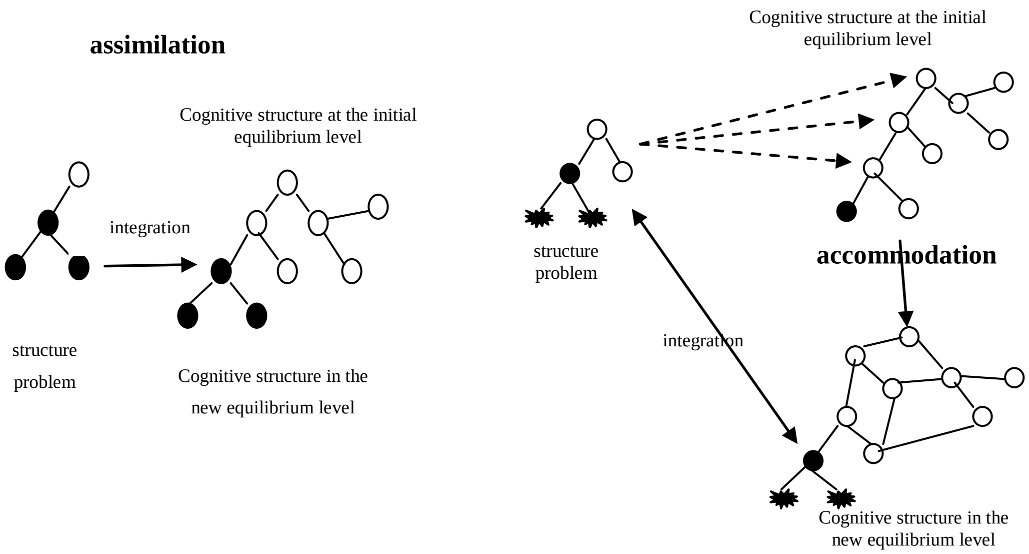


Figure 1. Illustration of the process of assimilation and accommodation adopted from Subanji (2007, p.6)

Subanji (2007, p. 39) said that the substructure incompleteness in the process of assimilation is a process of direct interpretation of the problem with more complex structure using a simple thinking structure. This thinking process was preceded by the imperfect assimilation process. The assimilation took place in the process of problem solving, but the complex problem was interpreted to the simple problem. Therefore, it produced an inappropriate answer.

In the process of problem solving (before the reflection), the students only conduct the assimilation process, but did not produce the appropriate structure to the structure of the problem. In this case, their thinking structure was still incomplete; nonetheless it had been used to interpret a complex problem structure. However, it produced an inappropriate answer (wrong). After receiving the answer, the students did not go through the reflection again.

Furthermore, when the opportunity for reflection was given, the disequilibration took place again in the students' thinking process, with the result that they continued to the assimilation and accommodation process. For the illustrations, see Figure 2.

