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BASICALLY DISCONNECTED COVERS OF THE EXTENSION kX OF A SPACE X

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Abstract. Observing that every Tychonoff space X has a weakly Lindelöf extension kX and the minimal basically diconneted cover ΛkX of kX is weakly Lindelöf, we first show that $\Lambda_{kX} : \Lambda kX \longrightarrow kX$ is a $z^{\#}$-irreducible map and that $\Lambda\beta X = \beta\Lambda kX$. And we show that $k\Lambda X = \Lambda kX$ if and only if $\Lambda_X^k : k\Lambda X \longrightarrow kX$ is an onto map and $\beta\Lambda X = \Lambda\beta X$.

1. Introduction

All spaces in this paper are assumed to be Tychonoff spaces and $\beta X(\upsilon X$, resp.) denotes the Stone-Čech compactification(Hewitt realcompactification, resp.) of X.

Iliadis constructed the absolute of a Hausdorff space X, which is the minimal extremally disconnected cover $(E(X), \pi_X)$ of X and they turn out to be the perfect onto projective covers([4]). To generalize extremally disconnected spaces, basically disconnected spaces, quasi-F spaces and cloz-spaces have been introduced and their minimal covers have been studied by various aurthors([2], [3], [5], [6], [8]). In these ramifications, minimal covers of compact spaces can be nisely characterized.

In particular, Vermeer showed that every space X has the minimal basically disconnected cover $(\Lambda X, \Lambda_X)$ and that for any compact space X, ΛX is given by the Stone space $\mathrm{S}(\sigma Z(X)^{\#})$ of a σ-complete Boolean algebra $\sigma Z(X)^{\#}$([7]). For any extension γX of a space X, relations of $\Lambda\gamma X$ and $\gamma\Lambda X$ have been studied([2], [3], [4], [5]). In fact, for any space X, $E(\beta X) = \beta E(X)$ and conditions on a space X that is equivalent to $E(\upsilon X) = \upsilon E(X)(\Lambda\upsilon X = \upsilon\Lambda X$, $\beta\Lambda X = \Lambda\beta X$, resp.) is known([7], [6]).

For any space X, there is an extension (kX, k_X) of X such that

(1) kX is a weakly Lindelöf space, and

(2) for any continuous map $f : X \longrightarrow Y$, there is a continuous map $f^k : kX \longrightarrow kY$ such that $f^k |_X = f$([9]).

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The purpose to write this paper is to find properties of the minimal basically connected cover ΛkX of kX and relations of ΛkX and $k\Lambda X$. For any space X, we first show that ΛkX is a weakly Lindelöf space and $\Lambda_{kX} : \Lambda kX \longrightarrow kX$ is a $z^{\#}$-irreducible map and that $\Lambda\beta X = \beta\Lambda kX$. And we show that $k\Lambda X = \Lambda kX$ if and only if $\Lambda_X^k : k\Lambda X \longrightarrow kX$ is an onto map and $\beta\Lambda X = \Lambda\beta X$.

For the terminology, we refer to [1] and [7].

2. Basically disconnected spaces

The set $\mathcal{R}(X)$ of all regular closed sets in a space X, when partially ordered by inclusion, becomes a complete Boolean algebra, in which the join, meet, and complementation operations are defined as follows : for any $A \in \mathcal{R}(X)$ and any $\{A_i \mid i \in I\} \subseteq \mathcal{R}(X)$,

$\vee\{A_i \mid i \in I\} = cl_X(\cup\{A_i \mid i \in I\})$,

$\wedge\{A_i \mid i \in I\} = cl_X(int_X(\cap\{A_i \mid i \in I\}))$, and

$A' = cl_X(X - A)$

and a sublattice of $\mathcal{R}(X)$ is a subset of $\mathcal{R}(X)$ that contains $\emptyset$, X and is closed under finite joins and meets.

Recall that a map $f : Y \longrightarrow X$ is called *a covering map* if it is a continuous, onto, perfect, and irreducible map.

