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An Overview on Performamce Control and Efficient Design of Lateral Resisting Moment Frames

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Abstract

This paper presents a brief overview of the recently developed performance-control method of moment frame design subjected to monotonously increasing lateral loading. The final product of any elastic-plastic analysis is a nonlinear load-displacement diagram associated with a progressive failure mechanism, which may or may not be as desirable as expected. Analytically derived failure mechanisms may include such undesirable features as soft story failure, partial failure modes, over-collapse, etc. The problem is compounded if any kind of performance control, e.g., drift optimization, material savings or integrity assessment is also involved. However, there is no reason why the process can not be reversed by first selecting a desirable collapse mechanism, then working backwards to select members that would lead to the desired outcome. This article provides an overview of the newly developed Performance control methodology of design for lateral resisting frameworks with a view towards integrity control and prevention of premature failure due to propagation of plasticity and progressive P-delta effects.

Keywords: Performance control, Plastic design, Lateral loading, Sway mechanism, Stiffness degradation, Minimum weight, Uniform response, P-delta effects

1. Introduction

There has been a surge of interest in Performance-Based-Plastic-Design (PBPD) methods of Earthquake Resistant Frameworks (ERF), e.g., (Fajfar, 2000), (Priestly et al., 1991, 2005), (Goel et al., 2010) and (Grigorian and Grigorian, 2011, 2012a, 2012b) in the past ten years. ERF are special classes of ductile structures that are expected to undergo large inelastic displacements, while sustaining some degree of structural integrity. The focus of the present work is on the design component of closed form PBPD methodologies, with emphasis on integrity control, structural degradation and material optimization rather than the analytic intricacies associated with such design procedures. The P-delta effect tends to accelerate the global loss of strength and stiffness of ERF due to premature formation of plastic hinges in the weakest elements of the structure, thus compounding a highly nonlinear behavior with an even higher degree of complexity. Performance Control (PC) introduces a relatively simple technique for circumventing the computational difficulties arising from such mathematical complications, with the ability to prevent and/or to control the loss of structural integrity in terms of postulated occurrences. All ERF are to be designed and constructed in strict compliance with pertinent code requirements. However, there are also many instances where engineers may opt for higher standards and enhanced design features such as those discussed in section 2.2

The conservative assumption made in this work is that the effects of strain hardening, yield over-strength and plastic hinge offsets from column center lines can be ignored for preliminary design purposes.

2. Design Requirements and Features

2.1. Basic design requirements

The minimum requirements for efficient ERF design may be summarized as follows, that;

- the design shall correspond to a kinematically admissible failure mechanism that satisfies the prescribed yield criteria, static equilibrium and boundary support conditions,
- the maximum inter-story drift of each level shall be that specified by the codes,
- all columns shall remain stable and essentially elastic through all phases of loading, and
- detailing, specifications and workmanship shall be in accordance with the requirements of the prevailing codes of practice.

The first three of these requirements are incorporated into the proposed design formulae, therefore eliminating the need for further checking and investigation of the final

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results.

2.2. Enhanced design features

In addition to minimum design requirements, the following performance enhancing features, may also be incorporated as part of the proposed design strategy, that;

- the maximum allowable drift angle shall be the same for all stories, (this reduces the effects of the P-delta forces),
- the theoretical total weight of the structure shall be a minimum, (this is associated with lower costs),
- gravity loads shall not reduce the ultimate carrying capacity of the structure, (this eliminates beam mechanisms),
- design formulae shall address stiffness degradation and strength deterioration due to large axial forces and formation of plastic hinges, (this provides hands on control and added insight for the designer)
- the solution shall address damage control and predetermined target decisions.

The implementation of these requirements by conventional methods of approach may entail several cycles of elastic-plastic analysis and code checks of continuously degrading systems, until a satisfactory convergence of checks and balances is established. Unlike conventional methods of design, where members are selected through preliminary sizing, the proposed method relies on system specific performance, where the elements of the framework are selected in accordance with the aforementioned criteria as pre-imposed conditions. In other words, failure mechanisms and stability conditions are *enforced rather than tested*, both strength and stiffness are induced *rather than investigated*, and material saving is achieved by selecting member demand-capacity ratios as *close to unity* as possible. The method is designed to avoid numerically massive, often theoretically complicated, analysis needed to estimate the lateral displacements of the structure under steadily increasing lateral forces, and, by focusing attention on system specific characteristics, it enables the designer to control the response of the structure at pre-selected performance stages such as before and at first yield, any fraction of the failure load or specified drift ratios up to and including incipient collapse. The methodology reduces the otherwise complicated task of structural optimization to direct design through rational member selection and observation of recommended rules of application. Most importantly, the proposed formulations help engineers gain insight into the structural behavior of ERF of Uniform Response (UR), (Grigorian 2013), and lend themselves well to manual as well as spreadsheet computations. In structures of UR, members of the same group, such as beams, columns and braces, share the same demand-capacity ratios regardless of their numbers and location within the system. In PC theory of structures is applied rather than followed. The focus of the present article is on indirect design optimization rather than analysis of the resulting system. While the proposed approach is general and may be extended, with minor modifications, to the efficient desig of all types of ERF, with different boundary conditions, such as eccentric and concentric braced frames, special truss moment frames, shear walls, etc., it was deemed instructive to introduce the basis of the proposed algorithm by exploring the performance of grade beam supported regular moment frames under lateral loading. However, some of these applications may be limited in nature to none slender frames, as the assumption of uniform drift my not be compatible with higher modes of natural vibrations of tall buildings, (BSSC, FEMA-356, 2000) and (Goel and Chopra, 2004). It has been assumed that the effects of shear, axial, panel zone, cracking and time dependant deformations, on the formation of plastic hinges can be ignored for the purposes of this presentation.

3. Moment Frame Response

3.1. The failure patterns

An understanding of the intricacies of the ultimate load behavior of moment frames and their failure patterns is a priori to appreciating the essence of integrity control and minimum weight design. While the plastic design of common types of moment frames is investigated in connection with several failure patterns, including the soft story mode of Fig. 1d, moment frames of UR are designed, to fail through a global sway mechanism involving beam ends only. In fact, the elements of such frames are

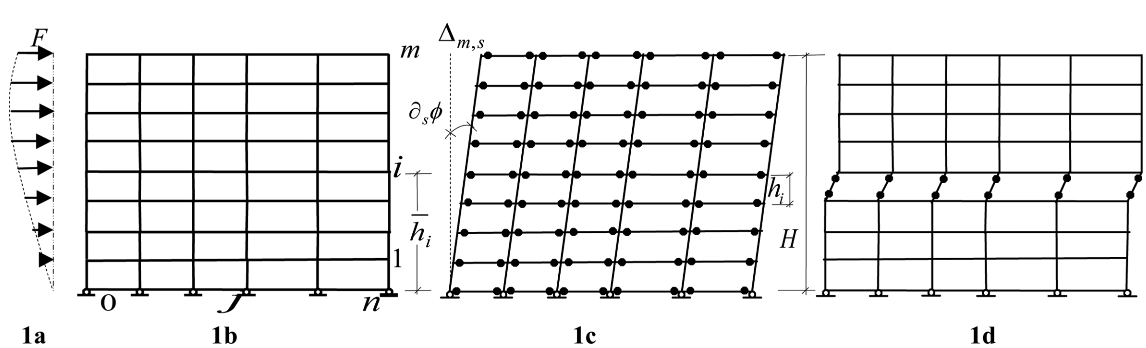

Figure 1. (1b) Laterally Loaded Moment Frame with, (1c) plausible and (1d), prevented failure mechanisms.