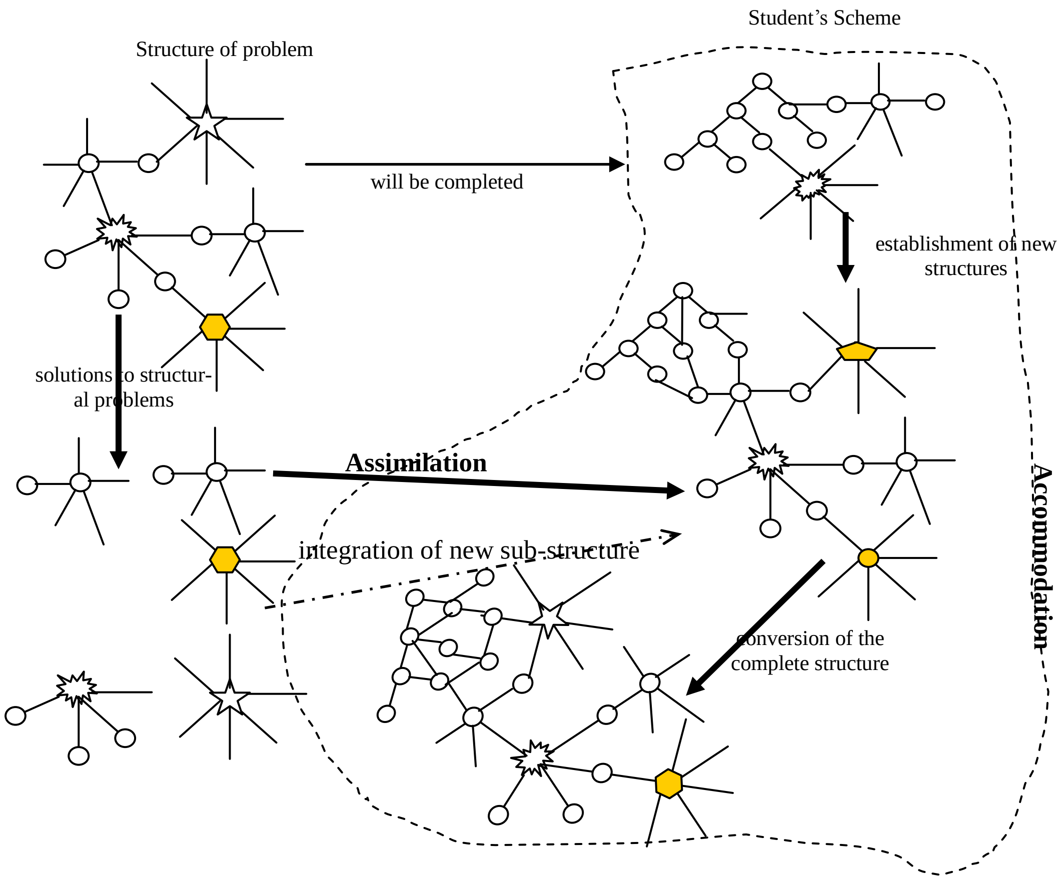


Figure 2. Structure of complex problem solving adopted from Subanji (2007)

1.2. Encoding Process of Thinking

Categorization is done to facilitate the interpretation of the data, simplifying analysis of the problems and the process of thinking of the object of research. It is related to the process of thinking including problems, relationships and strategies. As Gentner & Goldin-Meadow (2003, p. 6) shows the same view in cognitive linguistics that the coupling between language and cognition is strong enough to allow semantic structure to serve as a window on conceptual structure.

Furthermore Forbus, Gentner & Law (1995) habitual use of a given set of relational terms promotes uniform relational encoding; thereby the probability of transfer between relational situations is increasing. Then performed: when a given domain is encoded in terms of a stable set of relational terms, the likelihood of matching new examples with stored exemplars that share relational structure is increasing. Recoding involves a mental transformation of information into another code or format (Ashcroft, 1994).

1.3. Error Assimilation and Accommodation

In solving the problem, if the formation of cognitive structures is not perfect in the sense of the word: a cognitive structure to the structure of the problem is not the same, and then integrated it will produce the wrong answer (Subanji, 2007, p. 49). There is an example of model problem from Frederick (Kahneman, 2003, p. 451):

> "The price of baseball bat and ball is $12. Bat costs $ 10 more expensive than the ball. What is the price of the ball?"

Many students answered $2. Possibility of thought processes occurring imperfections assimilation. The model problem of Frederick structure can be described in Figures 3a and 3b.

