J Electr Eng Technol Vol. 8, No. 2: 206-214, 2013
http://dx.doi.org/10.5370/JEET.2013.8.2.206

ISSN(Print) 1975-0102
ISSN(Online) 2093-7423

A New Method for Monitoring Local Voltage Stability using the Saddle Node Bifurcation Set in Two Dimensional Power Parameter Space

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Abstract – This paper proposes a new method for monitoring local voltage stability using the saddle node bifurcation set or loadability boundary in two dimensional power parameter space. The method includes three main steps. First step is to determine the critical buses and the second step is building the static voltage stability boundary or the saddle node bifurcation set. Final step is monitoring the voltage stability through the distance from current operating point to the boundary. Critical buses are defined through the right eigenvector by direct method. The boundary of the static voltage stability region is a quadratic curve that can be obtained by the proposed method that is combining a variation of standard direct method and Thevenin equivalent model of electric power system. And finally the distance is computed through the Euclid norm of normal vector of the boundary at the closest saddle node bifurcation point. The advantage of the proposed method is that it gets the advantages of both methods, the accuracy of the direct method and simple of Thevenin Equivalent model. Thus, the proposed method holds some promises in terms of performing the real-time voltage stability monitoring of power system. Test results of New England 39 bus system are presented to show the effectiveness of the proposed method.

Keywords: Direct method, Thevenin equivalent, Saddle-node bifurcation, Static voltage stability boundary

1. Introduction

Now a day, the deregulation of electric power industry which has resulted in a significant increase in loading level of inter-area ties, which just designed for operating conditions in regulation environment. That makes power systems operate closer to their limits. Therefore stability problems, especially is voltage stability monitoring and control problem, although not a new issue, is now receiving a special attention again. With state of the art the approach ways of recent researches focus on how to make effective and flexible tools that support strongly operators in real time operation in general, real time voltage stability monitoring in special. Those tools try to build a security region that contains the current operating point in multi-dimensional power parameters space. Then voltage stability monitoring and control based on calculating distance and monitoring movement of the current operating point versus the boundary of the security region in real time [7]. It is also a part in works of this paper. All efforts to monitor voltage stability have to define extreme scenarios. To define the extreme scenarios, we have to answer three main questions. The first question is where the loads increase, at a single bus or all buses or combinations of some buses? The second question is how the loads increase or increasing direction of the loads? The third question relates to power factor at load buses, is constant or varying? Most existing simulation-based voltage stability monitoring methods focus on the following extreme scenario, it is increase load at all buses, load increases proportionally to normal operating level and power factor of each load is constant. Result of simulation following the scenario is P-V curve at each bus. Voltage stability assessment based on those P-V curves or some performance indices that associated with the scenario. Most of those performance indices are defined in the state space of power system models [24]. Therefore they cannot directly answer question such as: “Can the system withstand a 100 MVA increase on an arbitrary bus without encountering voltage collapse?” In order to answer directly the question the performance indices should be defined in the parameter space of power system model [24]. In this paper, parameter space based a new performance index is defined through the distance from the current operating point (at the critical bus) to the saddle node bifurcation set.

In addition in real time operation of power system, voltage stability monitoring based on the performance indices are insufficient. Those indices just provide information about the distance from the current state of system to critical point or proximity to voltage instability, the changing of the value of the indices represents the operating point coming towards or going away the critical point. The system loses stability when the operating point

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Received: April 17, 2012; Accepted: September 10, 2012

approaches and passes through the critical point. However they cannot provide visual monitoring about the changing of system. Therefore it supports not much for system operators in real time. A visual boundary separates the feasibility operating region and unstable region is necessary to monitor online voltage stability. It can provide more information that relate to voltage instability. Therefore the boundary together parameter space based the performance indices are useful tools to support the operators in collapse preventive control.

Voltage instability is a local phenomenon and it originates at buses within an area with high loads and low voltage profile. Those buses are called is critical buses or the weakest buses. Therefore the voltage instability monitoring typically associates with the monitoring the changing of load at the critical buses. From that, the requirement of a voltage stability monitoring method that can handle the whole loading level scenarios of the critical buses for operating power system, especially in real-time. This approach is flexible and visual in monitoring online the voltage stability of power systems. It is also a part in works of this paper.