designed and arranged in such a way as to fail through a failure pattern that is compatible with the pre-selected linearly varying drift function, as shown in Fig. 1c. Inter-story failure, such as that depicted in Fig. 1d is prevented from occurring during all stages of formations of plastic hinges. Similar descriptions of failure mechanisms can be found in the MCEER-00-0010 report. Consider the performance of the regular moment frame of Fig. 1b under monotonically increasing distribution of lateral forces F_i, such as those presented in Fig. 1a and axial joint forces $P_{i,j}$ at each joint i,j, as shown in Fig. 2a. Also consider the proposed sway type, beams-only failure mechanism of Fig.1c, with an idealized design drift function of the form; $\sigma_s\phi_{i,s} = \sigma_s\phi_s = \partial/\partial\bar{h}_i\,(\bar{h}_i/H)\Delta_{m,s}$ or $\phi_{i,s} = \phi_s = \Delta_{m,s}/H$, where the code prescribed inter-story rotations $\phi_{i,s}$ or the i^{th} story drift ratios at s^{th} loading stage, remain the same along the height of the structure. Symbol σ_s implies increment at s^{th} consecutive iteration. Equal joint rotations also imply imposition of points of zero moments at column midpoints. This in turn suggests that the most ideal lateral deformation profile for any frame, is that in which the code prescribed story level displacements fall along the same straight line defined by $\sigma_s\Delta_{i,s} = \partial_s\phi_{i,s}\bar{h}_i = \partial_s\phi_s\bar{h}_i$, where, $\Delta_{m,s}$ is the roof or m^{th} level lateral displacement at s^{th} response stage. H and $\bar{h}_i$ are the roof and i^{th} level vertical distances from the base respectively. Now if $\Delta_{m,s} \le \Delta_{m,Code}$ and $(N^P_{i,j}+N^P_{i,j+1}) \ge \lambda(M^P_{i,j}+M^P_{i+1,j})$ where, $\lambda > 1.0$ is the column over-strength factor, and, N^P and M^P are the column and beam plastic moments of resistance, magnified by the P-delta effects, respectively, then the proposed failure pattern would be in compliance with the requirements of the first three design conditions mentioned above. It therefore remains to select the properties of the constituent elements of the structure in such a way as to achieve the additional performance goals listed under subsection 2.2 above. However, since the members of the frame are to be selected in such a way as to enforce a beams only failure mechanism, through proportional imposition of element strengths, it will not be necessary to check the carrying capacity of the structure for the unlikely soft story collapse mode of Fig. 1d.

3.2. Load-displacement relationships

The sequential changes in the inter-story drift angles of any level i can be shown to be composed of two dominant components; beam rotations $\partial_s\theta_{B.i,s} = \partial_s\theta_{B,s}$ and, column rotations $\partial_s\theta_{C.i,s} = \partial_s\theta_{C,s}$, i.e.,

$$\partial_s\phi_{i,s} = \partial_s\theta_{C.i,s} + \partial_s\theta_{B.i,s} \tag{1a}$$

Indexes B and C refer to beams and columns respectively. In order to verify the validity of the selected failure pattern, three independent equilibrium equations, pertaining to the static equilibrium of beams and columns of a representative sub-frame, Fig. 2a, and the global equilibrium of the entire structure, Fig. 1b, need to be satisfied. The equilibrium equation of the continuous beams of the i^{th} level sub-frame, in terms of beam stiffness $k_{i,j}$ and additional beam rotation $\partial_s\theta_{B,s}$ at any stage s can be expressed as;

$$\partial_s M^A_{i,s}/f_{Cr.i,s} = \sum_{j=1}^{n} 2\partial_s M_{B.i,j,s} = 12E\partial_s\theta_{B,s}\sum_{r=1}^{n}\delta_r^{s-1}k_{i,r} \tag{2a}$$

Where, for the sake of expediency, beam relative stiffness factors $k_{i,j}$ are related to the sequence of formation of plastic hinges s, by means of the replacement subscript r, rather than their location j. This is achieved by replacing $k_{i,j}$ with $k_{i,s}$ and $\Sigma^n_{j=1} = k_{i,j}$ with $\Sigma^n_{r=1}k_{i,r}$ and incorporating the symbol δ_r^{s-1} and δ_j in all forthcoming equations in order to include the effects of formation or prevention of formation of plastic hinges at the ends of beams i, s. $M^R_{i,s} = V_{i,s}h_i$ and $M^A_{i,s} = (M^R_{i,s}+M^R_{i+1,s})/2$ are defined as the racking and average racking moments acting on i^{th} level beams at s^{th} response stage. $M^A_{1,s} = M^R_{1,s}/2 = V_{1,s}h_1/2$, and $M^A_{m,s} = M^R_{m,s}/2 = V_{m,s}h_m/2$ define the average racking moments of the grade and roof level beams respectively. Eq. (2a) gives the corresponding changes in $\theta_{B,s}$ due to changes in $M^A_{i,s}$ as;

$$\partial_s\theta_{B,s} = \partial_s M^A_{i,s}/12Ef_{Cr.i,s}\sum_{r=1}^{n}\delta_r^{s-1}k_{i,r} \tag{2b}$$

With $\partial_s\theta_{B,s}$ known, the magnitude of the corresponding end moments of beam i,j at s^{th} response stage may be computed as; $\partial_s M_{B.i,s} = 6Ek_{i,s}\partial_s\theta_{B,s}$, then by substituting for $\partial_s\theta_{B,s}$ from Eq. (2b), it gives (Grigorian and Grigorian., 2012c);

$$\partial_s M_{B.i,s} = \frac{\partial_s M^A_{i,s}\times\delta_r^{s-1}k_{i,s}}{2f_{Cr.i,s}\sum_{r=1}^{n}\delta_r^{s-1}k_{i,r}} \tag{2c}$$

Eq. (2c) is also referred to as the moment increment formula. The group of Eqs. (2) represent both the elastic as well as plastic response of the subject sub-frame, where by definition the subscript s also represents the stiffest member of the i^{th} level beams. $f_{Cr.i,s}$, is the capacity reduction or moment magnification function elaborated upon in subsection 2.3 below. $\delta_r^{s-1} = 0$ for $M_{B.i,s} = M^P_{B.i,s}$ and implies loss of stiffness with respect to member i, s. $\delta_r^{s-1} = 1$ for $M_{B.i,s} < M^P_{B.i,s}$. In mathematical terms, δ_r^{s-1}

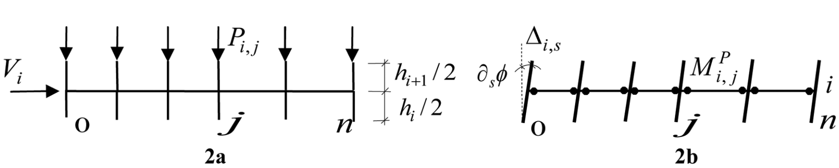

Figure 2. Laterally Loaded Representative Subframe with Constant Drift Profile.

