A Model Based on Average Investment for Solving Complex Annuity Problems of Sinking Fund

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Received: 11 November 2013 / Accepted 15 December 2013

ABSTRACT

Undoubtedly, the basic sinking fund formula gives the future value of a series of equal installments. The main underlying assumption for using this formula is that installment and compounding frequency must be in equal interval. But when installment for a deposit scheme or any other savings scheme and compounding frequency do not occur in an equal interval, which is treated as the complex annuity problems in Finance Literature, the basic sinking fund formula does not give the accurate result. As a result, the obtainable amount from different deposit schemes offered by different banks and financial institutions does not match with the amount of future value calculated through the basic sinking fund formula by the investors or savers. This study focuses the concealed facts for such type of mismatches in values and at the same time it provides a solution through developing a new formula by extending the basic formula intended not only to remove those mismatches but also get the accurate future value from a sinking fund provision in case of complex annuity. Besides, since banks and financial institutions calculate the interest on the average amount of equal installments deposited within a period of time due to complex annuity, the study also formulates an arithmetic formula for calculating the average amount of installment.

KEYWORDS

Compounding Frequency • Complex Annuity • Sinking Fund • Vellore District • Average Amount of Installment • New Extended Sinking Fund Formula

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1. INTRODUCTION

Through savings a little part of income, low and middle income level people try to stabilize and fulfill their future needs. Different offerings of banks and financial institutions help people to save a portion of their income by means of a sinking fund. Besides, different investors utilize this sinking fund provision to make decisions in terms of Capital Budgeting and return on equity (ROE) of a project. Investors including even the fixed-income level people consider the most beneficial sector from where they can earn more from their savings either through doing part-time business, investing into a new project, creating a new business venture, expanding the existing operations of any of their businesses, or saving their part of income through bank deposits. Whatever the option is considered, every investor must seek the opportunity cost, i.e. the benefit forgone from the next best alternative from his or her new investment because the opportunity cost considerations help people choose the best investment alternative. Clearly, it goes without saying that through the calculations of returns from different types of sinking fund available through different financial institutions such as banks, financial intermediaries, etc., anyone can take the decision of whether he or she should undertake the new project or not. Undoubtedly, basic sinking fund formula serves the purpose to calculate the future value of all deposited amount, i.e. it tells the total returns that will be found from this type of sinking fund.

But the future value for a specific equal series of payments found through the calculation by using the formula given in the basic finance Literature does not matches in many cases with the future value of that specific equal series of payments mentioned by different banks and financial institutions in their proposals. Therefore, the basic formula needs some adjustment to cope up with the value offered by banks and different financial institutions in their offers. But the adjustment demands to detect the problems and give solutions accordingly. Here, the study mainly focuses to find out the drawbacks prevailing to measure the future value of a series of equal payments deposited in banks or financial institutions and tries to give the solution by overcoming the drawbacks impeding to provide the accurate maturity values from the basic Sinking Fund formula.

2. RATIONALE OF THE STUDY

Basic Sinking Fund formula normally serves the purpose of calculating all the sinking fund related values. In case of use of the formula, the primary assumption is that the number of installment and compounding time period occurs with the same frequency. That is if the installment is taken monthly, compounding is to do on a monthly basis. But when installment and compounding frequency does not occur at the same frequency, the main difficulty arises. When the payment interval is different from compounding frequency, it is called complex annuity[1]. It is worthy to mention that when banks do compounding in different time period, but take the installment on a monthly basis, the basic formula of Sinking Fund does not provide the accurate results directly because banks use an average amount of the entire installment collected within a compounding time period and then apply the basic Sinking Fund formula for calculating the maturity value. To overcome such issues banks usually rely on calculation through the use of different software or spreadsheet program. For example, if an investor deposits Tk. 500 monthly basis in a bank throughout the next 10 years at 11.50% interest rate compounded quarterly basis, the bank's calculation tells that this investor will receive Tk. 1, 12,057[2] after 10 years, but through the direct use of basic formula this value never comes. Hence, to get the actual result, an investor or an analyst must rely on step by step calculation. However, for an investor as well as for a financial analyst, including the academician, such calculation becomes not only time consuming but also cumbersome. This study initiates to find out the problems inherent in the basic formula of sinking fund in calculating the maturity value accumulated from the installment deposit into the bank and to provide the solutions through the extension of the formula of sinking fund for such type of complex annuity.

3. LITERATURE REVIEW

Almost all the texts of Finance and Accounting Literature describe the sinking fund formula required for the calculation of accumulated value of a future amount of general annuity.

Ainsworth, Penne; Deines, Dan; Plumlee, R. David; & Larson, Cathy Xanthaky (1997) in "Introduction to Accounting: An Integrated Approach", Irwin / McGraw-Hill, describe the future value of an annuity is the amount of money that accumulates at some future dates as a result of making equal payments over equal intervals of time and earning a specified interest rate over that time period. They also mention that the amount of money that accumulates is a function of the size of the payments, the frequency of the payments, and the interest rate used over the life of the annuity. This discussion by them also includes that businesses and individuals use the future value of annuities to determine the amount to save on a regular basis (page: 483). The mentioned formula by them to calculate the future value of an ordinary annuity of \$1 is, or, in notation, $A_{n,i} = \frac{(\$1+r/c)^n-1}{r/c}$; where r = annual rate of interest, c = number of compounding in one year, n = number of payments and i = interest rate for annuity period (r/c). Then the future value of the annuity is determined through the multiplication of the Annuity Amount with $A_{n,i}$.