Lemma 2.1. ([5], [7]) *(1) Let $f : Y \longrightarrow X$ be a covering map. Then the map $\psi : R(Y) \longrightarrow R(X)$, defined by $\psi(A) = A \cap X$, is a Boolean isomorphism and the inverse map ψ^{-1} of ψ is given by $\psi^{-1}(B) = cl_Y(f^{-1}(int_X(B)))) = cl_Y(int_Y(f^{-1}(B)))$.*

(2) Let X be a dense subspace of a space K. Then the map $\phi : R(K) \longrightarrow R(X)$, defined by $\phi(A) = A \cap X$, is a Boolean isomorphism and the inverse map ϕ^{-1} of ϕ is given by $\phi^{-1}(B) = cl_K(B)$.

A lattice L is called *σ-complete* if every countable subset of L has join and meet. For any subset M of a Boolean algebra L, there is the smallest σ-complete Boolean subalgebra σM of L containing M. Let X be a space and $Z(X)$ the set of all zero-sets in X. Then $Z(X)^{\#} = \{cl_X(int_X(Z)) \mid Z \in Z(X)\}$ is a sublattice of $R(X)$. Note that for any zero-set A in X, there is a zero-set B is βX such that $A = B \cap X$. Hence, by Lemma 2.1, $\sigma Z(X)^{\#}$, $\sigma Z(\upsilon X)^{\#}$ and $\sigma Z(\beta X)^{\#}$ are Boolean isomorphic.

Definition 1. A space X is called *basically disconnected* if for any zero-set Z in X, $int_X(Z)$ is closed in X, equivalently, $Z(X)^{\#} = B(X)$, where $B(X)$ is the set of all clopen sets in X.

A space X is a basically disconnected space if and only if βX is a basically disconnected space. Suppose that X is a basically disconnected space. Then for any sequence (B_n) in $B(X)$, $\wedge\{B_n \mid n \in N\} = cl_X(int_X(\cap\{B_n \mid n \in N\})) \in Z(X)^{\#}$ and $\vee\{B_n \mid n \in N\} = cl_X(int_X(\cup\{B_n \mid n \in N\})) \in Z(X)^{\#}$. Hence X is a basically disconnected space if and only if $Z(X)^{\#}$ is a σ-complete Boolean algebra.

Definition 2. Let X be a space. Then a pair (Y, f) is called

(1) *a cover of* X if $f : Y \longrightarrow X$ is a covering map,

(2) *a basically disconnected cover of* X if (Y, f) is a cover of X and Y is a basically disconnected space, and

(3) *a minimal basically disconnected cover of* X if (Y, f) is a basically disconnected cover of X and for any basically disconnected cover (Z, g) of X, there is a covering map $h : Z \longrightarrow Y$ such that $f \circ h = g$.

Let X be a space and $\mathcal{B}$ a Boolean subalgebra of $R(X)$. Let $S(\mathcal{B}) = \{\alpha \mid \alpha$ is a $\mathcal{B}$-ultrafilter$\}$ and for any $B \in \mathcal{B}$, $\Sigma_B^{\mathcal{B}} = \{\alpha \in S(\mathcal{B}) \mid B \in \alpha\}$. Then the space $S(\mathcal{B})$, equipped with the topology for which $\{\Sigma_B^{\mathcal{B}} \mid B \in \mathcal{B}\}$ is a base, called *the Stone-space of* $\mathcal{B}$. Then $S(\mathcal{B})$ is a compact, zero-dimensional space.

Vermeer([8]) showed that every space X has a minimal basically disconnected cover $(\Lambda X, \Lambda_X)$ and that if X is a compact space, then ΛX is the Stone-space $S(\sigma Z(X)^{\#})$ of $\sigma Z(X)^{\#}$ and $\Lambda_X(\alpha) = \cap\{A \mid A \in \alpha\}$ $(\alpha \in \Lambda X)$.

Let X be a space. Since $\sigma Z(X)^{\#}$ and $\sigma Z(\beta X)^{\#}$ are Boolean isomorphic, $S(\sigma Z(X)^{\#})$ and $S(\sigma Z(\beta X)^{\#})$ are homeomorphic.

Let X, Y be spaces and $f : Y \longrightarrow X$ a map. For any $U \subseteq X$, let $f_U : f^{-1}(U) \longrightarrow U$ denote the restriction and co-restriction of f with respect to $f^{-1}(U)$ and U, respectively. For any space X, let $(\Lambda\beta X, \Lambda_\beta)$ denote the minimal basically disconnected cover of βX.

Lemma 2.2. ([5]) *Let* X *be a space. Then* $\Lambda_\beta^{-1}(X)$ *is a basically disconnected space if and only if* $(\Lambda_\beta^{-1}(X), \Lambda_{\beta_X})$ *is the minimal basically disconnected cover of* X.