= 1 for $r > s - 1$, and $\delta_r^{s-1} = 0$ for $r \leq s-1$. Similarly the equilibrium equation of the columns of the i^{th} level sub-frame, in terms of column stiffness $k_{i,j}$ and additional column rotation $\partial_s\theta_{C,s}$ at any stage s can be expressed as;

$$\partial_s V_{i,s} h_i / f_{Cr.i,s} = \partial_s M^R_{i,s} / f_{Cr.i,s} = \sum_{j=0}^{n} 2\partial_s M_{C.i,j,s} \tag{3a}$$

$$= 12E\partial_s\theta_{C,s}\sum_{j=0}^{n}\bar{\delta}_j\bar{k}_{i,j}$$

$$\partial_s\theta_{C,s} = \partial_s M^R_{i,s} / 12Ef_{Cr.i,s}\sum_{j=0}^{n}\bar{\delta}_j\bar{k}_{i,j} \tag{3b}$$

The additional end moments of column i,j at s^{th} response stage may be computed as; $\partial_s M_{C.i,s} = 6E\bar{k}_{i,s}\,\partial_s\theta_{C,s}$, then Eq. (3b) gives;

$$\partial_s M_{C.i,s} = \frac{\partial_s M^R_{i,s} \times \bar{\delta}_s\bar{k}_{i,s}}{2f_{Cr.i,s}\sum_{r=0}^{n}\bar{\delta}_r\bar{k}_{i,r}} \tag{3c}$$

$\bar{\delta}_j$ and $\bar{\delta}_r$ have been introduced to include the contribution or lack of contribution of column stiffness to overall stiffness of the structure due to formation of plastic hinges at the ends of the adjoining beams. Numerically, $\bar{\delta}_j = 0$ for $M_{B.i,j} = M^P_{B.i,j}$ and $M_{B.i,j-1} = M^P_{B.i,j-1}$, otherwise $\bar{\delta}_j = 1$. Substituting for $\partial_s\theta_{B.i,s}$ and $\partial_s\theta_{C.i,s}$ from Eqs. (2b) and (3b) respectively into Eq. (1a) and rearranging, yields;

$$\partial_s\phi_{i,s} = \frac{\partial_s V_{i,s} h_i}{12Ef_{CR.i,s}}\left[\frac{1}{\sum_{j=0}^{n}\bar{\delta}_j\bar{k}_{i,j}} + \frac{\partial_s\eta_{i,s}}{\sum_{r=1}^{n}\delta_r^{s-1}k_{i,r}}\right] = \frac{\partial_s V_{i,s}}{f_{CR.i,s}h_i K_{i,s}} \tag{4a}$$

as the total drift angle variation of the i^{th} level sub-frame at s^{th} response stage. $\eta_i = M^A_i / M^R_i$ is generally referred to as the story level racking ratio. $\eta_0 = \eta_m = 1$ for grade and roof level beams. And, since, $M^R_{m+1,s} = 0$, then the total additional drift angle of the uppermost level at $i = m$ could be expressed as;

$$\partial_s\phi_{m,s} = \partial_s\phi_s = \frac{\partial_s V_{m,s} h_m}{12Ef_{CR,m,s}}\left[\frac{1}{\sum_{j=0}^{n}\bar{\delta}_j\bar{k}_{m,j}} + \frac{1}{\sum_{r=1}^{n}\delta_r^{s-1}k_{m,r}}\right] \tag{4b}$$

$$= \frac{\partial_s V_{m,s}}{f_{CR.i,s}h_m K_{m,s}}$$

Eq. (4a) is also known as the drift increment formula and reduces to a previously established result for $i = j = 1$, (Grigorian, 1993). Eqs. (2c) and (4a) together describe the step-by-step response of the constituent elements of the structure caused by the phenomenon known as progressive collapse. The physical meanings of δ_j and δ_r in relation to existence or lack of existence of plastic hinges at beam ends are portrayed in Fig. 3. The existence of plastic hinges at the ends of the adjoining beams of Figs. 3a and 3b, render the stiffnesses of the subject columns inactive, whereas the lack of or the insufficiency of number of plastic hinges in Figs. 3c, 3d and 3e, allows the corresponding columns to contribute towards the global stiffness of the framework.

Plots of Eq. (4b), as presented in Fig. 5, for a generic single story frame, indicate three distinct modes of behavior: perfectly elastic, signified by the straight line running from $\phi = 0$ to first yield at ϕ_Y, the elastic-plastic curvilinear segment from ϕ_Y to ϕ_P at incipient collapse, and the perfectly plastic region corresponding to the fully ductile nature of the system. Goel and Chopra (2004) have shown that the plastic beam rotations predicted by the corresponding components of Eq. (4a) are in excellent agreement with the results of response history analysis conducted for a group of six buildings ranging from 9 to 20 stories in height.

3.3. Variations of the Capacity Reduction Function

The term $f_{Cr.i,s} = [1-(\Sigma^n_{j=0}P_{i,j})/P_{CR.i,s}]$ is also known as the moment magnification or capacity reduction function. $\Sigma^n_{j=0}P_{i,j}$ and $P_{Cr.i,s} = K_{i,s}h_i$ are the total axial load and the critical axial load of level i at s^{th} response stage respectively. The elastic failure load $P_{Cr.i,s}$ may also be interpreted as that total axial load at which the stiffness of the subframe becomes zero. The capacity reduction function is highly sensitive to variations in story level stiffness. While $\Sigma^n_{j=0}P_{i,j}$ remains constant for all s, the stiffness $K_{i,s}$ diminishes rapidly with advancing plasticity. If the original value of $f_{Cr.i,s=1} = (1 - \rho_{i,s=1})$ for $s = 1$, then the subsequent values of the capacity reduction function in terms of diminishing subframe stiffness may be expressed as;

$$f_{Cr.i,s} = [1-\rho_{i,s=1}(K_{i,1}/K_{i,s})] \tag{5a}$$

As demonstrated in the generic example of subsection 6.3, $f_{Cr.i,s}$ decreases rapidly with advancing plasticity and further deteriorates the global stiffness and load carrying capacity of the structure up to and including incipient collapse. It may be instructive to note that, for the subject example, $f_{Cr.i,s}$ decreases from 0.9, at zero lateral loading, to 0.41 at incipient collapse.

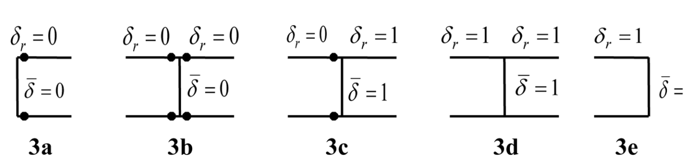

Figure 3. Pictorial Presentation of the Physical Meanings of $\bar{\delta}_j$ and δ_r.

4. The Rules of Proportionality

By definition, a structural framework of UR is that in which its sub-frames share the same drift ration and that its members share the same demand-capacity ratios regardless of their numbers and location within the structure. Therefore, in order to achieve uniform demand-capacity throughout the system and fulfill the condition of compatible drift angles along the height of the frame, the following conditions should be satisfied;

4.1. Global compatibility

The global equilibrium equation of the moment frame of UR, in terms of its incremental drift angles and beam moments can now be written as;

$$\sum_{i=1}^{m}\partial_s M_{i,s}^{R}/f_{Cr.i,s}=\sum_{i=1}^{m}\partial_s F_{i,s}\bar{h}_i/f_{Cr.i,s} \quad (6a)$$

$$=\sum_{i=0}^{m}\sum_{j=1}^{n}2\partial_s M_{B.i,j,s}=12E\partial_s\theta_{B,s}\sum_{i=0}^{m}\sum_{j=1}^{n}\delta_r^{s-1}k_{i,j}$$

Equating Eq. (2b) and (6a), gives;

$$\partial_s M_{i,s}^{A}/\left(f_{Cr.i,s}\sum_{j=1}^{n}\delta_r^{s-1}k_{i,j}\right) \quad (6b)$$

$$=\sum_{i=1}^{m}\partial_s M_{i,s}^{R}/\sum_{i=0}^{m}\sum_{j=1}^{n}(\delta_r^{s-1}k_{i,j}f_{Cr.i,s})$$