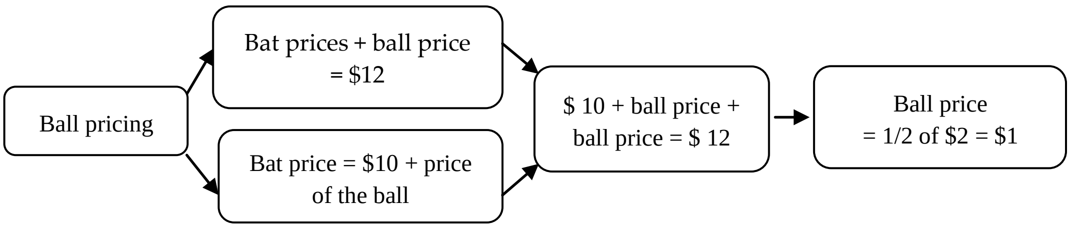


Figure 3a. Suitability: Structural problems with the structure of thinking

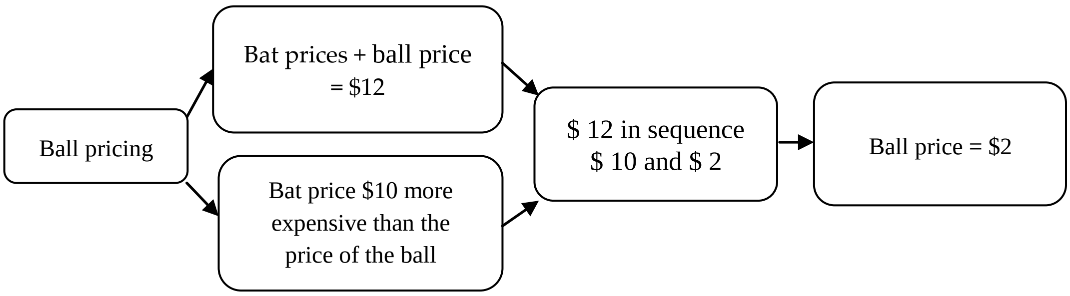


Figure 3b. Mismatches: structural problems with the structure of thinking

From the above illustration, it seems that there is no correspondence between the structures of a problem with the structure of student thinking. However, the assimilation process is already under way obtaining the answer $2. Frederick's real problem is a simple matter, even been able to use that mindset quickly without any control (reflection) then the answer to be incorrect.

Examples of accommodations mistake on elementary school students: e.g.

Today is Sunday. What day is it 2011 days later?

Basically, elementary school students are familiar with addition, multiplication, subtraction and division. However, when they are faced with the problems mentioned above in the absence of changes in cognitive structure namely linking multiple weekly with multiples 7, it will result in a wrong answer. Note Figure 4a.

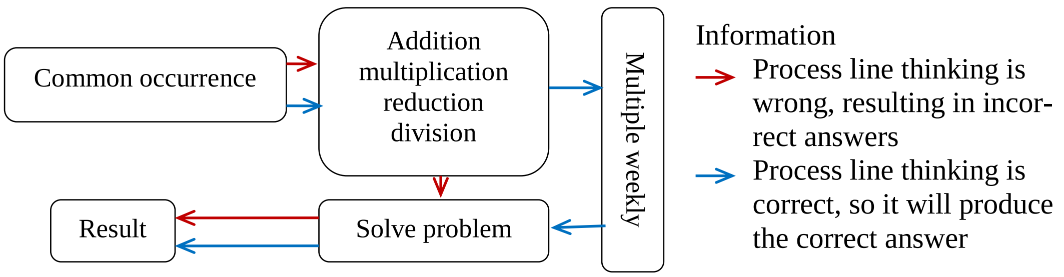
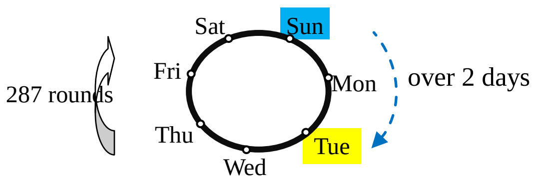


Figure 4a. Right mindset accommodation

On the contrary, when students firstly linked between weekly and multiples of 7, as well as more associated with the addition or day trip, there will be the right answer. The solution like that: 1 round = 1 week = 7 days, so the multiples will fall on the same day Sunday. 2011:7 = 287 remainder 2 or 2011 = 287 × 7 + 2, or 287x7 = Sunday. There is an excess of 2 days, so the answer is Tuesday. Figure 4b illustrated the problem solving.



Figur 4b. illustration of problem solving

1.4. Creative thinking and divergent thinking

Thinking can be divided into four categories, including *recall thinking, basic thinking,* *critical thinking, and creative thinking* (Krulik, Rudnick, & Milou, 2003, p. 89). Krulik et al. (2003) said that critical and creative thinking are higher-order thinking, and basic, critical, and creative thinking are reasoning. Figure 5 presents the hierarchy of thinking from Krulik at.al (2003).