With above mentioned reasons, this paper proposes the new method for monitoring local voltage stability based on the loadability boundary and distance from the operating point to the loadability boundary in two dimensional parameter space. The paper is organized as follows: Section 2 describes the new method. The next sections describe more detail about sub-steps in the new method and theirs mathematical formulation. In where section 3 discusses the Saddle Node Bifurcation (SNB) based direct method for identifying the critical buses and the closest SNB point. Section 4 and section 5 describe the building progress of the boundary through three sub-steps of step 2 that includes the Thevenin Equivalent two bus model and the displacing of the boundary. Section 6 is computation of security distance. The application of proposed method is illustrated in Section 7. Finally, section 8 provides the conclusion and some further discussions.

2. The Proposed Method

The proposed method includes three main steps. Step 1 we use right eigenvector corresponding to the zero eigenvalue obtained from the direct method that apply for a given instability scenario such as increasing load at all buses in system through a single loading parameter λ to identify the critical buses [1-5]. In step 2 corresponding to the critical buses that obtained from step 1, we build the actual shape saddle node bifurcation set or the loadability boundary for each bus. The boundary is built based on three sub-step process. Sub-step 2.1 finds exactly the closest point of collapse corresponding to the worst loading scenario at the critical bus (both real and reactive power increase independently) to calculate Thevenin impedance more accuracy. Sub-step 2.2 builds the rough (approximate) boundary from the new equivalent model seen from the critical bus. Due to having some small errors from estimating Thevenin equivalent parameters, therefore the boundary have to be displaced to build the actual boundary. Sub-step 2.3 displaces the rough boundary into the actual boundary along a displacing vector that connects two the closest SNB points, one is obtained exactly from direct method (exact point), and the other (approximated point) is obtained from the new equivalent model. And finally monitoring voltage stability via the boundary and security margin. The detail descrip-tion of the proposed method is showed in Fig. 1

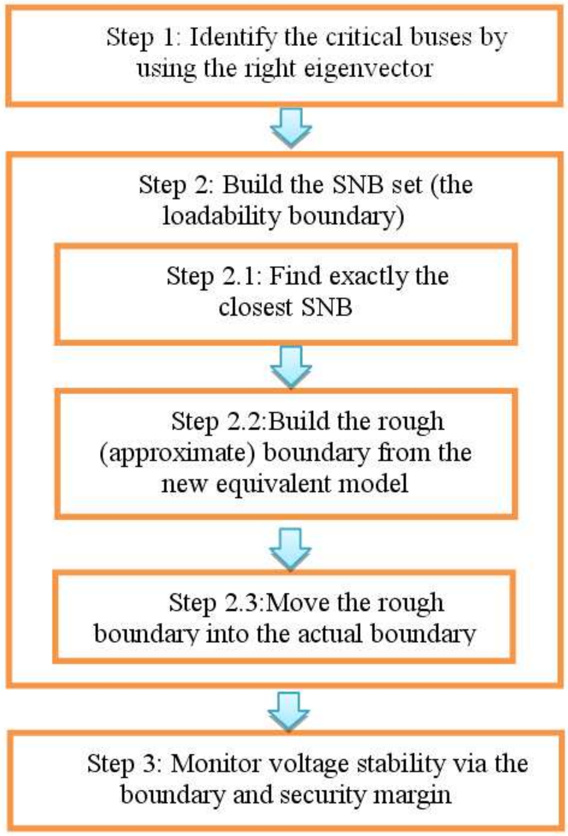

Fig.1. The proposed method

The main purpose of this paper is give a flexible and rapid method to build the local loadability boundary that can monitor online the volatile of load at the critical bus. To monitor continuously and efficiently, it is necessary to know the mathematical expression and the actual shape of the boundary. To monitor online efficiently, it is necessary to get the boundary expression as soon as possible. In addition, to monitor exactly, the built boundary must have the same shape as the actual boundary. The new equivalent model based method gives a better trade - off point among three above requirements in comparison with using directly direct method. Indeed, the direct method just finds saddle node bifurcation points discretely corresponding to scenarios.

The mathematical expression of the boundary just can be obtained by function approximation methods. Therefore the boundary is unsmooth and approximate. Besides that, the calculating for SNB points by direct method consumes time. Therefore it is not suitable for using direct method to monitor voltage instability online.