Spiceland, J. David; Sepe, James F; & Tomassini, Lawrence A (1949) in "Intermediate Accounting", 3rd Edition, McGraw-Hill Irwin, pages 286 to 301 presents the procedures of calculating the future value and the present value of general annuity. Here they tell that an annuity is a series of equal-sized cash flows occurring over equal intervals of time. An ordinary annuity exists when the cash flows occur at the end of each period. An annuity due exists when the cash flows occur at the beginning of each period. In addition, they mention that the future value of an ordinary annuity (FVA) is the future value of equal-sized cash flows with the first payment taking place at the end of the first compounding period and the last payment will not earn any interest since it is made at the end of the annuity period. The future value is calculated as FVA = Annuity $\times \frac{(1+i)^n-1}{i}$. The future value of an annuity due (FVAD) is the future value of a series of equal-sized cash flows with the first payment taking place at the beginning of the annuity period (i.e. the beginning of the first compounding period). FVAD is calculated as FVAD = Annuity $\times \frac{(1+i)^n-1}{i} \times (1+i)$. In both cases, i indicates the yearly interest rate and n indicates the number of years to maturity.

Kieso, Donald E; Weygandt, Jerry J; Warfield, Terry D (2001) in "Intermediate Accounting", 10th Edition, John Wiley & Sons, Inc., deliberates the idea concerning the

[1] Shao, Stephen P.; Shao, Lawrence P. in "Mathematics for Management and Finance"-4th Edition, South Western Publication Co., 1980, Ohio, P-423.

[2] Brochures of Trust Bank Ltd; Trust Smart Savers Scheme (TSSS)

future value of ordinary annuity and future value of an annuity due (P:288-290) as well as the idea of future value of a deferred annuity (P: 294).

Jr., Jay M. Smith; Skousen, K. Fred (1984) in "Intermediate Accounting: Comprehensive Volume", 8th Edition, South-Western Publishing Co. and Federal Publications Ltd., pages 1016-1020; Zelman, William N; McCue, Michael J; Glick, Noah D. (2009) in the text entitled "Financial Management of Health Care Organizations: An Introduction to Fundamental Tools, Concepts and Applications", 3rd Edition; Graham, John; Smart, Scott B. (2011) in "Introduction to Corporate Finance", 3rd Edition at page 82; Horne, James C. Van; Jr., John M. Wachowicz (2001) in "Fundamentals of Financial Management", 11th Edition, Pearson Education, at Page 60; Gitman, Lawrence J. (2007) in "Principles of Managerial Finance", 10th Edition, Pearson Education, pages 162 – 169; Keown, Arthur J; Martin, John D; Petty, J William; Jr, David F. Scott (2005) in their "Financial Management: Principles and Application", 10th Edition, pages 150-158; Schall, Lawrence D; Haley, Charles W (1991) in "Fundamentals of Financial Management" 6th Edition, McGraw-Hill, Pp: 84-86; Block, Stanley B; Hirt, Geoffrey A; Danielsen, Bartley R (2012) in "Foundations of Financial Management" 13th Edition, McGraw-Hill Irwin, page 258; Intermediate Accounting: IFRS Edition, Vol-1, page 303 (2010) including all other finance text address the same procedures and formula for the calculation of future value of general annuity but no text addresses the procedure for calculating the future value of complex annuity.

Whigham, Davidin (1998) in "Quantitative Business Methods Using Excel", Oxford University Press, provides spreadsheet calculation procedures of a sinking fund accumulation. Here, he shows that how future accumulation can vary for different interest rate along with different compounding frequency. He also shows the fact that what discrepancy may occur when an installment for a sinking fund and compounding frequency does not follow the same frequency.

Rose, Peter S.; and Hudgins, Sylvia C. (2005) in their "Bank Management & Financial Services" 6th Edition, McGraw Hill, page 407, have shown the manual calculation of the average amount of installment within a compounding period which gives the better ideas regarding how banks calculate the average installment amount.

Elliehausen, Gregory, and Barbara R. Lowrey (1997) observed the impact of the Truth in Savings Act on the cost of bank compliance in "The cost of implementing Consumer Financial Regulations: An Analysis of The Experience with the Truth in Savings Act", Staff Study No. 170, Board of Governors of the Federal Reserve System, December 1997. This undoubtedly reflects the necessity of accurate sinking fund calculation.

Though, none of the Finance text provides any specific format or model to calculate the accumulation in case of Sinking Fund for complex annuity offered by different banks, Shao, Stephen P.; Shao, Lawrence P. (1980) in "Mathematics for Management and Finance" 4th Edition, South Western Publication Co., Ohio, P-423 mentioned a formula in algebraic form as the Future Value of Annuity= $A \times \left(\frac{\left(1+\frac{r}{m}\right)^{m.n.c}-1}{\left(1+\frac{r}{m}\right)^{c}-1} \right) \left(1+\frac{r}{m}\right)^{c}$, which only focuses on such issues to solve the complex annuity problems of a sinking fund. Here, r = Rate of Interest per Year, m = No. of compounding Per Year, mn = No. of Payments, c = (Payment Period / Interest Period). But the calculation of the future value of complex annuity by using this formula is so cumbersome for common people.