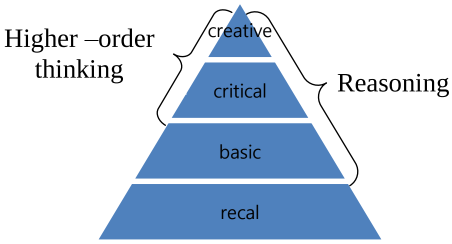


Figure 5. Hierarchy of thinking

Creative thinking is characterized as an ability to solve problems in not normal, unique, and various ways. The story of Gauss is one of the examples of creative thinking and it can also stimulate students' interest. When Gauss was a child, he and his class were asked to find the sum of the whole numbers from 1 through 100. Gauss was able to solve it in a minute. He placed the numbers in a row as follows: 1 + 2 + 3 + 4 + ... + 97 + 98 + 99 + 100. He then noticed that there were series of number pairs that summed to 101: 1 + 100 = 101; 2 + 99 = 101; 3 + 98 = 101; 4 + 97 = 101 and so on. Therefore, the answer is 50 pairs of 101, in other words 50 × 101 = 5,050.

According to Evans (1991: p. 41), divergent thinking component consists of problem sensitivity, fluency, flexibility, and originality. Evans gave further explanation of the components of divergent thinking, namely:

Problem sensitivity is the ability to recognize the existence of a problem or ignore the fact that less appropriate to recognize the real problem.

- Fluency is the ability to build a lot of ideas.
- Flexibility or resilience refers to the ability to build a diverse idea.
- Originality is the ability to generate ideas that are unusual, extraordinary, or unique.

A different set of mental habits characteristic of creativity according to Amabile (1983), and Parkins (1984) if you have mental habits that exemplify creative thinking, you tend to:

(1) Persevere;
(2) Push the limits of your knowledge and abilities;
(3) Generate, trust, and maintain your own standards of evaluation; and
(4) Generate new ways of viewing a situation that are outside the boundaries of standard conventions.

Padget (2012) realized that learning should touch on critical and creative thinking. Furthermore, Hadamard (1945) influenced by Gestalt psychology of his time theorized that mathematicians creative process followed the four stage Gestalt model (Wallas, 1926) of preparation-incubation-illumination-verification.

2. METHODOLOGY

2.1. Participants

A qualitative design was chosen for this study in order to investigate the intricate thinking process (Bogdan & Biklen, 1992). To see it, the data were gathered by the think aloud method (van Someren, Barnard & Sandberg, 1994) which was conducted by asking the research subjects to solve problems and to tell how their thinking process is at the same time. Think aloud was developed by the cognitive psychologists aiming to investigate how someone solves a problem. Using this method, the solver's cognitive process related to the problems can be recorded and analysed. The research subjects were 2 students who were in Mathematics Education academic year 2012/2013. They had not studied a conic section equation, but could express their thought process when they solve the problems.

2.2. Questionnaire

To investigate the creative thinking of students, researchers gave questionnaires can open students explore the characteristics of creative thinking to solve problems with a central question:

> "Finding the set of points where the ratio of fixed distance to one of the lines, the lines are perpendicular to each other, and to the point that lies on the other line! "

For the complete information about the thinking process of students, investigator conducted interviews to student during students working for the task and after that. In accordance with the opinion of Guba & Lincoln (1994) the received view of science pictures the Inquirer as standing behind a one-way mirror, viewing natural phenomena as they happen and recording them objectively. The researchers called the students one by one to work construction tasks of conic section equation. We explored several students, until finding at least two students, who were able to answer perfectly, and explained their thought processes when solving problems.

3. RESULTS

After exploring 9 students, we found 2 students, named Subject 4 (S4) and Subject 9 (S9), who were able to answer perfectly. We interviewed them to know their mindset such as 'what is his way of thought to solve problems'. As for the answer as follows:

Firstly, S4 made two lines, which are perpendicular to the x-axis and y-axis. Then put point A between the x-axis and y-axis. The next line drawn perpendicular to the y-axis of point A he calls B, and line drawn from the x-axis to the point A, he called C. So, that distance comparisons between the distance of AB is equal to the distance AC.

Subject 4 has been constructed of conic section equations with various positions, namely:

(1) The comparison same distance between the AC and AB (e = 1) will be obtained parabolic equation, as shown in Figure 6a.

(2) The comparison: distance AC < distance AB (e < 1, taken e = $\frac{1}{2}$) will be obtained ellips equations, as shown in Figure 6b.

(3) The comparison: distance AC > distance AB (e > 1, taken e = 2) will be obtained hyperbolic equation, as shown in Figure 6c.