Advantage of the proposed method is it takes the main advantages of both two efficient methods; the accuracy of the direct method and the simple of Thevenin equivalent based method. From Thevenin equivalent two bus model, the actual shape of the boundary is easy to be obtained. This shape is natural shape; it is either convex or non-convex quadratic curve depend on the value of Thevenin equivalent parameters. In addition the drawback of direct method is overcome in the proposed method. The main drawback of direct method is the high computational cost as the number of equations increases two fold with respect to the system steady state equations [3], especially in case of multi parameters. In the proposed method both two steps that apply the direct method just use a single parameter in step 1 or two parameter in step 2.1. Therefore the number of additional equations is negligible. However in this paper we assume that ignoring the reactive power limit of generators and the other reactive power compensate devices to the result from saddle node bifurcation calculation is correct.

3. The Saddle Node Bifurcation Based Direct Method

3.1. The standard direct method for identifying the weakest bus

The purpose of step 1 is identifying the critical buses (or the weakest buses). There are some methods to do that. In [2] Ajjarapu and in [4] Souza propose to use the tangent vector to identify the weakest bus. In [5] Musirin use a voltage stability index to rank weak buses. Most of them have some drawbacks such as slow computation or ranking criterion is not strict. In this paper we use the right eigenvector corresponding to the zero eigenvalue of Jacobian matrix of power flow model to identify the critical buses.

In physical meaning the right eigenvector represents the response of the system and shows the direction in state space along which voltage instability will evolve. The components of the right eigenvector are proportional to bus sensitivities that indicate how weak a particular bus is near the critical point and help determine the areas close to voltage instability. The greater the bus sensitivity value, the weaker the bus is [2]. Therefore the largest com-ponents in magnitude of the right eigenvector are useful in identifying the weak area of the power system, especially a bus in that area in which the voltage collapse is occurs initially, that bus is called as the weakest bus or critical bus. In addition the right eigenvector computation is more efficient the above mentioned methods.

The saddle node bifurcation conditions of steady state model of power system are represented through the following equations:

$$\begin{cases} f(x,\lambda)=0 & \text{(1a)} \\ f_x(x^*,\lambda^*).v=0 & \text{(1b)} \\ \|v\|=1 & \text{(1c)} \end{cases}$$

Eq. (1a) represents a set of power flow equations, $\boldsymbol{x}$ is a vector of system state variables, such as bus voltage magnitudes and angles, $\lambda \in R^p$ is parameters vector such as real and reactive power at buses. Eq. (1b) represents the saddle node bifurcation at voltage collapse point, at that point, the power flow Jacobian matrix $\boldsymbol{f_x}$ is singular and is vanished by a right eigenvector $\boldsymbol{v}$ corresponding to the zero eigenvalue. Eq. (1c) is a normalization condition that shows the right eigenvector $\boldsymbol{v}$ is not a zero vector. The whole equations characterize the conditions of the generic static voltage collapse point. The information from the right eigenvector can be used to identify the critical buses. In this paper, we use equation systems (1) in step 1 to define the critical buses in case of single parameter, that mean p=1. Loading scenario for step 1 is load at all buses increase proportionally to the initial loading level. After that in Step 2.1, we use a variation of equation systems (1) in case of p=2 to find exactly the closest collapse point corresponding loading levels at each critical bus. Note equation systems (1) can be obtained by KKT conditions in constraint Optimization problems with objective function is maximization of the Distance to Voltage Collapse such as [11-13].

Due to equation system (1) is non-linear system; we use Newton Raphson – Sydel to solve it.

$$\begin{bmatrix} f_x & 0 & f_\lambda \\ f_{xx}.v & f_x & 0 \\ 0 & \dfrac{\partial\|v\|}{\partial v} & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta v \\ \Delta\lambda \end{bmatrix} = -\begin{bmatrix} f(x,\lambda) \\ f_x.v \\ \|v\|-1 \end{bmatrix} \quad (2)$$

In Eq. (2) Hessian $f_{xx}$is a N×N×N tensor and that $f_{xx}.v$is a N×N matrix, where N is the dimension of Jacobian matrix of power flow problem. Direct method requires good initial conditions, particularly for v. A efficient mean is to initiate the eigenvector of the power flow Jacobian matrix corresponding to the minimal eigenvalue by using inverse power method [14] at the current operating point. In this paper we use Eq. (3).

$$v=\frac{x_i-x_{i-1}}{\|x_i-x_{i-1}\|} \quad (3)$$

In power flow problem, $v = \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix}$

The reason for using Eq. (3) is based on the fact that the trajectory of the state variables tends to the right eigenvector v in a small neighborhood of the PoC [7].