4. OBJECTIVES

This study searches for whether the basic Sinking Fund Formula in the Finance Literature directly serves the purpose of calculating the accumulated sum of money in a future period of time or not. At the same time this study initiated to know whether the calculated value of the formula matches with the maturity values provided by the banks operating in Bangladesh or not. So, the main or specific objectives of the study become under the main headings expressed as follows:

i. To search for any discrepancy evolving between the calculated value found from the prevailing Sinking Fund formula and the maturity value provided or offered by banks through their offerings.
ii. To find out ways that help to solve the discrepancy arisen due to the mismatches found i.e. to develop new formula or new model to get an accurate calculation.

Besides those primary (or main) objectives, the study has some secondary objectives. These are:

i. To simplify the sinking fund calculation and help the bankers' community through saving their time and efforts they provide in calculating the future value of deposit schemes while making proposals and doing consultancy and thereby increase their efficiency.
ii. To give a platform to the academician and financial analyst for the real and meaningful analysis.

5. METHODOLOGY

The study is exploratory in nature. All the data come from secondary sources. Among 52 commercial banks operating in Bangladesh, those which publicly disclose the yearly interest rate clearly becomes under consideration for analysis and finally 5 banks comprised of three private commercial banks, one nationalized commercial bank and one Islamic commercial bank serve as sample. Printing materials, website of respective banks, brochures and advertising materials have been analyzed in this study. Shortly to tell, all of the data presented and analyzed in this paper are from respective publicly available brochures and website of respective banks. Mainly, different Deposit Scheme Plan in the name of Monthly Savings Scheme, Deposit Pension Plan, and Deposit Savings Scheme etc. of the sample banks are browsed over and utilized to collect the required data. Besides, for analysis purpose, some undisclosed data such as compounding frequency have been derived through the spread sheet calculation by utilizing the published data and at the same time these derived data were justified by the corresponding bank officials. Finally, the results found through the findings of the study were verified with the value found from the banks' proposals and all the comparison were presented through tabular presentation.

6. PROBLEMS WITH THE SINKING FUND FORMULA OF GENERAL ANNUITY

The formula given in the basic Finance Literature concentrates on general annuity. But in case of complex annuity, i.e. when compounding frequency and installment does not occur at the same frequency, this basic formula does not provide results correctly. Table 1 compares between the value offered by Sinking Fund proposal in the name of the Monthly Deposit Scheme (MSDS) by AB Bank Ltd. and the value calculated through the prevailing Sinking Fund formula, given that the annual interest rate is 12.25%[3] and AB Bank compound the deposited amount of installments semiannually[4]. This table clearly shows that there are mismatches between the maturity values offered by AB Bank Ltd. and the value calculated through the basic Sinking Fund formula. These mismatches arise because the bank compounds semiannually i.e. two times in a year on the average of the deposited amount but takes the installment on a monthly basis. Here, it is found that the basic formula of sinking fund does not serve the purpose properly.

The main reason why the basic sinking fund formula result discrepancies is that a bank or financial institutions take deposits, monthly basis, but does not provide interest on a monthly basis. Rather, most of the banks provide interest after a few periods on the average amount of the collected deposits. But to work properly with the basic sinking fund formula, it is mandatory that number of deposits must be equal to the number of compounding frequencies. However, when average amount is adopted to provide the interest, the amount of accumulation will obviously will be different from the accumulated amount normally found from the prevailing formula.

7. A SOLUTION TO THE PROBLEM IN SINKING-FUND CALCULATION

Based on the average amount of the entire installment collected within a compounding time period, banks calculate the maturity value manually. The following analysis provides the way of calculating such value:

7.1. DEVELOPMENT OF FORMULA TO CALCULATE THE AVERAGE AMOUNT OF INSTALLMENT

Table 1 shows that AB Bank Ltd. takes Tk. 1000 monthly installments and provides interest semiannually. The bank normally will have the very first installment for 6 months or 180 days, 2nd installment for 5 months or 150 days, 3rd installment for 4 months or 120 days, 4th installment for 3 months or 90 days, 5th installment for 2 months or 60 days, 6th installment for 1 month or 30 days at its hands. Hence, bank provides interest only on the average amount it has throughout the six months periods. Banks normally calcu-

TABLE 1: Comparison between Bank's Value and Calculated Value of Accumulated Money.

Value According to	Monthly Installment	Pre-tax Maturity Value After			
		3 Years	5 Years	7 Years	10 Years
AB Bank's Proposal*	500	21,742	41,196	65,873	1,15,848
Prevailing Sinking Fund Formula**	500	21,842	41,529	66,651	1,17,914
AB Bank's Proposal*	1000	43,484	82,393	1,31,748	2,31,695
Prevailing Sinking Fund Formula**	1000	43,684	83,058	1,33,301	2,35,829

SOURCE: * AB Bank's Website. ** Calculated through the Formula Using Spread Sheet.