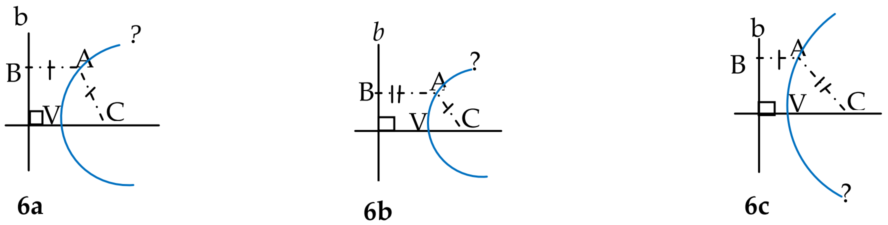


Figure 6. Sketch the graph of a conic section generated S4 and S9

6a	**6b**	**6c**
V(1, 0) and C(2, 0)	V(2, 0) and C(3, 0)	V(1, 0) and C(3, 0)
CA = BA $\sqrt{(x-2)^2+(y-0)^2} = x$ $\sqrt{x^2-4x+4+y^2} = x$ $x^2-4x+4+y^2 = x^2$ $y^2-4x+4 = 0$	2CA = BA $2\sqrt{(x-3)^2+(y-0)^2} = x$ $2\sqrt{x^2-4x+4+y^2} = x$ $4(x^2-4x+4+y^2) = x^2$ $3x^2+4y^2-16x+16 = 0$	CA =2BA $\sqrt{(x-2)^2+(y-0)^2} = 2x$ $\sqrt{x^2-4x+4+y^2} = 2x$ $x^2-4x+4+y^2 = 4x^2$ $y^2-3x^2-4x+4 = 0$

If A(x, y), and b coincides with the y-axis and C on the x-axis, then b $\equiv$ x = 0, consequently S9 used the comparison distance between two points, for solving the problem. So it does not produce a conic section equation. After the reflection, he did with comparison a combination of distance calculations point to the line, and the distance between two points. As S4 and S9 did as in Figure 6. For the structure of creative thinking when building a conic section equation is shown in Figure 7.