3.2. The variation of standard direct method for computing the closest saddle node bifurcation point.

We modify the equation systems (1) little as following

$$\begin{cases} f(x,\lambda) = 0 & (4a) \\ [w.f_x(x^*,\lambda^*)]^T = 0 & (4b) \\ k.\left(\lambda^* - \lambda^0\right) - f_\lambda^T.w^T = 0 & (4c) \\ \|w\| = 1 & (4d) \end{cases}$$

where, w is left eigenvector, $f_\lambda^T.w^T$ is the normal vector at saddle node bifurcation point, k is parallel parameter. Instead of using the right eigenvector v we use the left eigenvector w corresponding to the zero eigenvalue in Eq. (4b) to represent the singular condition at the saddle node bifurcation point. The left eigenvector provides valuable information regarding the geometry of the bifurcation. Geometrically, it parallels with the normal vector of the saddle node bifurcation set at λ^* in parameter space, this is represent in Eq. (4c). Therefore the closest SNB point is intersection point between the line is perpendicular with the boundary and the boundary, that means it is solution of equation systems (4).In step 2.3.of the proposed method in this paper we claim that the actual boundary obtained from displacing the rough boundary along a direction vector should contain the closest saddle node bifurcation point (detail explains for why we claim such that will be represented in session5). Therefore to compute the closest saddle node bifurcation point equation systems (4) is solved by Newton Raphson – Sydel method similar to the above one.

4. The New Thevenin Equivalent Two Bus Model

4.1. The static voltage stability boundary in parameter space of two bus system

Due to the static voltage stability boundary is built through the Thevenin Equivalent two bus model seen from the critical bus. Now, we consider the simple two bus system as shown in Fig. 2. The generator at bus 1 transfers power through a transmission line having an impedance of Z = R+jX to a load at bus 2. Bus 1 is considered as the swing bus where both the voltage magnitude E and angle $\delta_1 = 0$ are kept constant.

From the simple two-bus system in Fig. 2, the voltage equation and static voltage stability boundary in power space at bus 2 can bedefined as follows:

$$V\angle-\delta = E\angle 0 - \left(R + jX\right)\frac{P - jQ}{V\angle\delta} \quad (5)$$

After some mathematical manipulations we arrive at the voltage equation at bus 2 as follows:

$$V^4 + (2RP + 2XQ - E^2)V^2 + \left(R^2 + X^2\right)(P^2 + Q^2) = 0 \quad (6)$$

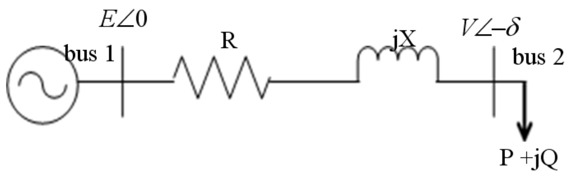

Fig. 2. A simple two bus system

The Eq. (6) is a biquadratic equation that represents the dependence of the voltage at load bus on the power injection at that bus. When the load increases to the maximum value, the voltage reaches to the corresponding critical value or voltage collapse point. The critical point is obtained by finding the singularity condition of Eq. (6) and is also voltage stability constraint being Δ=0. Since

$$\Delta = (2RP + 2XQ - E^2)^2 - 4\left(R^2 + X^2\right)(P^2 + Q^2)$$
$$= E^4 - 4(RP + QX)E^2 - 4X^2P^2 - 4R^2Q^2 + 8RXPQ$$

$\Delta = 0$ can be written as

$$X^2P^2 + R^2Q^2 - 2RXPQ + RE^2P + XE^2Q - \frac{E^4}{4} = 0 \quad (7)$$

where, Eq. (7) is a quadratic equation that represents the boundary of voltage stability region in power injection space at bus 2 of the system. Each point on the boundary is power injection limit for load at that bus. The critical voltage corresponding to the critical power is then calculated as follows:

$$V_{cr} = \sqrt{\frac{E^2}{2} - \left(RP_{cr} + XQ_{cr}\right)} \quad (8)$$

where, (P_{cr}, Q_{cr}) represents the critical point on the boundaries and satisfies Eq. (7), and V_{cr} is the critical voltage corresponding to this critical point.