Assuming that the ratio $\Sigma_{j=0}^{n}P_{i,j}/P_{Cr.i,s}$ is constant for all i, then $f_{Cr.i,s}$ would also be constant along the height of the structure. The constancy of $\partial_s\theta_{C.i,s}=\partial_s\theta_{C,s}$, implies a state of uniform stiffness demand-capacity for all beams of the frame. Next, substituting for $\Sigma_{i=1}^{m}\partial_s M_{i,s}^{R}=\Sigma_{i=0}^{m}\partial_s M_{i,s}^{A}$ Eq. (6b) becomes;

$$\partial_s M_{i,s}^{A}/\left(f_{Cr.i,s}\sum_{j=1}^{n}\delta_r^{s-1}k_{i,j}\right) \quad (6c)$$

$$=\sum_{i=1}^{m}\partial_s M_{i,s}^{A}/\sum_{i=0}^{m}\sum_{j=1}^{n}(\delta_r^{s-1}k_{i,j}f_{Cr.i,s})$$

Eq. (6c) can be satisfied only if the following proportionality conditions are met;

$$\sum_{j=1}^{n}k_{1,j}=\frac{\partial_s M_{1,s}^{A}}{\partial_s M_{m,s}^{A}}\sum_{j=1}^{n}k_{m,j}\;...\;\sum_{j=1}^{n}k_{i,j}=\frac{\partial_s M_{i,s}^{A}}{\partial_s M_{m,s}^{A}}\sum_{j=1}^{n}k_{m,j}\;... \quad (6d)$$

It follows therefore, that a similar rule of proportionality can be deducted for beam moments, i.e.,

$$M_{B.1,j}=\frac{\partial_s M_{1,s}^{A}}{\partial_s M_{m,s}^{A}}M_{B.m,j}\;...\;M_{B.i,j}=\frac{\partial_s M_{i,s}^{A}}{\partial_s M_{m,s}^{A}}M_{B.m,j}\;... \quad (6e)$$

In other words, the condition of uniform drift requires that the sum of the stiffnesses of beams of each story be selected in proportion with the average racking moments of that story. The practical implications of Eq. (6e) is, that if a suitable solution can be found for any subframe, say that at $i = m$, then the corresponding solutions of all other sub-frames can be computed in proportion with their average racking moments. A similar set of rules could be derived for the columns, i.e.,

$$\sum_{j=0}^{n}\bar{k}_{1,j}=\frac{\partial_s M_{1,s}^{R}}{\partial_s M_{m,s}^{R}}\sum_{j=0}^{n}\bar{k}_{m,j}\;...\;\sum_{j=0}^{n}\bar{k}_{i,j}=\frac{\partial_s M_{i,s}^{R}}{\partial_s M_{m,s}^{R}}\sum_{j=0}^{n}\bar{k}_{m,j}\;... \quad (6f)$$

$$M_{C.1,j}=\frac{\partial_s M_{1,s}^{R}}{\partial_s M_{m,s}^{R}}M_{C.m,j}\;...\;M_{C.i,j}=\frac{\partial_s M_{i,s}^{R}}{\partial_s M_{m,s}^{R}}M_{C.m,j}\;... \quad (6g)$$

4.2. Applications

Eqs. (6e) through (6g) collectively represent the proportionality rules for moment frames of UR. $\partial_s C_{B.i,j}=\partial_s M_{i,s}^{A}/\partial_s M_{m,s}^{A}$ and $\partial_s C_{C.i,j}=\partial_s M_{i,s}^{R}/\partial_s M_{m,s}^{R}$ are the proportionality multipliers for the beams and columns of the i^{th} level subframe at s^{th} loading stage respectively. Once the multipliers $C_{B.i,j}$ and $C_{C.i,j}$ are known, the required design items $I_{i,j}$, $J_{i,j}$, $M_{i,j}^{P}$ and $N_{i,j}^{P}$ can be computed in terms of their corresponding quantities, that bear the subscript $i = m$. Therefore, by direct proportioning;

$$I_{i,j}=C_{B.i,j}\times I_{m,j} \quad \text{and} \quad I_{i,j}=C_{C.i,j}(h_i/h_m)\times J_{m,j} \quad (6h)$$

$$M_{ij}^{P}=C_{B.i,j}\times M_{m,j}^{P} \quad \text{and} \quad N_{i,j}^{P}=C_{C.i,j}\times N_{m,j}^{P} \quad (6k)$$

In other words, if the drift ratio is to remain constant, i.e., $\phi_i=\phi_m$, then the stiffness $K_i=(V_i/V_m)(h_m/h_i)K_m$ of all other levels can also be determined by simple proportioning. The use of these multipliers is shown in section 7.2 below.

4.3. The energy absorption capacity

Since the load-displacement relationship (4b) of any level i can be expressed in terms of a single variable $\phi_{i,s}$, for all phases of the monotonically increasing lateral loading, then the entire subfloor may be construed as a statically determinate, single degree of freedom (SDOF) system, for which the total internal energy absorption capacity may be expressed as;

$$U_i=\sum_{r=1}^{n}(F_{i,r-1}+F_{i,r})\partial_r\phi_{i,r}/2 \quad (6m)$$

U_i may be computed as the area under the corresponding load-displacement curves, such as those depicted in Fig. 4. The energy absorption capacity, U_i, becomes more meaningful when used in conjunction with the equivalent dynamic energy, U_E, due to Housner (1992) for the base shear determination of SDOF ductile systems, under seismic conditions. It is instructive to note that the term U_i of Eq. (5b) contains the capacity reduction factors $\bar{\delta}_j$, δ_r^{s-1} and $f_{Cr.i,s}$. The effects of these factors on the magnitude of U_i are discussed under subsection 6.3 and 7.2 below. The same rules of proportionality also apply to the computation of the energy absorption capacity, U_i of any level i, in terms of the corresponding quantity of any other level, say that of the roof level sub-frame, U_m, i.e.,

$$(U_i/U_m)=(V_i/V_m)^2(K_m/K_i)=M_i^R/M_m^R \tag{6n}$$

Eq. (6n) can now be used to compute the total energy absorption capacity of the entire structure in terms of the roof level capacity U_m, racking moment M_m^R and the total overturning moment M_O, i.e.

$$U=(U_m/M_m^R)M_O \tag{6p}$$

While the term U_i is valid for all typical subframes discussed in this article, the summation Eq. (6p) is applicable only to moment frames of UR where the energy absorption capacity of each subframe is proportional to the demand imposed upon it. The global energy capacity is used in conjunction with the equivalent elastic input energy from the ground motion to estimate the corresponding base shear in terms of structure and site specific data.

5. Plastic Limit State

5.1. Plastic design and minimum weight solution

An important characteristic of proportionately designed subframes of UR (Grigorian, 1989) is that the weight of each segment is a minimum with respect to the demand imposed upon it. Minimum weight segments are, obviously, more flexible than their regular counterparts and need to be safeguarded against local instabilities at large axial loads. The softening or loss of stiffness of such frames may be estimated, to a very good degree of accuracy by including the P-delta effects in the corresponding plastic limit state analysis, as presented in this section. Since Eq. (6f) is valid for all loading stages, then its final form at incipient collapse may be expressed as;

$$M_{i,j}^P=(M_i^A/M_m^A)M_{m,j}^P \tag{7a}$$

The virtual work equation corresponding to failure pattern of Fig. 1c, may be expressed as;

$$\sum_{i=1}^{m}F_i\theta\bar{h}_i=\sum_{i=1}^{m}V_ih_i\theta=\sum_{i=1}^{m}\sum_{j=1}^{n}(2M_{i,j}^P+P_{i,j}\delta_i)\theta \tag{7b}$$

where, θ is an auxiliary virtual rotation, corresponding to $\delta_i=\theta h_i$. Eq. (7b) may be simplified, in terms of the average racking moments and the moment magnifying factor as;

$$\sum_{i=0}^{m}M_i^A=\sum_{i=1}^{m}\sum_{j=1}^{n}2f_{Cr.i}M_{i,j}^P \tag{7c}$$

The virtual work equation corresponding to the typical subframe failure mechanism of Fig. 2b, with uniform floor moment of resistance $M_{i,j}^P=M_i^P$ for all j, can be written down as;

$$\sum_{j=1}^{n}2f_{Cr.i}M_{i,j}^P=2nf_{Cr,i}M_i^P=M_i^A \tag{7d}$$

Substituting for $M_m^P=M^P$ and $M_m^A=F_Ph_m/2f_{Cr.m}$, assuming $f_{Cr.i}$ is constant for all practical purposes, and combining Eqs. (7a), (7c) and (7b) gives;

$$F_P=4nM^Pf_{Cr.m}/h_m \tag{7e}$$

as the plastic collapse load of the structure in terms of the plastic moment of resistance of the roof level beams and is used to verify the validity of the long hand solutions presented in the forthcoming sections.