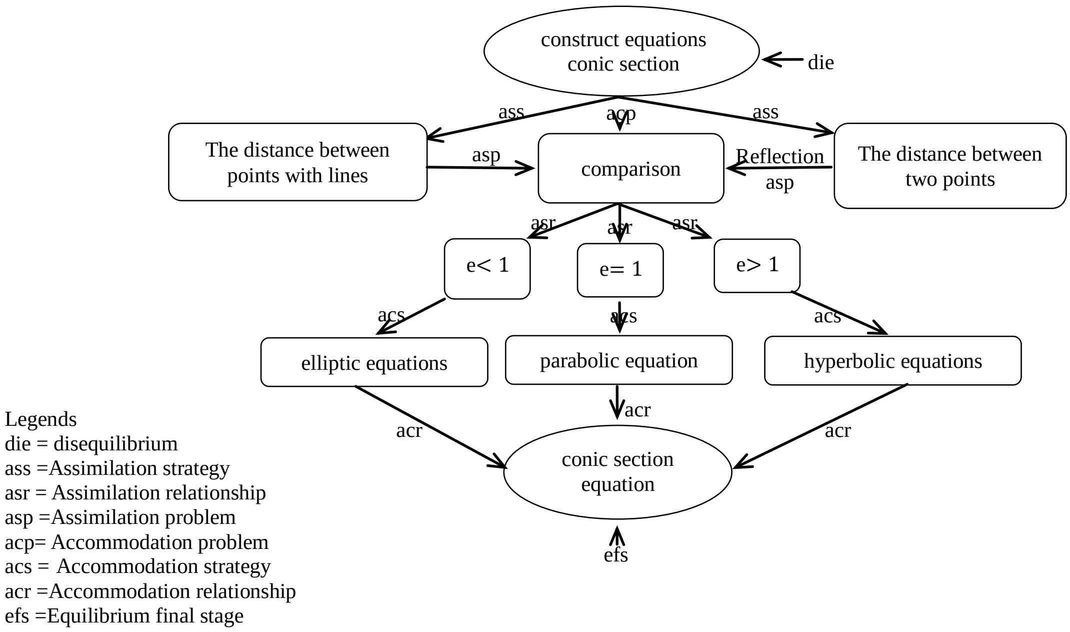
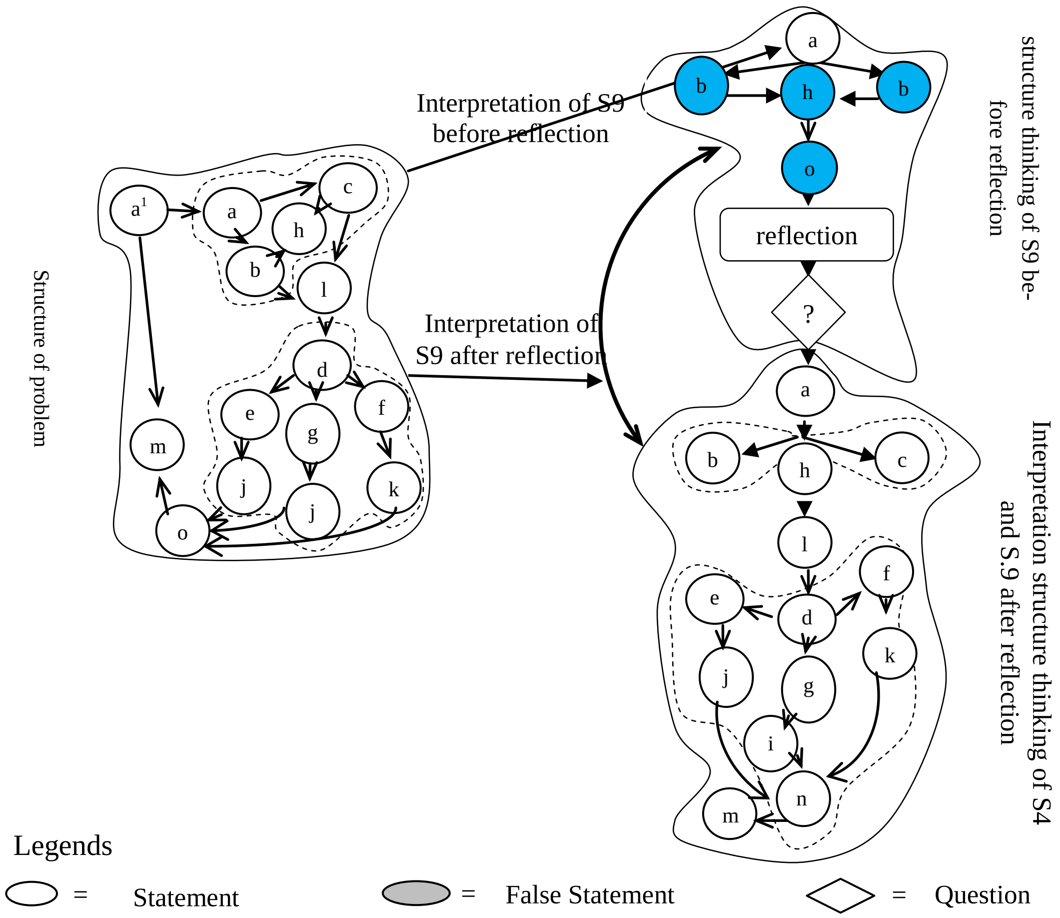


Figure 7. Sequence of creative thinking as students construct a conic section equation

Problem solving means answering a question for which one does not directly have an answer available. This can be because the answer cannot be directly retrieved from memory but must be constructed from information that is available in memory or that can be obtained from the environment. Another possibility is that finding the answer involves exploring possible answers none of which is immediately recognized as the solution to a problem. Problem solving then means that new information must be inferred from givens and knowledge in memory to accept or reject possible answers. Most of the problem solving involves a combination of these two types of reasoning: constructing solutions and constructing justifications of these solutions (van Someren et. al, 1994).