Because of the changing of the power factor of load, the critical voltage is also changed to follow Eq. (8). The

Equality constraint in Eq. (7) corresponds to the quadratic voltage stability boundary for the simple two-bus system. Inside the boundary, the voltage equation has two solutions; one is the solution corresponding to the normal operating condition of system while the other one is the unacceptable. low voltage solution. On the boundary, two solutions merge and this point represents the voltage collapse point, i.e. the nose point in P-V curve. There are no solutions outside the boundary. The envelope of operating condition becomes the loci of voltage collapse points where the quadratic curve with two axes correspond to the real and reactive power injections at the node. The distance from operating point to the boundary is called security margin.

4.2. The review of local voltage stability indices based on thevenin equivalent models

Now a day with the deploying the Phasor Measurement Unit (PMU) in power system, Thevenin equivalent model is receiving some attentions related to the local voltage stability assessment [8-10, 22-23].Based on the equivalent model of the original system, some voltage stability indices (VSI) are defined such as

$$VSI_i = 1 - \frac{|Z_{Load,equi,i}|}{|Z_{The,i}|} \tag{9a}$$

$$\text{or } \; VSI_i = \frac{(E_{th} - Z_{th}.I_{th})^2}{4Z_{th}} \tag{9b}$$

$$\text{or } \; VSI_i = \frac{P_{i,\max} - P_{i,0}}{P_{i,\max}} \tag{9c}$$

Voltage instability occurs when the indices approach the critical value. The voltage stability assessment based on those indices has some drawbacks. The firstly, the critical criterion of voltage instability based on those indices is that the absolute value of equivalent load impedance becomes equal to the absolute value of the Thevenin equivalent impedance [1, 9, 22]. That just considers the maximum deliverable real power condition that means it just mentions the role of real power in voltage instability problem. That is not directly applicable to power systems because in power systems both real and reactive powers affect to the voltage instability, especially the role of reactive power is more clearly and overwhelmed that of real power. Secondly, they are inconsistent in model of load. They use the impedance model to represent load in calculating, but in the final criterion and conclusion (the definition of indices), they use the power load model. Those drawbacks will be overcome thoroughly by the new proposed method in this paper.

There are many methods to calculate the Thevenin equivalent parameters. In [10] Khoi Vu applied the traditional curve-fitting technique that uses data in two consecutive measurements; Thevenin equivalent parameters are calculated by using data from at least two consecutive measurements of voltage and current. If more than two sets of voltage and current measurements are obtained, the Thevenin equivalent parameters can be estimated by using the least square method, this method depend strongly on the reliability of phasor measurements that are inherently stochastic and errors. This is one of fundamental drawbacks of this method. [8-9] reviewed almost existing methods to calculate the Thevenin equivalent parameters, most of them use impedance load model. Again, remind that the methods use the impedance load model that is not suitable for static voltage instability.

4.3. New method to calculate thevenin equivalent parameters

In this paper we propose the new method to define Thevenin parameters based on the power model and information of load flow in two base cases, one without load and the other with the closest margin at the critical bus (the candidate bus).

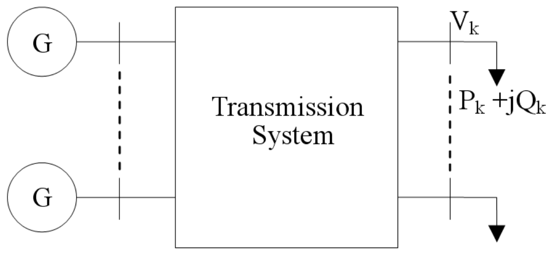

Fig. 3. The Initial General Power System

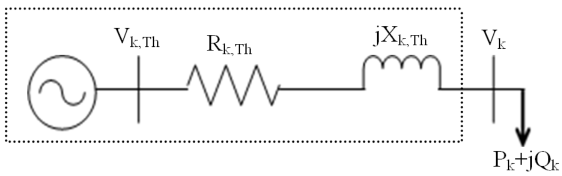

Fig. 4. The Thevenin Equivalent System seen from Bus k

In the equivalent two-bus system shown in Fig. 4, the critical bus of the original system is kept intact but the rest of the system is replaced by its Thevenin equivalent circuit. We claim that when the system is represented by such an equivalent circuit, a generator can be modeled by a terminal with constant voltage source having zero reactance (X=0) and given real power, i.e., PV bus, while a load can be modeled as having constant real and reactive powers, i.e., PQ bus. That means that we use load flow model for calculating Thevenin parameters.