5.2. On soft story failure

Theoretically, an independent soft story failure, involving columns only, can also be envisaged for a typical subframe at level *i*. The virtual work equation for the shear type collapse mechanism of Fig. 2c, may be expressed in terms of column moments of resistance as;

$$M_i^R\theta=V_ih_i\theta=\sum_{j=0}^{n}(2N_{i,j}^P+P_{i,j}\delta_i)\theta \tag{7f}$$

With $N_{i,0}^P=N_{i,n}^P=N_i^P$ and $N_{i,j}^P=2N_i^P$ for all other j, Eq. (7f) reduces to; $V_ih_i=8nN_i^Pf_{Cr.i}$, which, when compared with $V_mh_{mi}=8nN_m^Pf_{Cr.m}$ gives $N_{i,j}^P=(V_ih_i/V_mh_m)N_{m,j}^P=C_{C.i,j}\times N_{m,j}^P$ a result, previously established through direct proportioning. Now if $N_m^P=\lambda M^P$ and as indicated in section 3.1, $\lambda>1.0$, then no soft story mechanism can develop within the subframes of the system. Eq. (7a) describes a state of uniform demand-capacity for all members of the structure at collapse. However, since uniform demand-capacity means providing just as much capacity as demanded, then the entire framework constitutes a structure of UR and therefore of minimum weight. In other words, since the proposed yield pattern satisfies both the kinematic as well as the static conditions of plastic failure, is compatible with the boundary support conditions and does not violate the prescribed yield criteria, then the design is *unique* and represents a minimum weight solution, (Foulkes, 1953) and (Neal, 1963) and Since the factor $f_{Cr.m}$ in Eq. (7e), is independent of the mode of propagation of plasticity, then it can also be associated with an equivalent single stage loading, $s=1$, i.e., when all plastic hinges form simultaneously. This limits the use of the virtual work method to the analysis of moment frames of single stage UR under combined axial and lateral forces and/or similar frames under negligible gravity loading. This problem is resolved, as described in the next section, by allowing for the stiffness degradation of the yielding members and controlling the sequential formation of the plastic hinges.

5.3. On boundary support conditions

It is rather significant that the plastic failure load, Eq. (7e), of the generic moment frame of UR, with moment resisting grade beams, subjected to an arbitrary distribution of lateral forces with an apex value F at $i=m$, is independent of the number of stories, m, and the type of boundary support conditions at $i = 0$. The grade beams provided at the base of the generalized moment frame of Fig. 1b, not only constitute integral parts of the lowermost sub-frame but also act as natural control devices used to prevent formation of plastic hinges at the lower ends of the base level columns. The intuitive substitution of $N^P_{i=0} = 0$ for hinged supports, and $N^P_{i=0} = N^P_{i=1}$ for fixed end conditions, could lead to violation of the prescribed yield criteria as well as incorrect distribution of internal forces at the most critical sub-frame of the system. However, in order to incorporate the effects of column supports at the base, without raising major theoretical issues, it was deemed expedient to rewrite the proportionality rules (6n) for the members of the base level subframe in terms of the column over strength factor λ and symbols defining the standard boundary conditions, i.e.,

$$M^P_{0,j} = \delta_{beam} C_{B.0,j} M^P_{m,j} = \delta_{beam}(V_1 h_1/4nf_{Cr}) \tag{7g}$$

$$M^P_{1,j} = \{C_{C.2,j} + [\delta_{fix} + \delta_{beam} + \delta_{pin}] (2 - \delta_{beam} - \delta_{fix}) C_{C.1,j}\} f_{Cr} M^P_m \tag{7h}$$

$$N^P_{1,j} = (2 - \delta_{beam} - \delta_{fix}) C_{C.1,j} N^P_{m,j} = (2 - \delta_{beam} - \delta_{fix}) \lambda (V_1 h_1/4nf_{Cr}) \tag{7k}$$

Where, the boundary symbols $\delta_{fix..}$, $\delta_{pin..}$ and $\delta_{beam..}$ are numerically equal to unity with respect to their own case, and are zero for the other two cases. However, in order to show that in moment frames of UR, the representative failure load, $F_P = 4nM^P f_{Cr.m}/h_m$ is independent of both the number of stories and the boundary support conditions, it would be sufficient to substitute Eqs. (7g), (7h) and (7k) into the expanded virtual work Eq. (7m) and perform the corresponding summations for any given moment frame of UR. It will be seen that the right hand side of Eq. (7m) remains the same for the different boundary support types discussed in this work.

$$\sum_{i=1}^{m} F_i \bar{h}_i = 2\sum_{j=1}^{n}(M^P_{0,j} + M^P_{i,j}) + 2\delta_{fix}\sum_{j=0}^{n} N^P_{i,j} + 2\sum_{i=2}^{m}\sum_{j=1}^{n} M^P_{i,j} + \sum_{i=1}^{m}\sum_{j=0}^{n} P_{i,j}\,\delta_i \tag{7m}$$

5.4. Introductory example I

Assuming $f_{Cr.i} = f_{Cr.}$, $\lambda_i = \lambda = 1$, $h_1 = h_2 = h_3 = h = L$ and $\rho = (JL\,/\,Ih) = 1$, generate moment frames of UR, with hinged, fixed and grade beam supported first floor columns and compare their performance for the same uniform drift angle ϕ. For the loading and frame configuration see Fig. 4. Using Eq. (6k) and the following drift angle formulae for the lowermost subframes of the three alternatives at incipient collapse; $\phi_{beam} = Fh^2/12f_{Cr.}EI_{beam}$, $\phi_{pin} = Fh^2/4f_{Cr.}EI_{pin}$ and $\phi_{fix} = Fh^2/12f_{Cr.}EI_{fix}$, it gives; $I_{beam} = I$, $I_{pin} = 3I$, and $I_{fix} = I$.Next, using Eq. (7e) as the starting point, for plastic moment calculations and employing Eq. (6k) in conjunction with directives of Eqs. (7g) through (7m) and (6h), the desired solutions are worked out as shown in Figs. 4a through 4c. The virtual work equation for all thee cases of Example I may be written as;

$$\left(3 + \frac{4}{3} + \frac{1}{3}\right)\frac{Fh}{f_{Cr.}} = 2\left[\frac{3}{3} + \frac{8}{3} + \frac{5}{3} + \frac{6}{3} + \frac{6}{3}\right]M^P \tag{7n}$$

which reduces to Eq. (7e). Similarly, the lateral displacement equation for the all three cases may be expressed as; $\delta_i = Fh^3 i/12f_{Cr.}EI$, where $i = 1$, 2 and 3. To compare the efficiencies of the three basic boundary support conditions, their total weight functions in terms of their section properties may be expressed as;

$$G = C[\Sigma^3_{i=0} LM^P_i + 2h\Sigma^3_{i=1} N^P_i] \tag{7p}$$

where, C is an arbitrary constant of proportionality. Assuming $\lambda = 1.1$ and substituting for M^P_i and N^P_i from the frames of Fig. 4 into Eqn. (7p), it gives; $G_{beam} = 58.8\,CLM^P/3$, $G_{pin.} = 72CLM^P/3$ and $G_{fix} = 52.8CLM^P/3$. As expected the stiffer boundary support conditions result in higher economies with respect to the same external loading.