We recall that a covering map $f : Y \longrightarrow X$ is called $z^{\#}$*-irreducible*($\sigma z^{\#}$-irreducible, resp.) if $f(Z(Y)^{\#}) = Z(X)^{\#}$($f(\sigma Z(Y)^{\#}) = \sigma Z(X)^{\#}$, resp.). Let $f : Y \longrightarrow X$ be a covering map and Z a zero-set in X. By Lemma 2.1, $f(cl_Y(int_Y(f^{-1}(Z)))) = cl_X(int_X(Z))$ and $cl_Y(int_Y(f^{-1}(Z))) \in Z(X)^{\#}$. Hence $Z(X)^{\#} \subseteq f(Z(Y)^{\#})$ and so $f : Y \longrightarrow X$ is $z^{\#}$-irreducible if and only if $f(Z(Y)^{\#}) \subseteq Z(X)^{\#}$. Using these we have the following :

Proposition 2.3. *Let* $f : Y \longrightarrow X$ *and* $g : W \longrightarrow Y$ *be covering maps. Then*

(1) if $f : Y \longrightarrow X$ *is* $z^{\#}$*-irreducible, then* $f : Y \longrightarrow X$ *is* $\sigma z^{\#}$*-irreducible,*

(2) $f \circ g : W \longrightarrow X$ *is* $z^{\#}$*-irreducible if and only if* $f : Y \longrightarrow X$ $g : W \longrightarrow Y$ *are* $z^{\#}$*-irreducible , and*

(3) if $f : Y \longrightarrow X$ *is* $z^{\#}$*-irreducible and* X *is a basically disconnected space, then* f *is a homeomorphism.*

Definition 3. A space X is called *a weakly Lindelöf space* if for any open cover $\mathcal{U}$ of X, there is a countable subset $\mathcal{V}$ of $\mathcal{U}$ such that $\cup\{V \mid V \in \mathcal{V}\}$ is dense in X.

It is well-known that if X is a weakly Lindelöf space, then $\beta\Lambda X = \Lambda\beta X$([3]), that is, there is a homeomorphism map $h : \beta\Lambda X \longrightarrow \Lambda\beta X$ such that $\Lambda_\beta \circ h = \Lambda_X^\beta$, where $\Lambda_X^\beta : \beta\Lambda X \longrightarrow \beta X$ is the Stone-extension of $\beta_X \circ \Lambda_X$. Moreover, if X is a weakly Lindelöf space, then $(\Lambda_\beta^{-1}(X), \Lambda_{\beta_X})$ is the minimal basically disconnected cover of X([3]).

3. A minimal basically disconnected cover of kX

A z-filter $\mathcal{F}$ on a space X is called *real* if $\mathcal{F}$ is closed under the countable intersection.

For any space X, let $kX = \upsilon X \cup \{p \in \beta X - \upsilon X \mid$ there is a real z-filter $\mathcal{F}$ on X such that $\cap\{cl_{\upsilon X}(F) \mid F \in \mathcal{F}\} = \emptyset$ and $p \in \cap\{cl_{\beta X}(F) \mid F \in \mathcal{F}\}\}$. Then kX is a extension of a space X such that $\upsilon X \subseteq kX \subseteq \beta X$([9]).

Lemma 3.1. ([9]) *For any space X, kX is a weakly Lindelöf space.*

It is well known that a space X is weakly Lindelöf if and only if for any $Z(X)^{\#}$-filter $\mathcal{A}$ with the countable meet property, $\cap\{A \mid A \in \mathcal{A}\} \neq \emptyset$.

Let X be a space. For any $A \in \sigma Z(\beta X)^{\#}$, let $\Sigma_A^{\sigma Z(\beta X)^{\#}} = \Sigma_A$ and $\Sigma_A \cap \Lambda kX = \lambda_A$. Then for any $A \in \sigma Z(\beta X)^{\#}$, $\Lambda_\beta(\Sigma_A) = A$ and $\Lambda_{kX}(\lambda_A) = A \cap kX$, because $\Lambda kX = \Lambda_\beta^{-1}(kX)$ and $\Lambda_{kX} = \Lambda_{\beta_{kX}}$.