"Equivalent" term represents the calculated objective values have to equal in both of systems, the original system and the equivalent system. Old Thevenin equivalent model with constant equivalent impedance is linear equivalent model that just guarantees the same calculated values as in both systems in two extreme cases, open circuit and short circuit. The calculated values from the equivalent system in other cases by using constant equivalent parameters will make large error in result because of non-linear native of power system. Therefore in this paper we propose a new method to calculate equivalent parameters with varying equivalent impedance that represents the change of load and current operating condition of system.

As the definition of Thevenin voltage is open-circuit voltage at the candidate bus. In power flow applications of power system, open-circuit at one bus means that no-load (or no-power injection) at that bus. Therefore Thevenin voltage is the voltage at the candidate bus obtained by the solution of power flow in base case without load at that bus.

$$V_{k,Th} = V_k^0 \tag{10}$$

To improve the accuracy of calculating of Thevenin impedance, its value should change following the change of system conditions and load at that bus. Therefore The Thevenin impedance will be calculated as following. From Fig. 4, we have

$$\begin{aligned} R_{k,Th} + jX_{k,Th} &= \frac{V_k\angle-\delta.(V_k^0\angle\delta^0 - V_k\angle\delta)}{P_k - jQ_k} \\ &= \frac{V_k.V_k^0.S_k.\cos(\delta^0-\delta+\phi) - V_k^2.P_k}{P_k^2+Q_k^2} \\ &\quad + j\frac{V_k.V_k^0.S_k.\sin(\delta^0-\delta+\phi) - V_k^2.Q_k}{P_k^2+Q_k^2} \end{aligned}$$

Therefore

$$\begin{aligned} R_{k,Th} &= \frac{V_k.V_k^0.S_k.\cos(\delta^0-\delta+\phi) - V_k^2.P_k}{P_k^2+Q_k^2} \\ X_{k,Th} &= \frac{V_k.V_k^0.S_k.\sin(\delta^0-\delta+\phi) - V_k^2.Q_k}{P_k^2+Q_k^2} \end{aligned} \tag{11}$$

In where $S_k\angle\phi = P_k + jQ_k$ is load at bus k, $V_k^0\angle\delta^0$ is no-load (open-circuit) voltage at bus k, $V_k\angle\delta$ is voltage at bus k when consumed load is P_k+jQ_k. $V_k\angle\delta$ is obtained by load flow calculation in case the consumed load at bus k is P_k+jQ_k. Therefore, in general Thevenin impedance is a function of load and system conditions. In addition, since the shape of voltage stability boundary depends on the exact of Thevenin parameters, the accurate calculation of Thevenin parameters becomes an important key to the proposed method. There are some advantages of the new equivalent model. It guarantees the consistent of load model in voltage instability problem (power load model). Therefore it preserves the non-linear native of voltage instability phenomena. Beside that it represents both the role of real and reactive powers on the voltage instability phenomena. It is easy to update the equivalent impedance when knowing the currently consumed load, the more accuracy the result will be if the larger the currently consumed load, because the extrapolation point will be closer to the SNB point, especially when the consumed load is the closest margin, the result is the most accuracy for applying to build the loadability boundary (or the static voltage stability boundary). This is suitable to interpret development process of voltage collapse in point of view gradual increasing of load from no load to normal load and eventually maximum load.

5. The Displacement of the Boundary

Expression (7) is represented in P-Q coordinate with (0,0) is origin point. Two collapse points, one is obtained from direct method (exact point), and the other (approximated point) is obtained from Thevenin equivalent model are lied in the coordinate. The vector that link two collapse points called is displacing vector to move the rough boundary (expression (7)) obtained from the equivalent two bus model to become the actual static voltage stability boundary of the critical bus.

$$\vec{D} = \left(P_1^* - P_2^*\right).\vec{i_P} + \left(Q_1^* - Q_2^*\right).\vec{i_Q} \tag{12}$$