5.5. Sequential propagation of plasticity

Fig. 1c depicts the final stage of formation of plastic hinges corresponding to a kinematically admissible failure mechanism, but furnishes no information regarding the history of formation of plastic hinges before collapse. While the minimum number of sequences of formations of plastic hinges could be as small as $s_{min} = 1$, the maximum number of such formations, leading to the same failure pattern could be as high as; $s_{max} = (m + 1)n$, or the total number of beams in the structure. However, experi-

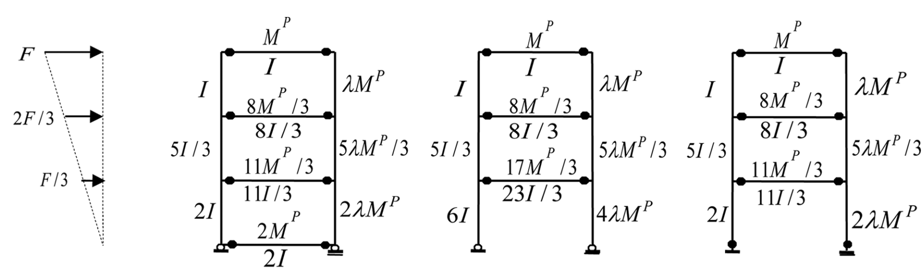

Figure 4. Moment Frames of UR with Different Boundary Support Condition.

ence has shown that, $s_{max} = n =$ number of bays or $s_{max} = m =$ numbers of stories offer more practical options for design purposes. For instance, if option $s_{max} = n$ is selected for the purposes of this section, then Eq. (2c) can be employed to establish the first, $r = 1$, increment of loading that causes formation of the first set of plastic hinges in the beams of the stiffest elements of any representative story, say the roof level beams. i.e.,

$$\partial_1 F_{m,1} = \frac{4M^P}{k_{m,1}h_m} f_{Cr.m,1} \sum_{r=1}^{n} k_{m,r} \tag{8a}$$

Obviously, the balance of bending moment needed to elevate the moment of resistance of the next stiffest beam, $s = 2$, to M^P can be computed as $[1 - (k_{m,s=2}/k_{m,s=1})]M^P$. Therefore, the amount of additional force required to generate plastic hinges at the ends of the next stiffest beam may be generalized as;

$$\partial_s F_{m,s} = \frac{4M^P}{k_{m,s}h_m} f_{Cr.m,s}\left(1 - \frac{k_{m,s}}{k_{m,s-1}}\right) \sum_{r=1}^{n} \delta_r^{s-1} k_{m,r} \tag{8b}$$

Interestingly enough, the sum of the incremental forces, $\partial_s F_{m,s}$ for constant $f_{CR,m}$, can be shown to add up to the ultimate load, $F_m^P = F^P$, i.e.,

$$F_m^P = \sum_{s=1}^{n} \partial_s F_{m,s} = \sum_{s=1}^{n} \frac{4M^P}{k_{m,s}h_m} f_{Cr.m,s}\left(1 - \frac{k_{m,s}}{k_{m,s-1}}\right) \sum_{r=1}^{n} \delta_r^{s-1} k_{m,r}$$

$$= 4nf_{Cr.m} M^P / h_m \tag{8c}$$

a result, that as expected, coincides with Eq. (7e) above. Obviously, since $f_{CR.m,s} < 1.0$ then for any value of the capacity reduction function, $F^P < 4nM^P/h_m$. Eqs. (4a) and (8a) indicate that each stage of propagation of plastic hinges characterized by $s = 1, 2\ldots, n$ may be construed as a target design point or a state of stable damage with respect to fully elastic or fully plastic conditions of the structure. The final stage characterized by Eqs. (8c) or (7e) also represents a *minimum* weight, *unique* state plastic design since it satisfies the prescribed yield criteria, and static equilibrium as well as the selected boundary support conditions at incipient collapse. This implies that the proposed scheme also provides an envelope of several initial designs within which member sizes could be rearranged for any purpose while observing the prescribed performance conditions. For long hand numerical solutions the interested reader is referred to (Grigorian and Grigorian, 2012c).

6. On Structural Integrity

6.1. On degradation of structural integrity

In a severe seismic event, ERF have the potential to fail in a side sway mode due to diminishing structural integrity. Structural integrity may be defined as the measure by which a stable system performs under severe or extraordinary loading conditions**.** Depending upon the functional use of a structure, its integrity may be related to a demand-capacity quotient or a load-displacement relationship. In either case, structural integrity is always associated with global strength F_s and stiffness K_s, at any loading stage s. An appreciation of the nature of degradation of the sub-frame stiffness $K_{i,s}$, and sub-frame ultimate capacity $F_{i,s}^P$, under monotonically increasing lateral forces, is essential to understanding the performance of ERF in general, and moment frames in particular. Progressive plasticity and +increasing P-delta effects tend to rapidly diminish the global strength and stiffness and modify the static as well as dynamic characteristics of ERF under seismic conditions. The effects of structural degradation are more pronounced in multistory frames since many members with similar characteristics either, fail, become inactive or develop plastic hinges simultaneously. Sequential degradation of the global strength and stiffness, in structures sustaining incremental forces can, in general, manifest itself through several or combination of several effects, namely by;

- reducing the energy absorption capacity of the structure, depending upon the sequences and patterns of formations of the plastic hinges,
- increasing the risks of local, racking and global instabilities due to overburdening of the remaining active elements,
- increasing the natural period of vibration of each stage of global loss of stiffness with advancing stages of loading until the structure ceases to resist external forces,
- reducing the load carrying capacity of the structure during all loading stages, and at plastic collapse, due to ever increasing P-delta effects.
- increasing the magnitude of the lateral drift ratio throughout the loading history of the framework.

6.2. On quantification of structural integrity

The ability to define and quantify structural integrity for ERF may lead to prevention of disproportionate, global and/or progressive collapse of earthquake resisting systems. The global loss of structural integrity of earthquake resisting moment frames, amongst other causes, can be attributed to any one or combinations of the following member related factors;

- Loss of individual beam stiffness, due to formation of plastic hinges at beam ends, symbolized by $\delta_r^{s-1} = 0$ in Eqs. (2c) and (4a),
- Loss of individual column stiffness, due to formation of plastic hinges at column ends, symbolized by $\overline{\delta} = 0$ in Eqs. (3c) and (4a),
- Inactiveness of individual column stiffness, due to formation of plastic hinges at the ends of the adjoining beams, symbolized by $\overline{\delta} = 0$, and as shown in figure 3 above,
- Loss of subframe racking stiffness $K_{i,s}$, due to collective P-delta effects symbolized by $f_{CR.i}$ and, $\overline{\delta}$ and δ_r^{s-1}, appearing together in all equations of this article.