[3] AB Bank Website

[4] Found through Spread Sheet Calculation and also justified by Bank Official.

late the average amount as follows[5]:

$$Avg.Installment = \frac{(1000\times180)+(1000\times150)+(1000\times120)+(1000\times90)+(1000\times60)+(1000\times30)}{180}$$
$$= \frac{1000\times(180+150+120+90+60+30)}{180}$$
$$= \frac{1000\times630}{180} = 3500$$

However, to calculate the average amount more efficiently of any installment for compounding, the 2nd line can be re-arranged as $= \frac{1000}{180}\times(180+150+120+90+60+30)$

Here is $(180+150+120+90+60+30)$ is an arithmetic progression where 1st element, a=180; No. of elements, n=6; Common Difference, d=Successive Element-Previous Element=150-180=-30; So, by applying the formula of arithmetic progression i.e. $S_n = \frac{n}{2}\{2a+(n-1)d\}$ to calculate the sum of those items it can be written as:

$$180+150+120+90+60+30 = S_6 = \frac{6}{2}\{(2\times180)+(6-1)\times(-30)\}$$

Hence, Avg. Installment =

$$\frac{1000}{180} = \times \left[\frac{6}{2}\{(2\times180)+(6-1)\times(-30)\} \right] = 3500$$

But here, 1000 = Monthly Installment; 180 = Total Compounding Days; 6 = Number of Installment within a Compounding Frequency; 30 = Days Interval between Two Installment.

So, the average installment can be written as a formula of:

$$Avg.\ Installment = \frac{Installment}{Total\ Compounding\ Days} \times \frac{No.of\ Installment}{2} \times \{2 \times Total\ Compounding\ Days + (No.of\ Installment - 1)(-Days\ Interval\ Between\ Two\ Installment)\}$$

7.1.1. ASSUMPTIONS

The above mentioned formula must base on the following assumptions:

1) Number of days in a year is 360 days and each month contains 30 days.
2) Number of installments comes more than the compounding frequency used by banks.
3) When a number of installments is equal to the compounding frequency, then the average installment amount is equal to the each installment amount.
4) If any bank applies the actual number of days in a month, then manual average calculation must be used instead of the written formula.

7.2. Calculating the Future Value of Installment

Again, going back to table 1, it shows that AB Bank Ltd. takes Tk. 1000 monthly installments and provides interest 12.25% semiannually. A step by step spread sheet calculation done in table 2 tells that after 3 years Tk. 1000 monthly installments at 12.25% interest rate compounded semiannually becomes Tk. **43,483.92887 ≈ Tk. 43,484** which exactly matches with the **table 1 value of Tk. 43,484.** Similarly, the future value after 5 years Tk. 82392.83225 ≈ Tk. 82,393; after 7 years Tk. 1,31,746.5421 ≈ Tk. 1,31,747; after 10 years Tk. 2,31,695.1207≈ Tk. 2,31,695 found in the above table exactly matches with the future value provided by AB Bank Ltd mentioned in table 1.

7.2.1. ANALYSIS OF THE MANUAL CALCULATION

As the maturity value calculated in table 2 exactly matches with the maturity value provided by AB Bank Ltd mentioned in table 1, it provides a great framework to simplify the process of calculation. Here, in table 2, it is noticeable that in the first phase, the interest comes only on average amount (i.e. Tk. 3500) of the installment deposited (i.e. Tk. 6000) within the compounding frequency. In the 2nd compounding time noticeable at 2nd raw, interest comes on the average amount i.e. Tk. 9714.375 (i.e. average amount of installment within this phase Tk. 3500 + interest bearing amount of Tk. 3500 in the 1st phase + non-interest bearing amount in the 1st phase i.e. Tk. 2500 + interest earned in 1st phase i.e. Tk. 214.375). In the 3rd raw it is found that inter-

[5] Rose, Peter S; Hudgins, Sylvia C; Bank Management & Financial Services; 6th Edition, McGraw-Hill, 2005, P: 407

est for 3rd compounding time comes on the average amount i.e. Tk. 16309.38047 (i.e. which is equal to average amount of installment within this 3rd phase Tk. 3500+ interest bearing amount of Tk. 3500 in the 2nd phase + non-interest bearing amount in the 2nd Phase i.e. Tk. 2500 + interest bearing amount of Tk. 3500 in the 1st phase + non-interest bearing amount in the 1st phase i.e. Tk. 2500 + interest earned in 2nd phase i.e. Tk. 595.0054688+ interest earned in 1st phase i.e. Tk. 214.375).

In the same manner, from the manual calculation found in the table 2, it is clear that interest comes to the average amount of installment i.e. on Tk. 3500 up-to the maturity period and though non-interest bearing amount i.e. Tk. 2500 does not earn any interest in the first compounding

TABLE 2: Calculation of Future Value (Before Tax) Based on Average Amount of Installment