Therefore the new origin is

$$\begin{cases} x_P^{new} = P_1^* - P_2^* \\ x_Q^{new} = Q_1^* - Q_2^* \end{cases} \tag{13}$$

where (P_1^*,Q_1^*) is the exact collapse point obtained by direct method, (P_2^*,Q_2^*) is the collapse point obtained by Thevenin equivalent model of the critical bus. Substitute (11) into (7), we get the expression of actual boundary in two dimensional parameter space as following:

$$\begin{aligned} &X^2\left(P-x_P^{new}\right)^2 + R^2\left(Q-x_Q^{new}\right)^2 \\ &-2RX\left(P-x_P^{new}\right)\left(Q-x_Q^{new}\right) + RE^2\left(P-x_P^{new}\right) \\ &+XE^2\left(Q-x_Q^{new}\right) - \frac{E^4}{4} = 0 \end{aligned} \tag{14}$$

or $$X^2P^2 + R^2Q^2 - 2RXPQ + d.P + e.Q + f = 0 \tag{15}$$

In where $d = -2X^2.x_P^{new} + 2RX.x_Q^{new} + RE^2$

$$e = -2R^2.x_Q^{new} + 2RX.x_P^{new} + XE^2$$

$$\begin{aligned} f &= X^2.(x_P^{new})^2 + R^2.(x_Q^{new})^2 - 2RX.x_P^{new}.x_Q^{new} \\ &\quad - RE^2.x_P^{new} - XE^2.x_Q^{new} - \frac{E^4}{4} \end{aligned}$$

Eq. (15) is the actual boundary of the static voltage boundary in two dimensional parameter space corresponding to the critical bus.

Which we should define two collapse points?

Because the static voltage stability boundary is saddle node bifurcation set, each point on the boundary is a saddle node bifurcation point corresponding to a given instability scenario (that means power factor vary), The question is which two collapse points (each point corres-ponding to each method, direct method and Thevenin equivalent model) we should choose to create the displace vector D? The answer is we should choose the closest saddle node bifurcation point because it guarantees that the boundary obtained above surely contains that point because direction along normal vector at the closest saddle node bifurcation point that is local loading margin in the worst scenario at the critical bus for voltage collapse.

6. Computing a Closest Saddle Node Bifurcation Point and Security Distance based on Thevenin Equivalent Model

The distance from a given operating point to the voltage stability boundary in parameter space gives a security margin regarding voltage collapse. Thus, in order to preserve a safe power system operation, the distance to the boundary from the current loads (P_0,Q_0) can be monitored; from that operators can make preventive control decisions to avoid a possible collapse of the system.

The Voltage Static Stability Boundary is two dimensional quadratic curve in parameter space, the general quadratic equation is

$$Q(\mathbf{x}) = \mathbf{x}^T\mathbf{A}\mathbf{x} + \mathbf{b}^T\mathbf{x} + c = 0 \tag{16}$$

where, **A** is a symmetric n×n matrix (in this paper, n=2), **b** is an n×1 vector and c is scalar. The parameter **x** is an n×1 vector.

Given the curve Q(x) = 0 and an operating point **y**, find the distance from **y** to the boundary curve and compute a closest point **x**. Geometrically, the closest point **x** on the curve to **y** must satisfy the condition that $\|\mathbf{y} - \mathbf{x}\|$ is normal to the curve. Since the curve gradient $\nabla Q(x)$ is normal to the curve, the algebraic condition for the closest point is:

$$\mathbf{x} = (\mathbf{I} + 2t\mathbf{A})^{-1}(\mathbf{y} - t\mathbf{b}) \tag{17}$$

And the distance from a given operating point to the boundary curve can be calculated as following:

$$\| \mathrm{y} - \mathrm{x} \| = t\nabla Q(\mathrm{x}) = t(2\mathrm{A}\mathrm{x} + \mathrm{b}) \tag{18}$$

7. Simulation Results

The proposed method is tested on New England 39 bus system. Step 1 with the scenario is the loads at buses are increased proportional to their initial load levels to define the critical buses. In the New England 39-bus test system, there 29 load buses, 9 generation buses and bus 31 is chosen as the slack bus. Therefore the dimension of right eigenvector is 67×1. The simulation result for step 1 is showed in Fig. 5. From Fig. 5, we can see that $\Delta\delta_8 = -0.2130$ (index in the right eigenvector is 8th) and $\Delta V_8 = -0.1164$ (index in the right eigenvector is 46th) are the biggest components in magnitude in comparison with the corresponding others of the right eigenvector. Therefore bus number 8 is considered as the critical bus.