In order to manage the myriad factors that influence structural integrity, it seems rational to define a single, uniform criterion that can address each form of deterioration separately, on the same scale, say from one to zero. Since there are no commonly accepted definitions or measures for structural integrity, the authors propose to compare any intermediate state of structural integrity with its original, undamaged, perfectly elastic condition, in terms of such meaningful, quantities as; $(K_s/K_{s=1})$ as a measure of stiffness degradation, (F_s/F_P) as an indication of diminishing strength, (U_s/U_P) as the remaining energy absorption capacity quotient and (ϕ_s/ϕ_P) as the rotational capability of the remaining, undamaged system, etc,. Therefore the proposed integrity ratio at any loading stage may be expressed as;

Integrity Ratio = Remaining Capacity / Original Capacity (9a)

The parametric example, 6.3, has been designed to demonstrate how rapidly $f_{Cr.i,s}$ decreases with advancing plasticity and deteriorates the general integrity of the structure during all loading stages up to and including incipient collapse. It may be seen from the numerical results of example 5.3, Table 1, that as $f_{CR.i,s}$ decreases from an initial value of 0.90 to 0.87 at first yield, the corresponding integrity ratios, in terms of remaining stiffness, load carrying capacity, energy absorption, and drift allowance become 0.76, 0.27, 0.68 and 0.47 respectively. Apparently, while the structure has retained over 75% of its global stiffness, K, it has lost over the same percentage of its load carrying capacity F_P at first yield. Finally, since the rate of degradation of structural integrity is a function of the pattern and sequence of formation of plastic hinges, all subscripts j have been replaced with subscript s to relate the loss of structural integrity to sequential formation of plastic hinges. However, while the proposed integrity measures could be used effectively in connection with simple ERF, it is felt that more research is needed before a consensus can be reached on the subject.

6.3. Introductory example II

Consider the incremental lateral displacements of a singly story $(m = 1) \times (n = 10)$ moment frame with grade beams, subjected to a roof level, monotonically increasing lateral force F and axial nodal loads P. The roof and the grade level beams have identical properties. The geometric and sectional properties of the frame are given as; $L_1 = L_2 = L = h$, $L_3 = L_4 = 1.25L$, $L_7 = L_8 = 1.75L$, $L_9 = L_{10} = 2.0L$. $I_j = I$, $J_j = 2J_1 = 2.2I$, $J_1 = J_{10} = J = 1.1I$, $\rho_1 = 10P/P_{CR} = 0.1$, $M_j^P = M^P$ and $N_j^P > 2M^P$. i.e., $k_1 = k_2 = I/L$, $k_5 = k_6 = I/1.5L$, $k_3 = k_4 = I/1.25L$, $k_7 = k_8 = I/1.75L$, $k_9 = k_{10} = I/2L$, $\bar{k}_1 = \bar{k}_{10} = J/h = 1.1I/L$ and $\bar{k}_j = 2.2I/L$ for all other "j". Since for $s = 1$, $\bar{\delta}_j = \delta_r^{s-1}$, then; $\Sigma_{j=0}^{10}\bar{\delta}_j\bar{k}_j = 20J/h = 22I/L$ and $2\Sigma_{j=1}^{10}\delta_1^0 k_j = 2 \times 7.0762I/L = 14.1524I/L$.

The purpose of the long hand solution presented herein is to demonstrate the influences of the failing and/or inactive members in deteriorating the structural integrity of the system, due to propagation of plasticity and continuously increasing effects of the capacity reducing function.

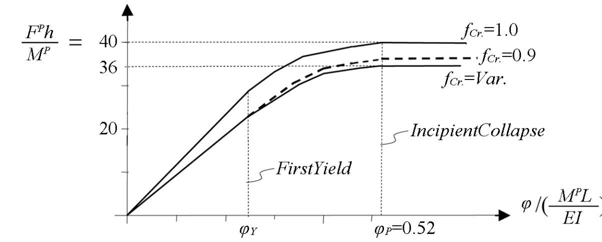

Figure 5. Structural Deterioration due to Effects of the Capacity Reduction Factors.

As there are at least five distinct sets of beam stiffnesses, then $s = 5$. Eqs. (2c), (4b), (5a), and (8a) are then used to provide the complete solution to the progressive plasticity of the subject moment frame as presented in Fig. 5 and Table 1 below, where $K_0 = EI/Lh^2$, $\phi_0 = M^P L/EI$ and $F_0 = M^P/h$. Figure 5 displays the load-displacement relationship for three distinct values of the capacity reduction function. The upper curve, corresponding to $\rho = 0$ and $f_{Cr} = 1.0$, i.e. no P-delta effects, has been provided for comparison only. The dashed curve corresponds to a constant value of $\rho = 0.1$ and $f_{Cr} = 0.9$, a number favored by most codes, e.g., ACI 318-02 Ch.21, and has been presented to illustrate the effects of constant magnifying factor on the performance of the system. The lowest curve corresponds to a naturally varying f_{CR} with an initial $\rho = 0.1$, and becomes smaller with advancing plasticity. The implications of the three conditions may be studied by comparing their effects on the limit state performance of the subject structure, i.e.; $F^P_{\rho=0.1} = 40.00M^P/h$, $F^P_{\rho=0.1,Const} = 36.00M^P/h$ and $F^P_{\rho=0.1,Var.} = 34.67M^P/h$. Loss of capacity can also be computed by comparing the actual capacities of the system with respect to different conditions of f_{Cr}. The internal energy absorption capacity $U_{\rho=0}$ for the preceding example can be worked out as the total area under the upper force-displacement curve of Fig. 4, i.e., $U_{m,4} = 12.54(M^P)^2L/EI$. Similarly, the quantity $U_{\rho=0.1,\text{Var.}} = 11.04(M^P)^2L/EI$ can be computed as the reduced capacity of the entire system due to progressive P-delta effects.

7. Design Strategy

7.1. Preliminary member selection

While the preliminary sizing of the members of ERF, due to iterative processes, has little or no effect on the outcome of linear design solutions, it may impact the mode of formation of the plastic hinges as well as the ultimate carrying capacity of the subject frameworks. In the proposed methodology the entire design strategy is based on the rational selection of the constituent elements of the system, with a definite outcome in mind, rather than investigating the results of numerical computations.

Perhaps the most useful utility of the proposed procedure is the ability to control the manner and sequences of formation of plastic hinges with respect to predetermined force levels or stipulated drift angles. Practically speaking, there are several preliminary selection options and their combinations for the beams and columns of representative subframes. However, the following three options lend themselves well to practical performance control and strategic decision making, namely;

1-Beams and columns of uniform stiffness and strength, where, $k_{i,j} = k_i$, $\bar{k}_{i,j} = \bar{k}_i$, $M^P_{i,j} = M^P_i$ and $N^P_{i,j} = N^P_i$. This corresponds to a state of single stage UR where all members of the same group share the same demand-capacity ratio. In single stage structures of UR all members develop plastic hinges simultaneously, as if acting as a statically determinate single degree of freedom system. While this particular characteristic of single stage structures of UR deprives the moment frame from its inherent redundancies, it allows the designer to either delay or accelerate the modes of formation of the plastic hinges of selected groups of beams by using such basic technologies as Reduced Beam Sections (RBS) and/or Added Flange Plates (AFP). Depending on the statement of the problem, anyone of these two control options and their combinations may be utilized to modify the ultimate carrying capacity of the system in a positive or negative sense. RBS modifications tend to decrease, while AFP enhancements tend to increase the plastic collapse capacity of ductile systems. By the same token, RBS treatments tend to compromise while AFP modifications tend to enhance both local and global drift ratios. Naturally, the two systems can also be combined to control the number of sequences of formations of plastic hinges, and therefore the ultimate response of the structure as desired. This option is also suited for reinforced concrete (RC) frames, where sectional dimensions and rebar sizes can be adjusted rather freely.