[1]	[2]	[3]	[4]	[5]=[3][4]	[6]=[4]*12.25%	[7]=[3]+[6]
Period (Years)	Installment Amount	Cumulative Amount Deposited	Average Amount*	Extra Amount	Interest Amount	Total Value
0.5	1000	6000	3500	2500	214.375	6214.375
1	1000	12214.375	9714.375	2500	595.0054688	12809.38047
15	1000	18809.38047	16309.38047	2500	998.9495537	19808.33002
2	1000	25808.33002	23308.33002	2500	1427.635214	27235.96524
2.5	1000	33235.96524	30735.96524	2500	1882.577871	35118.54311
3	1000	41118.54311	38618.54311	2500	2365.385765	43483.92887
3.5	1000	49483.92887	46983.92887	2500	2877.765643	52361.69452
4	1000	58361.69452	55861.69452	2500	3421.528789	61783.2233
4.5	1000	67783.2233	65283.2233	2500	3998.597427	71781.82073
5	1000	77781.82073	75281.82073	2500	4611.01152	82392.83225
5.5	1000	88392.83225	85892.83225	2500	5260.935975	93653.76823
6	1000	99653.76823	97153.76823	2500	5950.668304	105604.4365
6.5	1000	111604.4365	109104.4365	2500	6682.646738	118287.0833
7	1000	124287.0833	121787.0833	2500	7459.45885	131746.5421
7.5	1000	137746.5421	135246.5421	2500	8283.850705	146030.3928
8	1000	152030.3928	149530.3928	2500	9158.73656	161189.1294
8.5	1000	167189.1294	164689.1294	2500	10087.20917	177276.3386
9	1000	183276.3386	180776.3386	2500	11072.55074	194348.8893
9.5	1000	200348.8893	197848.8893	2500	12118.24447	212467.1338
10	1000	218467.1338	215967.1338	2500	13227.98694	231695.1207
10.5	1000	237695.1207	235195.1207	2500	14405.70114	252100.8219
11	1000	258100.8219	255600.8219	2500	15655.55034	273756.3722
11.5	1000	279756.3722	277256.3722	2500	16981.9528	296738.325
13	1000	353011.3826	350511.3826	2500	21468.82219	374480.2048
13.5	1000	380480.2048	377980.2048	2500	23151.28755	403631.4924
14	1000	409631.4924	407131.4924	2500	24936.80391	434568.2963
14.5	1000	440568.2963	438068.2963	2500	26831.68315	467399.9794
15	1000	473399.9794	470899.9794	2500	28842.62374	502242.6032

SOURCE: Spread Sheet Calculation Based on the Data Found from Bank Brochures and Web Site.

*Average Amount found from the Manual Calculation based on total amount of installment within the compounding frequency plus starting balance of the periods.

cycle, but as this Tk. 2500 stays the whole time throughout the other compounding cycle it also earns interest in its all compounding cycle except the 1st compounding cycle. Hence, the essence comes in this regard that the average amount gets compounded by all compounding frequency, but the extra amount which does not earn any interest in the 1st compounding period gets compounded by 1 less period of all compounding frequencies. At the same time, the value of the non-compounded amount, i.e. the extra amount of the 1st compounding time stays with the other amount.

7.2.2. DEVELOPMENT OF FORMULA TO CALCULATE THE FUTURE VALUE

To express the above analysis mathematically, consideration of calculating maturity value after 3 years of Tk. 1000 monthly installments at 12.25% interest rate compounded semiannually is undertaken.

Here, interest rate, i=12.25%, number of years, n=3; compounding frequency, m=2; so, total compounding periods, mn=3*2=6 periods; $\frac{i}{m}=\frac{0.1225}{2}=0.06125$. It is found that Tk. It is found that Tk. 3500 will be compounded throughout the whole 6 periods but Tk. 2500 will be compounded 1 less period, i.e. through out the whole last 5 periods.

Now, Future Value of TK. 3500 average amount =

$$3500 \times FVIFA_{i/m,\, mn} \times \left(1+\frac{i}{m}\right)$$

And, Future Value of TK. 2500 will be =

$$2500 \times FVIFA_{i/m,\, mn} \times \left(1+\frac{i}{m}\right)$$

As the installment comes at the beginning of each month from the starting of the sinking fund, $\left(1+\frac{i}{m}\right)$ is multiplied with both amounts. Besides, Tk. 2500 of the first compounding cycles remains with the compounded value. So, the total maturity value will be the summation of all future accumulated sums of money plus the 1st compounding cycle's uncompounded Tk. 2500. Hence, it can be written as:

$$FVA = 3500 \times FVIFA_{i/m,\, mn} \times \left(1+\frac{i}{m}\right) + 2500 \times FVIFA_{i/m,\, (mn\text{-}1)} \times \left(1+\frac{i}{m}\right) + 2500$$

$$= \left(1+\frac{i}{m}\right)\left(3500 \times FVIFA_{i/m,\, mn} + 2500 \times FVIFA_{i/m,\, (mn\text{-}1)}\right) + 2500$$

$$= \left(1+\frac{i}{m}\right)\left(3500 \times FVIFA_{6.125\%,\, 6periods} + 2500 \times FVIFA_{6.125\%,\, (6\text{-}1)periods}\right) + 2500$$

Now,

$$FVIFA_{i/m,\, mn} = \frac{1}{i/m}\left\{\left(1+\frac{i}{m}\right)^{mn} \text{-}1\right\}$$

$$or;\ FVIFA_{6.125\%,\, 6periods} = \frac{1}{0.06125}\{(1 + 0.06125)^{6}\text{-}1\}$$

$$= 6.9973133054200714111328125$$

And,

$$FVIFA_{i/m,\, (mn\text{-}1)} = \frac{1}{i/m}\left\{\left(1+\frac{i}{m}\right)^{(mn\text{-}1)} \text{-}1\right\}$$

$$or;\ FVIFA_{6.125\%,\, (6\text{-}1)periods} = \frac{1}{0.06125}\{(1 + 0.06125)^{(6\text{-}1)}\text{-}1\}$$