Theorem 3.2. *Let X be a space. Then we have the following :*
(1) $(\Lambda_\beta^{-1}(kX), \Lambda_{\beta_{kX}})$ is the minimal basically disconnected cover of X,
(2) ΛkX is a weakly Lindelöf space, and
(3) $\Lambda_{kX} : \Lambda kX \longrightarrow kX$ is a $z^{\#}$-irreducible map.

Proof. (1) By Lemma 3.1, kX is a weakly Lindelöf space and by Lemma 2.4, $(\Lambda_\beta^{-1}(kX), \Lambda_{\beta_{kX}})$ is the minimal basically disconnected cover of X.

(2) Let $\mathcal{A}$ be a z-filter on ΛkX with the countable meet property and $\cap\{A \mid A \in \mathcal{A}\} = \emptyset$. Suppose that $\cap\{\Lambda_{kX}(A) \mid A \in \mathcal{A}\} \neq \emptyset$. Pick $x \in \cap\{\Lambda_{kX}(A) \mid A \in \mathcal{A}\}$. Since $\mathcal{A}$ has the countable meet property, $\mathcal{A}$ has the finte intersection property. Hence $\{A \cap \Lambda_{kX}^{-1}(x) \mid A \in \mathcal{A}\}$ is a family of closed sets in $\Lambda_{kX}^{-1}(x)$ with the finite intersection property. Since $\Lambda_{kX}^{-1}(x)$ is a compact subset in ΛkX, $\cap\{A \cap \Lambda_{kX}^{-1}(x) \mid A \in \mathcal{A}\} \neq \emptyset$ and so $\cap\{A \mid A \in \mathcal{A}\} \neq \emptyset$. This is a contracdiction. Thus $\cap\{\Lambda_{kX}(A) \mid A \in \mathcal{A}\} = \emptyset$.

Since kX is a weakly Lindelöf space, there is a sequence (A_n) in $\mathcal{A}$ such that $cl_{kX}(\cup\{kX - \Lambda_{kX}(A_n) \mid n \in N\}) = kX$. Let $A \in \mathcal{A}$. Then $\Lambda_{kX}^{-1}(\Lambda_{kX}(\Lambda kX - A)) \supseteq \Lambda kX - A$ and hence $\Lambda_{kX}(A') \supseteq \Lambda_{kX}(\Lambda kX - A) \supseteq kX - \Lambda_{kX}(A)$. Thus $cl_{kX}(\cup\{\Lambda_{kX}(A'_n) \mid n \in N\}) = kX$. Note that

$$\begin{aligned} kX &= cl_{kX}(\cup\{\Lambda_{kX}(A'_n) \mid n \in N\}) \\ &= cl_{kX}(\Lambda_{kX}(\cup\{A'_n \mid n \in N\})) \\ &= \Lambda_{kX}(cl_{kX}(\cup\{A'_n \mid n \in N\})) \\ &= \Lambda_{kX}(\vee\{A'_n \mid n \in N\}). \end{aligned}$$

Since Λ_{kX} is an irreducible map, $\vee\{A'_n \mid n \in N\} = \Lambda kX$ and so $(\vee\{A'_n \mid n \in N\})' = \wedge\{A_n \mid n \in N\} = \emptyset$. Since $\mathcal{A}$ has the countable meet property, it is a contradiction. Hence $\cap\{A \mid A \in \mathcal{A}\} \neq \emptyset$ and so ΛkX is a weakly Lindelöf space.

(3) Take any zero-set Z in ΛkX. Since ΛkX is a weakly Lindelöf space, $\Lambda kX - Z$ is an open weakly Lindelöf subspace of ΛkX. By (1), there is a sequence (Z_n) in $\sigma Z(\beta X)^{\#}$ such that for any $n \in N$, $\Sigma_{Z_n} \cap \Lambda kX \subseteq \Lambda KX - Z$ and

$$\begin{aligned}\Lambda kX - Z &= cl_{\Lambda kX}(\cup\{\Sigma_{Z_n} \cap \Lambda kX \mid n \in N\}) \cap (\Lambda kX - Z)\\ &= cl_{\Lambda kX}(\cup\{\lambda_{Z_n} \mid n \in N\}) \cap (\Lambda kX - Z)\\ &= (\vee\{\lambda_{Z_n} \mid n \in N\}) \cap (\Lambda kX - Z).\end{aligned}$$