2-Beams and columns of uniform section and strength, where, $I_{i,j} = I_i$, $J_{i,j} = J_i$, $M^P_{i,j} = M^P_i$ and $N^P_{i,j} = N^P_i$. This corresponds to a state of uniform group response where members of selected groups of elements, such as beam of equal spans, may develop plastic hinges simultaneously, depending upon the relative identical stiffnesses of the group and magnitude of the applied loading. The sequence of formation of the plastic hinges of any series of beams is the same as the sequence of decreasing order of stiffness of the beams of the same series or floor level. Each distinct stage of loading would then correspond to a corresponding sequence of formation of plastic hinges, such as the simultaneous yielding of the beams of the shortest bay at first yield. The number of sequences of formations of plastic hinges, or the number of distinct control stages would then be the same as the number of different bays/spans in the subframe. This scenario is suited to both RC as well as steel moment frames, provided that differences in span lengths do not exceed 50%.

3-Beams and columns of uniform stiffness and length proportional moments of resistance i.e., $k_{i,j} = k_i$, and $M^P_{i,j} = (L_j/L_1)M^P_1$ for all j, $\bar{k}_{i,j} = \bar{k}_i$ and $N^P_{i,j} = N^P_i$ for $j = 1, 2, ..., n-1$. $\bar{k}_{i,0} = \bar{k}_{i,n} = \bar{k}_i/2$ and $N^P_{i,0} = N^P_{i,n} = N^P_i/2$. This option is better suited for reinforced concrete (RC) frames and tends to result in the less material consumption than other choices available for the same purpose. However least material consumption does not necessarily imply the least overall cost. All such strategies lead to direct member selection and admissible plastic design solutions. Traditional, best-guessed selection options lead to investigative processes and less efficient member design. The importance of the initial selection of such members using anyone of these strategies is best demonstrated through generic deign examples presented below.

7.2. Introductory example III

Utilize the member selection strategy, option 2 described above and consider the optimal design of a regular $(m = 15) \times (n = 10)$ moment frame, such as that shown in Fig. 1, under triangular distribution of lateral forces $F_i = F(h_i/H)$ and axial nodal loads $P_{i,j}$, provided that target drift ratios are limited to $\phi_Y \leq 0.01$ radians and $\phi_P \leq 0.0175$ radians at first yield and at incipient collapse respectively. The geometric and material properties of the uppermost level subframe at $i = m$ are the same as those described for the single story frame of subsection 6.3. The material properties of all other levels, as those for level $i = m$, are to be selected in accordance with the recommendations of selection strategy 2, i.e., $I_{i,j} = I_i$, $J_{i,j} = 2J_i = 2.2I_i$, $\Sigma^n_{j=0} P_{i,j}/P_{CR\,i,1} = 0.1$, $J_{i,0} = J_{i,n} = J_i = (\rho = 1.1) \times I_i$, $M^P_{i,j} = M^P_i$ and $N^P_{i,j} > 2M^P_i$. Let for the sake of generality, $h_1 - h_4 = 1.5h$, $h_5 - h_8 = 1.25h$, $h_9 - h_{12} = 1.15h$ and h_{13}–$h_{15} = 1.0h$. Since the rules of proportionality, for minimum material use apply, then the results of the numerical solutions obtained for level $i = m = 15$ can be used to establish the minimum values of $I_{m=15} = I$ and $M^P_{m=15} = M^P$ in conformance with the prescribed design conditions. The complete solution to the progressive plasticity of the uppermost level subframe of the subject multistory structure, which coincides with that of the single story moment frame of the previous example, is presented in Table 1.

This solution offers a long range of control points, in addition to the five distinct stages, which may serve as target displacement for performance assessment of the subject single story or multilevel framework. From Eq. (4b) and line 1, Table 1, $I = I_m = I_Y = 0.2739M^PL/0.01E = 27.39 M^Pl/E$, and from line 5, Table 1, $I = I_m = I_P = 0.5167M^PL/0.0175E = 29.49M^Pl/E$, therefore, $I_P > I_Y$ governs. The required design quantities $I_{i,j}$, $J_{i,j}$, $M^P_{i,j}$ and $N^P_{i,j}$ can now be computed in terms of their corresponding quantities, bearing the subscript m, by direct proportioning, in accordance with Eqs. 6(h) and (6k). The complete design of the introductory example II is summarized in Table 2 below.

Table 1. Numerical solutions of the one story frame of example I and the uppermost story of example II

s	K_s/K_0	ρ_s	$f_{Cr.s}$	$\partial_s F_s/F_0$	F_s/F_0	$\partial_s \phi_s/\phi_0$	ϕ_s/ϕ_0	comments
1	103.35	0.100	0.900	25.4743	25.4743	0.2739	0.2739	First yield
2	78.96	0.131	0.869	4.4177	28.8920	0.0644	0.3383	2nd stage loading
3	56.14	0.184	0.816	2.8363	31.7283	0.0619	0.4002	3rd stage loading
4	35.89	0.288	0.712	1.5254	34.2537	0.0597	0.4599	4th stage loading
5	17.60	0.587	0.413	0.4128	34.6665	0.0568	0.5167	Plastic collapse

Table 2. Summary of numerical solutions of the multi-story frame of introductory example III

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
I_i / I_m	25.66	25.30	24.70	21.92	19.00	17.94	16.72	14.74	12.68	11.10	9.36	7.08	4.80	2.96	1.00
M_i^P/M_m^P	25.66	25.30	24.70	21.92	19.00	17.94	16.72	14.74	12.68	11.10	9.36	7.08	4.80	2.96	1.00
J_i / J_m	92.34	19.16	18.80	18.26	12.18	11.56	10.85	10.04	7.72	6.85	5.90	4.87	2.84	1.95	1.00
$N/_i^P/N_m^P$	12.89	12.77	12.53	12.17	9.74	9.25	8.68	8.03	6.71	5.96	5.13	4.23	2.84	1.95	1.00

8. Conclusions

The purpose of this article was to advance the notion that it is possible to design, theoretically efficient earthquake resisting moment frames, without resorting to complicated analysis or sophisticated computer-aided solutions. A goal was set to develop closed form load-displacement and moment-distribution relationships that would address the nonlinear response of such frames throughout their monotonic loading history, starting from zero to first yield, followed by successive formations of plastic hinges up to and including incipient collapse. It was stipulated, that these solutions, as preliminary design tools, were to satisfy the fundamental strength, stiffness and stability requirements of the codes as well as addressing such important issues as integrity control, minimum weight arrangements, propagation of plasticity and the P-delta effects.

The stated goals were achieved by focusing attention, primarily on the elastic-plastic flexural performance of regular moment frames under purely lateral and axial forces, i.e., by;

- Enforcing desirable collapse mechanism and stability conditions, as opposed to ruling out undesirable failure patterns and instability scenarios.
- Selecting the groups of members, such as beams and columns in accordance with pre-determined selection strategies instead of arbitrary element sizing.
- Inducing the strength and the stiffness of individual members with a view toward their performance rather than investigating their suitability for the purpose.
- Employing material saving strategies in which demand-capacity ratios of the members of the same group are as close to unity as possible.
- Imposing a linearly varying drift angle along the height of the structure.

Structural frameworks inherently possessing these characteristics are collectively referred to as structures of uniform response. The proposed methodology not only results in a practical tool for the efficient design of earthquake resisting moment frames, but also provides means of assessing the structural integrity of the system in terms of such tangible quotients as the remaining capacity/original capacity or the remaining drift angle/original drift angle etc. Several generic examples were provided to illustrate the applications of the proposed formulae. It was shown by parametric solutions that the most useful utility of the proposed procedure could be the ability to control the manner and sequences of formation of plastic hinges with respect to predetermined force levels and/or stipulated drift angles. However, the applications of the proposed methodology are restricted to monotonically increasing, constant profile loading only. It should also be emphasized that the assumption of uniform drift may not be compatible with higher modes of vibrations of very tall buildings.

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