$$or;\ FVIFA_{6.125\%,\, 5periods} = \frac{1}{0.06125}\{(1 + 0.06125)^{5}\text{-}1\}$$

$$= 5.65117861523681640625$$

Hence, by putting the values calculated above, the equation can be written as

$$FVA = \left(1+\frac{i}{m}\right)(3500 \times FVIFA_{6.125\%,\ 6periods} + 2500 \times FVIFA_{6.125\%,\ (6\text{-}1)periods}) + 2500$$

$$= (1 + 0.06125)(3500 \times 6.997313305420071411132812 5 + 2500 \times 5.6511786152368164062 5 + 2500$$

$$= 43483.928872369856275558471679688 \quad \approx 43483.92887$$

This future value is for 3 years maturity period compounded semiannually and it exactly matches with the maturity value after 3 years manually calculated in the table 2 found in row number 6. So, it can be said that the equation established from the table 2 serves the purpose cent percent directly and efficiently. To make applicable this formula for any amount of installment with any compounding frequency, it can be written as, 6000 = total installment within the compounding period = Installment No. of Months within Compounding Period; 3500 = The average amount of the installment within the compounding cycle; 2500 = Extra amount that is not compounded in the first compounding cycle = total amount of installment within the compounding cycle less the average amount of the installment within the compounding cycle.

Now, if it is denoted that, X = Total Installment Amount within the Compounding Period, and $\overline{X}$ = Average Installment Amount in the Compounding Period; then instead of total installment within the compounding period, it is possible to write X instead of 3500, $\overline{X}$ instead of 2500, and (X - $\overline{X}$) instead of (6000-3500). Finally, the equation can be rewritten as follows:

$$FVA = \overline{X} \times FVIFA_{i/m,\ mn}\left(1+\frac{i}{m}\right) + (X-\overline{X}) \times FVIFA_{i/m,\ mn-1}\left(1+\frac{i}{m}\right) + (X-\overline{X})$$

$$= \left(1+\frac{i}{m}\right)\{\overline{X} \times FVIFA_{i/m,\ mn} + (X-\overline{X}) \times FVIFA_{i/m,\ mn-1}\} + (X-\overline{X})$$

7.2.3. ASSUMPTIONS

This extended formula of the Sinking Fund is initiated based on the following assumptions:

1) Applicable only for the calculation of future value of a Sinking Fund.
2) The installment no. and the compounding frequency does not match i.e. complex annuity condition prevails.
3) Installment occurs at the beginning of the creation of Sinking Fund.
4) The deposited installment remains in the Sinking Fund up-to the maturity period.
5) The maturity value comes from the formula on a before tax basis.

7.3. FURTHER VERIFICATIONS OF THE EXTENDED MODEL

From the brochure of the Sylhet Corporate Branch of Trust Bank Ltd. (Table 3 & Table 4), the information relating to Trust Smart Savers Scheme is verified by using the new formula portrayed below:

For verification purpose, at first Tk. 500 deposits in each month for 10 year period is considered. Trust Bank Provides interest on all sinking fund amount quarterly basis. So interest applicable average amount can be calculated as follows:

Here, Installment is = Tk. 500; No. of Installment within Each Compounding Frequency = 3; Total Compounding Days = 90 days; Days Laps in Each Installment = 30 days.

So, the Interest Applicable Average Amount will be:

$$Avg.Installment = \frac{Installment}{TotalCompoundingDays} \times \frac{No.ofInstallment}{2} \times \{2 \times TotalCompoundingDays + (No.ofInstallment-1)(-DaysIntervalBetweenTwoInstallment)\}$$

$$or,\ \bar{x} = \frac{500}{90} \times \frac{3}{2} \times \{2 \times 90 + (3-1) \times (-30)\} = \frac{500}{90} \times \frac{3}{2} \times (180-60) = \frac{500}{90} \times \frac{3}{2} \times 120 = 1000$$

Now by putting the value of in the extended formula, it is found that

$$FVA = \overline{X} \times FVIFA_{i/m,\, mn}\left(1+\frac{i}{m}\right) + (X-\overline{X}) \times FVIFA_{i/m,\, mn-1}\left(1+\frac{i}{m}\right) + (X-\overline{X})$$

$$-\left(1+\frac{i}{m}\right)\{\overline{X} \times FVIFA_{i/m,\, mn} + (X-\overline{X}) \times FVIFA_{i/m,\, mn-1}\} + (X-\overline{X})$$

Here, $X = 500 \times 3 = 1500$; i = 11.50% ; n =10years ; m = 4 ; So,

$$i/_m = \frac{0.1150}{4} = 0.02875, \text{ and } mn = 4 \times 10 = 40 \; periods$$

$$\text{Now, } FVIFA_{2.875\%,\, 40Periods} = \frac{1}{i/_m}\{(1+\frac{i}{m})^{mn} - 1\} = \frac{1}{0.02875}\{(1+0.02875)^{40}-1\} = 73.300065069370056882652833151398$$

$$\text{And, } FVIFA_{2.875\%,\, 39Periods} = \frac{1}{i/_m}\{(1+\frac{i}{m})^{(mn-1)} - 1\} = \frac{1}{0.02875}\{(1+0.02875)^{39}-1\} = 70.279528621501877893222681070618$$