Hence $\cup\{\lambda_{Z_n} \mid n \in N\} \subseteq \Lambda kX - Z \subseteq \vee\{\lambda_{Z_n} \mid n \in N\}$ and so $cl_{\Lambda kX}(int_{\lambda kX}(\Lambda kX - Z)) = \vee\{\lambda_{Z_n} \mid n \in N\}$. Thus $cl_{\Lambda kX}(int_{\Lambda kX}(Z)) = \wedge\{\lambda_{Z'_n} \mid n \in N\}$. Note that for any $A \in \sigma Z(\beta X)^{\#}$, $\Lambda_{\Lambda kX}(\lambda_A) = A \cap kX$. By Lemma 2.1,

$$\begin{aligned}\Lambda_{\Lambda kX}(cl_{\Lambda kX}(int_{\Lambda kX}(Z))) &= \Lambda_{\Lambda kX}(\wedge\{\lambda_{Z'_n} \mid n \in N\})\\ &= (\wedge\{\Lambda_{\Lambda kX}(\lambda_{Z'_n}) \mid n \in N\})\\ &= \wedge\{Z'_n \cap kX \mid n \in N\}\\ &= (\wedge\{Z'_n \mid n \in N\}) \cap kX\end{aligned}$$

and hence $\Lambda_{\Lambda kX}(cl_{\Lambda kX}(int_{\Lambda kX}(Z))) \in \sigma Z(kX)^{\#}$. Thus $\Lambda_{\Lambda kX}$ is a $\sigma z^{\#}$-irreduci -ble map. □

Let X be a space. Then $\beta\Lambda X = \Lambda\beta X$ if and only if Λ_X is $z^{\#}$-irreducible([3]). Using this, we have the following :

Corollary 3.3. *For any space, $\Lambda\beta X = \beta\Lambda kX$.*

Lemma 3.4. ([8]) *For any continuous map $f : X \longrightarrow Y$, there is a unique continuous map $f^k : kX \longrightarrow kY$ such that $f^k \circ k_X = k_Y \circ f$.*

Let X be a space. Then there is a covering map $h : \beta\Lambda X \longrightarrow \Lambda\beta X$ such that $\Lambda_\beta \circ h \circ \beta_{\Lambda X} = \beta_X \circ \Lambda_X$. By Lemma 3.4, there is a continuous map $\Lambda_X^k : k\Lambda X \longrightarrow kX$ such that $\Lambda_X^k \circ k_{\Lambda X} = k_X \circ \Lambda_X$. Hence there is a continuous map $t_X : k\Lambda X \longrightarrow \Lambda kX$ such that $\beta_{\Lambda kX} \circ t_X = h \circ \beta_{k\Lambda X}$ and $\Lambda_{\Lambda kX} \circ t_X = \Lambda_X^k$. If t_X is a homeomorphism, then we write $k\Lambda X = \Lambda kX$

Corollary 3.5. *Let X be a space. If $k\Lambda X = \Lambda kX$, then $\beta\Lambda X = \Lambda\beta X$.*

Proof. Since $t_X : k\Lambda X \longrightarrow \Lambda kX$ is a homeomorphism and $\Lambda_{kX} : \Lambda kX \longrightarrow kX$ is $\sigma z^{\#}$-irreducible, $\Lambda_X^k : k\Lambda X \longrightarrow kX$ is $\sigma z^{\#}$-irreducible. Take any zero-set Z in $\beta\Lambda X$. Then, by Lemma 2.1, $cl_{\beta\Lambda X}(int_{\beta\Lambda X}(Z)) \cap k\Lambda X \in Z(k\Lambda X)^{\#}$. Hence

$$\begin{aligned}\Lambda_X^k(cl_{\beta\Lambda X}(int_{\beta\Lambda X}(Z)) \cap k\Lambda X) &= \Lambda_\beta(h(cl_{\beta\Lambda X}(int_{\beta\Lambda X}(Z)))) \cap kX\\ &\in \sigma Z(kX)^{\#}.\end{aligned}$$

By Lemma 2.1, $\Lambda_\beta(h(cl_{\beta\Lambda X}(int_{\beta\Lambda X}(Z)))) \in \sigma Z(\beta X)^{\#}$ and so $\Lambda_\beta \circ h$ is a $\sigma z^{\#}$-irreducible map. Thus $h : \beta\Lambda X \longrightarrow \Lambda\beta X$ is a $\sigma z^{\#}$-irreducible map. Since $\beta\Lambda X$ and $\Lambda\beta X$ are basically disconnected spaces, h is a homeomorphism. □