After define the critical buses, we continue the step 2. The result for step 2 is the actual boundary expression in quadratic form of P and Q parameters. In Fig. 6, the inside small curve is the rough boundary that obtained from Thevenin Equivalent two bus seen from bus 8, with Thevenin voltage and Thevenin impedance parameters are calculated following Eq. (9, 10) and Eq. (11), the outside bold curve is the actual boundary that obtained from moving the rough boundary along the direction vector $\vec{D}$,

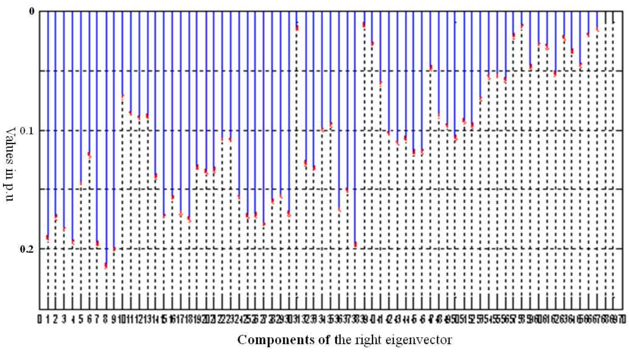

Fig. 5. The right eigenvector corresponding to zero eigenvalue in the given scenario

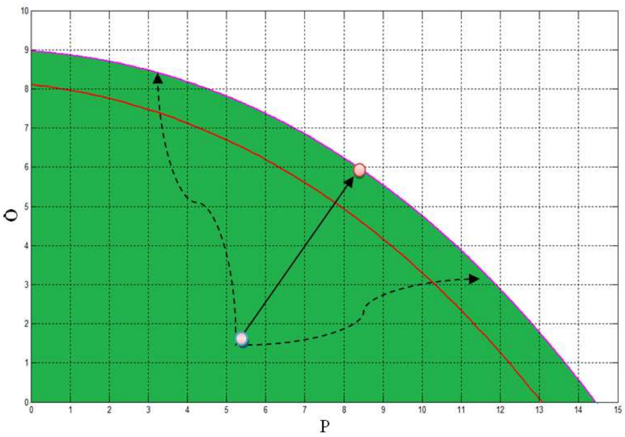

Fig. 6. The rough and actual static voltage stability boundaries at the critical bus (bus 8)

and the analysis expression of the actual boundary is:

$$(0.8556P^2 - 0.399PQ + 0.0465Q^2).10^{-3} + 0.0055P + 0.0284Q - 0.2582 = 0$$

From the rough and actual boundaries, we can see that the Thevenin equivalent model underestimate the shape of the boundary. The error is corrected by the displace vector $\vec{D}$. Step 3 is assessing the voltage stability via the distance from the current operating point to the actual boundary. Appling (17) and (18) equations, it is easy to get result. Simulation result for security distance from the current operating point (522 MW; 176.6 MVAr) to the actual boundary at bus 8 is 529.01 MVA, the corresponding closest saddle node bifurcation point is (836.28 MW, 602.14 MVAr). The worst load increasing scenario is the direction along the red bold vector. In addition we can be easy to monitor the volatile of load at bus 8, any changing direction, any changing of power factor such as dash arrows via the boundary in Fig. 6.

Table 1. The closest SNB points obtained by both methods

	Direct Method	The new equivalent method	Error %
The closest SNB point (MW,MVAr)	(836.28; 602.14)	(789.12; 583.25)	4.78
The collapse voltage (pu)	0.5079	0.5258	3.52

8. Conclusion

The voltage stability monitoring is one of the most importance problems in the electric power system operation. It is desirable that this monitoring can be performed in real-time while the boundary of voltage stability region is determined accurately. In this paper we propose a new method for assessing static voltage stability that is simple, fast and relatively accurate. The static voltage stability boundary is determined based on the combination of direct method and Thevenin equivalent two bus model of the original system seen from the critical bus. The proposed method makes use of the quadratic equality that represents the static voltage stability boundary for the critical bus in system and that may be easily plotted in two dimensional power parameter space. It is also shown that it is sufficient to determine the voltage stability margin at the critical bus for the given initial operating conditions. The Thevenin voltage and Thevenin impedance are defined based on the power model of load, which keeps the integrity of model in finding voltage stability boundary. The effectiveness of the proposed method is demonstrated on New England 39 bus system.

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