By substituting the values of different variables it is found that

$$FVA = \left(1+\frac{0.115}{4}\right)\{1000 \times FVIFA_{2.875\%,\, 40periods} + (1500-1000) \times FVIFA_{2.875\%,\, 39periods}\} + (1500-1000)$$

$$= 1.02875\{(1000 \times 73.300065069370056882652833151398) + (500 \times 70.279528621501877893222681070618)\}$$

$$+ 500 = 1{,}12{,}057.47 \quad \approx 1{,}12{,}057$$

Table 3 also shows that at 11.50 annual interest rate with a quarterly compounding frequency, Trust Bank provides Tk. 1, 12,057 for Tk. 500 monthly installments in a Smart Savers Scheme (i.e. in a Sinking Fund) for 10 years. The extended formula in the above calculation also provides the same amount from the sinking fund calculation. Same procedure goes true for other amount of the installment.

Table 5, 6 and 7 also provide a more extensive proof regarding the maturity value found from the already developed and discussed extended formula through the comparison with the maturity value found from offerings by the respective banks for different installment amount for different maturity periods. All the calculated values exactly match with the respective banks' proposal values except Shah Jalal Islami Bank. Since Shah Jalal Islami Bank offers the profit rate and the proposed value is the proposed (estimated) value which may vary based on the earnings of profit, so this bank has presented the amount of FV only by the rounded figure expressed to the nearest thousands taka in its proposal. It is here worthy to mention that if the calculated value found by utilizing the new formula is rounded to the nearest thousand taka figures, it also gets matched with this Islamic bank's proposal value (Table 7).

8. FINDINGS

TABLE 3: Trust Savers Scheme

Amount	3 Years	5 Years	7 Years	10 Years
500	21,543	40,560	64,417	1,12,057
1000	43,085	81,119	1,28,834	2,24,115
2000	86,171	1,62,239	2,57,669	4,48,230
3000	1,29,256	2,43,358	3,86,503	6,72,345
4000	1,72,341	3,24,478	5,15,337	8,96,460
5000	2,15,426	4,05,597	6,44,172	11,20,575

SOURCE: Brochure of Trust Bank

TABLE 4: Interest Rate for Different Deposit Term for Trust Saver Scheme

Deposit Term	Interset Rate	Compounding Frequency
3 Years	11.50%	Quarterly
5 Years	11.50%	Quarterly
7 Years	11.50%	Quarterly
10 Years	11.50%	Quarterly

SOURCE: Brochure of Trust Bank

* Compounding frequency was determined through Spread Sheet by analyzing the proposal value with the installment amount for respective time period.

The study clearly tells that when the basic sinking fund formula does not provide the actual accumulated value due to the discrepancy between the number of deposit receiving and the compounding frequency, i.e. complex annuity, the model developed as an extension form of the prevailing sinking fund formula gives the accurate results. Besides, as banks and financial institutions compound the average amount of its all deposit amounts within the particular compounding frequency in sinking fund provision, this study has also presented an arithmetic formula for calculating the average amount of installment on which banks will apply the compounding procedures. So, the findings of the study are twice.

i. The first and most important one is the extended version of the basic Sinking Fund Formula and it is written as follows:

$$FVA = \overline{X} \times FVIFA_{i/m,\ mn}\left(1+\frac{i}{m}\right) + (X-\overline{X}) \times FVIFA_{i/m,\ mn-1}\left(1+\frac{i}{m}\right) + (X-\overline{X})$$

Where, $\overline{X}$ = Average of the Installment Amount in the Compounding Period; X = Total Installment Amount in the Compounding Period; i =Interest Rate on Sinking Fund; n = No. of Years; m = Compounding Frequency.

ii. The second one is the arithmetic formula for calculating the average amount of the installment.

$$\overline{X} = \frac{Installment}{TotalCompoundingDays} \times \frac{No.of\,Installment}{2} \times \{2\times TotalCompoundingDays + (No.of\,Installment-1)(-DaysIntervalBetweenTwoInstallment)\}$$

9. CONCLUDING REMARKS

The necessity of basic Sinking Fund formula is worth mentioning. When the number of installment receiving and the compounding frequency take place at the same frequency, the given formula in the basic Finance literature delivers the accurate sinking fund accumulated value. But in case of complex annuity, i.e. when the compounding frequency and number of installments do not occur with the same frequency, the accurate value which gets accumulated from the sinking fund of a bank does not come through this basic formula. In that case, the formula developed throughout this study delivers the accurate results. Since, banks and financial institutions normally do not follow the same frequency in installment receiving and interest giving when they compound the sinking fund deposit of their customers, the use of the extended formula will allow them to get the accurate value by the use of this developed model. Having avoided the manual step by step calculation of sinking fund, officials of banks and financial institutions can find out the accumulated value now easily and ultimately not only bank officials undoubtedly will increase their efficiency by utilizing the findings of this study properly for any type of sinking fund calculation but also investors and financial analysts must get a strong platform for their decision making activities. Last but not least, it goes without saying that whether to take decision in Capital Budgeting, or ROE from a project or even comparing actual opportunity cost, through the use of this extended formula along with the formula derived to calculate the average amount, the respective banks, individuals or investors can take the accurate decisions.