In the process of problem solving, S9 only conducted the assimilation process, but did not produce the appropriate structure to the structure of the problem. In this case, his thinking structure was still incomplete; nonetheless it had been used to interpret a complex problem structure. Yet, it produced an inappropriate answer (wrong). After receiving the reflection, S9 could answer to solve problem for construction of conic equation. The cognitive structure of S4 & S9 creative thinking can be seen in Figure 8.



a	Problem, the proposed construction of conic section equation	g	b < *c*
a[1]	Problem, often encountered in daily life	h	quadratic
b	The distance between two points	i	elliptic equations
c	The distance between points on a line	j	parabolic equation
d	compare	k	hiperbolic equations
e	b = c	l	graph
f	b>c	m	completed
o	equation of the line for a corner	n	conic section equation

Figure 8. The cognitive structure of S4 & S9 creative thinking

4. DISCUSSION

From the results, it can be concluded that the process of assimilation and accommodation when creative thinking starts from students' awareness of the existence of complex problems. The study revealed that there are three characteristics of creative thinking processes:

(1) The existence of sub-structure perfection of thought that will be used in generalizing the solution,
(2) Capable of reflecting on their own to the fullest, and
(3) The existence of consciousness to explore the possibility of another solution.

In addition it was found that Imperfections of the process of assimilation or accommodation that produce sub-structure formation imperfection of thought will produce the wrong answers. According to the results of research Subanji (2007, p. 155) that:
Assimilation or accommodation imperfections can occur in three forms:

(1) The incompleteness of the sub-structure of thought in the process of assimilation,
(2) Incompleteness think sub-structure in the process of accommodation, and
(3) Mismatch sub-structures thinking in the process of assimilation or accommodation. But needs more study.

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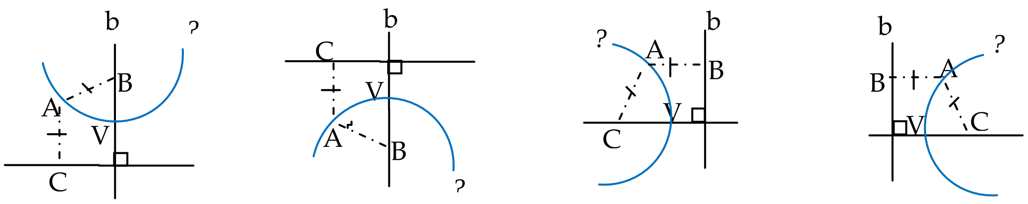
APPENDIX
ATTACHMENT OF STUDENTS' WORK



1a 1b 1c 1d

Figure 1. Sketch the graph of parabola with a variety of positions

Calculation of parabola, S4 and S9 stated as follows for Figure 1:

	B	**V**	**c**	**The parabolic equations in a variety of positions**
1a	(0, 0)	(0,–1)	y = –2	$x^2 = 4y + 4$
	(0, 1)	(0,0)	y= –1	$x^2 = 4y$
	(0, 2)	(0,1)	y = 0	$x^2 = 4y – 4$
1b	(0, 0)	(0, 1)	y = 2	$x^2 = –4y + 4$
	(0,–1)	(0, 0)	y= 1	$x^2 = –4y$
	(0,–2)	(0,–1)	y = 0	$x^2 =–4y –4$
	C	**V**	**b**	
1c	(0, 0)	(1, 0)	x = 2	$y^2 = –4x + 4$
	(–1, 0)	(0, 0)	x= 1	$y^2 = –4x$
	(–2 ,0)	(–1, 0)	x = 0	$y^2 = –4x – 4$
1d	(0, 0)	(–1, 0)	x = –2	$y^2 = 4x + 4$
	(1, 0)	(0, 0)	x= –1	$y^2 = 4x$
	(2, 0)	(1, 0)	x = 0	$y^2 = 4x – 4$