Let X be a space such that $\beta\Lambda X = \Lambda\beta X$. By Corollary 3.5, there is a homeomorphsim $m_X : \beta\Lambda X \longrightarrow \beta\Lambda kX$ such that $\beta_{\Lambda kX} \circ t_X = m_X \circ \beta_{k\Lambda X}$. Since $m_X \circ \beta_{k\Lambda X}$ is an embedding, t_X is an embedding.

A subspace X of a space Y is called C^**-embedded in* Y if for any real-valued continuous map $f : X \longrightarrow R$, there is a continuous map $g : Y \longrightarrow R$ such that $g\,|_X = f$. For any space X, X is C^*-embedded in βX and if $X \subseteq Y \subseteq W \subseteq \beta X$, then Y is C^*-embedded in W([1]). Hence we have the following

Corollary 3.6. *Let X be a space such that $\beta\Lambda X = \Lambda\beta X$. Then $k\Lambda X$ is a C^*-embedded subspace of ΛkX.*

Theorem 3.7. *Let X be a space. Then the following are equivalent:*
(1) $k\Lambda X = \Lambda kX$,
(2) t_X is an onto map and $\beta\Lambda X = \Lambda\beta X$, and
(3) Λ^k_X is an onto map and $\beta\Lambda X = \Lambda\beta X$.

Proof. (1) ⇒ (2) By Corollary 3.5, it is trivial.

(2) ⇒ (3) Since Λ_{kX} and t_X are onto maps, Λ^k_X is an onto map.

(3) ⇒ (1) Let $f = \Lambda^k_X$. Take any $x \in kX$. Since f is an onto map and Λ_X is a covering map, $f(k\Lambda X - \Lambda X) = kX - X$([7]). Since $\beta_{kX} \circ f = \Lambda_\beta \circ h \circ \beta_{k\Lambda X}$, $f^{-1}(x) = (\Lambda_\beta \circ h)^{-1}(x) \subseteq k\Lambda X - \Lambda X$. Since $\Lambda_\beta \circ h$ is a covering map, $f^{-1}(x)$ is a compact subset of $k\Lambda X$ and hence f is a compact map. By Corollary 3.6, $f^{-1}(x) = \Lambda_\beta^{-1}(x) \subseteq \Lambda kX$.

Let F be a closed set in $k\Lambda X$ and $x \in kX - f(F)$. Then $f^{-1}(x) \cap F = \emptyset$. Since $f^{-1}(x)$ is a comact space and $\Lambda\beta X$ is the Stone space of $S(\sigma Z(\beta X)^\#)$, there is a $B \in \sigma Z(\beta X)^\#$ such that $f^{-1}(x) \subseteq \Sigma_B$ and $F \subseteq \Sigma_{B'}$. Since $\Lambda_\beta(\Sigma'_B) = B'$ and $\Lambda_\beta^{-1}(x) \cap \Sigma_{B'} = f^{-1}(x) \cap \Sigma_{B'} = \emptyset$, $x \notin B'$. Since $cl_{kX}(f(F)) \subseteq B'$, $x \notin cl_{kX}(f(F))$. Thus f is a closed map and so f is a perfect map.

Since $m_X \circ \Lambda_\beta \circ \beta_{k\Lambda X} = \beta_{kX} \circ \Lambda^k_X$ and $m_X \circ \Lambda_\beta$ is a covering map, Λ^k_X is a covering map. Since $k\Lambda X$ is a basically disconnected space, there is a covering map $l : k\Lambda X \longrightarrow \Lambda kX$ such that $\Lambda_{\Lambda kX} \circ l = \Lambda^k_X$. Since $\Lambda X = \Lambda_\beta^{-1}(X)$ and $\Lambda kX = \Lambda_\beta^{-1}(kX)$, $l \circ k_{\Lambda X} = t_X \circ k_{\Lambda X}$, where $k_{\Lambda X} : \Lambda X \longrightarrow k\Lambda X$ is the inclusion map. Since $k_{\Lambda X}$ is a dense embedding, $l = t_X$ and t_X is a homeomorphism. □

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