10. RECOMMENDATIONS

This study offers the following recommendations for the fruitful utilization of the extended model of the Sinking Fund along with the arithmetic formula of the average amount calculation:

- Banking sector should utilize this extended model of sinking fund to save the time and increase the efficiency.
- Financial analyst and investors should calculate the future sum of money of any series of equal payments or receipts by this formula to evaluate any project or investment proposal and get the accurate picture of the return.
- Through the uses of this formula, personal savers can compare their proposed return accurately among various alternative offerings.
- Banks, individuals, investors etc. can have proper financial planning through the proper utilization of this new extended formula.
- The arithmetic formula for calculating the average amount must be utilized prior to utilize the extended formula because when compounding frequency and number of installment in a sinking fund do not match, the compounding done by banks or financial institutions bases on average amount of installment.

TABLE 5: Comparison of Pre-tax Maturity Value found from New Extended Formula with the Respective Bank's Pre-tax Maturity Value found from Brochures & Website.

Monthly Installment	Bank's Proposal	Calculated Value	Bank's Proposal	Calculated Value	Bank's Proposal	Calculated Value
	5 Years	5 Years	7 Years	7 Years	10 Years	10 Years
AB Bank Ltd						
Interest Rate 12.25%[I] Compounded Semiannually*						
500	41,196	41196.15	65,873	65873.27	1,15,848	115847.56
2000	1,64,786	164785.66	2,63,496	263493.08	4,63,390	463390.24
5000	4,11,964	411964.16	6,58,733	658732.71	11,58,476	1158475.60
10000	8,23,930	823928.32	13,17,480	1317465.42	23,16,950	2316951.21
Trust Bank Ltd (Trust Savers Offer)						
Interest Rate 11.50[I]% Compounded Quarterly*						
2000	1,62,239	162239.00	2,57,669	257668.66	4,48,230	448229.90
3000	2,43,358	243358.50	3,86,503	386502.99	6,72,345	672344.85
4000	3,24,478	324477.99	5,15,337	515337.32	8,96,460	896459.80
5000	4,02,597	405597.49	6,44,172	644171.65	11,20,575	1120574.74
Pubali Bank Ltd (Pubali Pension Scheme)						
Interest Rate 11%[I], 11[I]% & 12[I]% for 5 years, 7 years & 10 years respectively Compounded Annually*						
1000	79,186	79186.49	1,24,394	124394.33	2,24,273	224272.83
2000	1,58,373	158372.99	2,48,789	248788.66	4,48,546	448545.67
3000	2,37,559	237559.48	3,73,183	373182.99	6,72,819	672818.50
5000	3,95,932	395932.47	6,21,972	621971.65	11,21,364	1121364.17
10000	7,91,865	791864.95	12,43,943	1243943.30	22,42,728	2242728.34

SOURCES:

[I] In case of AB Bank Ltd: Bank's Web-site.

[I] In case of Trust Bank Ltd & Pubali Bank Ltd: Banks' published brochures.

[II] Calculated through the newly developed extended formula and *Compounding frequency was determined through Spread Sheet and justified also by the respective bank's officials.

TABLE 6: Comparison of Pre-tax Maturity Value Found from New Extended Formula with the Respective Nationalized Bank's Pre-tax Maturity Value found from Brochures.

Sonali Bank Limited				
Interest Rate 8.50% [I]for Sonali Deposit Scheme(SDS) for 5 Years Compounded Annually[*]				
Monthly Installment	**Bank's Proposal with Bonus[I]**	**Bonus Amount[I]**	**Calculated Value[II]**	**Calculated Value including Bonus**
500	38,689	1,500	37,189.12	38,689.12
1000	76,378	2,000	74,378.24	76,378.24
5000	3,77,891	6,000	3,71,891.21	3,77,891.21
10000	7,54,782	11,000	7,43,782.42	7,54,782.42
Interest Rate 8.00% for Education Deposit Scheme (EDS) for 10 Years Compounded Annually				
500	92,185	1,500	90,685.88	92,185.88
1000	1,83,371	2,000	1,81,371.76	1,83,371.76
5000	9,12,858	6,000	9,06,858.81	9,12,858.81
10000	18,24,717	11,000	18,13,717.62	18,24,717.62

[I] Bank's published brochure.
[II] Calculated through the newly developed extended formula and
[*] Compounding frequency was determined through Spread Sheet and justified by bank's officials

TABLE 7: Comparison of Pre-tax Maturity Value found from New Extended Formula with an Islami Bank's Pre-tax Maturity Value found from Brochures & Website.

Shah Jalal Islami Bank						
Profit Rate 12.23%[I], 12.07%[I] & 12.05%[I] (Proposed) for 5 years, 8 Years & 10 years Respectively Compounded Monthly[*]						
Monthly Installment	**Bank's Proposal[I]**	**Calculated Value[II]**	**Bank's Proposal[I]**	**Calculated Value[II]**	**Bank's Proposal[I]**	**Calculated Value[II]**
	5 Years	5 Years	8 Years	8 Years	10 Years	10 Years
1000	83,000	83012.29	1,62,000	162050.28	2,33,000	233032.01
5000	4,15,000	415061.45	8,10,000	810251.39	11,65,000	1165160.07
10000	8,30,000	830122.90	16,20,000	1620502.79	23,30,000	2330320.13

Bank: Bank's Web-site
[II] Calculated through the newly developed extended formula and *Compounding frequency was determined through Spread Sheet and justified by bank's officials.

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