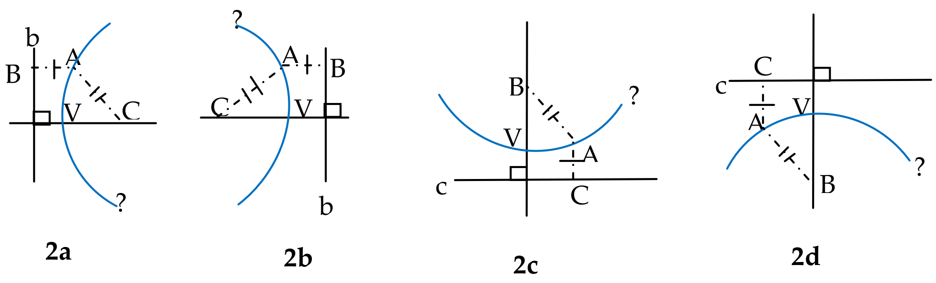


Figure 2. Sketch the graph of hyperbole in various positions

Calculation of hyperbole, S4 and S9 stated as follows for Figure 2

	C	**V**	**b**	**The hyperbole equations in a variety of positions**
2a	(0, 0)	(–2, 0)	$x = -3$	$3x^2 - y^2 + 24x + 36 = 0$
	(2, 0)	(0, 0)	$x = -1$	$3x^2 - y^2 + 12x = 0$
	(1, 0)	(3, 0)	$x = 0$	$3x^2 - y^2 + 6x - 9 = 0$
2b	(0, 0)	(2, 0)	$x = 3$	$3x^2 - y^2 - 24x + 36 = 0$
	(–2, 0)	(0, 0)	$x = 1$	$3x^2 - y^2 - 12x = 0$
	(–3, 0)	(–1, 0)	$x = 0$	$3x^2 - y^2 - 6x - 9 = 0$
	B	**V**	**c**	
2c	(0, 0)	(0,–2)	$y = -3$	$3y^2 - x^2 + 24y + 36 = 0$
	(0, 2)	(0, 0)	$y = -1$	$3y^2 - x^2 + 12y = 0$
	(0, 3)	(0, 1)	$y = 0$	$3y^2 - x^2 + 6y - 9 = 0$
2d	(0, 0)	(0, 2)	$y = 3$	$3y^2 - x^2 - 24y + 36 = 0$
	(0,–2)	(0,0),	$y = 1$	$3y^2 - x^2 - 12y = 0$
	(0,–3)	(0,–1)	$y = 0$	$3y^2 - x^2 - 6y - 9 = 0$

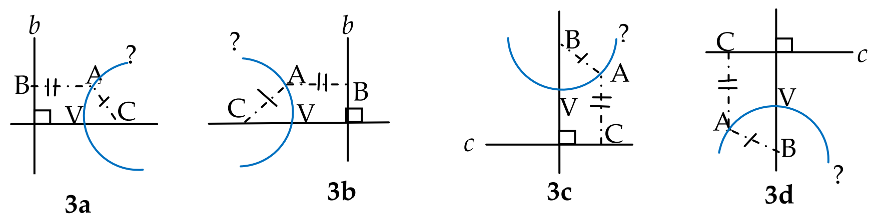


Figure 3. Sketch the graph of an ellipse with a variety of positions

Calculation of ellipse, S4 and S9 stated as follows for Figure 3:

	C	**V**	**b**	**An ellipse equations in a variety of positions**
3a	(0, 0)	(–1,0)	$x = -3$	$3x^2 + 4y^2 - 6x + 9 = 0$
	(1,0)	(0, 0)	$x = -2$	$3x^2 + 4y^2 - 12x = 0$
	(3, 0)	(2, 0)	$x = 0$	$3x^2 + 4y^2 - 24x - 36 = 0$
3b	(0, 0)	(1,0)	$x = 3$	$3x^2 + 4y^2 + 6x + 9 = 0$
	(–1, 0)	(0,0)	$x = 2$	$3x^2 + 4y^2 + 12x = 0$
	(–3,0)	(–2,0)	$x = 0$	$3x^2 + 4y^2 24x + 9 = 0$
	B	**V**	**c**	
3c	(0, 0)	(0,–1)	$y = -3$	$4x^2 + 3y^2 + 6y - 9 = 0$
	(0, 0)	(0, 1)	$y = -2$	$4x^2 + 3y^2 - 12y = 0$
	(0, 3)	(0, 2)	$y = 0$	$4x^2 + 3y^2 - 24y + 36 = 0$
3d	(0, 0)	(0, 1)	$y = 3$	$4x^2 + 3y^2 + 6y - 9 = 0$
	(0, 0)	(0,–1)	$y = 2$	$4x^2 + 3y^2 + 12y = 0$
	(0,–3)	(0,–2)	$y = 0$	$4x^2 + 3y^2 + 24y + 36 = 0$