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Roles of Teachers in Learning Study: A Case Study in Teaching Fractions[1]

WONG, Tak Wah*
Department of Mathematics and Information Technology, Hong Kong Institute of Education, Hong Kong SAR, China; Email: twwong@ied.edu.hk

LAI, Yiu Chi
Department of Mathematics and Information Technology, Hong Kong Institute of Education, Hong Kong SAR, China; Email: yiuchi@ied.edu.hk

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This paper aims to explore whether Learning Study improves teachers' subject content knowledge, pedagogical content knowledge, and attitude toward teaching mathematics. A Learning Study was conducted in a Hong Kong primary school for a research lesson on comparing the size of fractions to explore the new teacher roles.

Keywords: subject content knowledge, pedagogical content knowledge, attitude toward mathematics teaching, roles of teachers, learning study
MESC Classification: D49
MSC2010 Classification: 03-01, 97D40

BACKGROUND

In recent years, many new initiatives with impressive slogans have been introduced in schools in Hong Kong, for example: learning to learn, project learning, life-long learning, using information technology in education and caring, and learning diversity in education. However, the implementation details and outcomes are always vague and difficult to observe in real classrooms. Lo (2004) concluded that these educational initiatives have only limited effects on students' learning and claimed that in order to improve student's learning outcomes, teachers should be capable of creating high-quality teaching plans and teaching materials, and also an ideal learning environment. For the teaching and learning

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* Corresponding author

of mathematics, actual classroom practice and achieving effective mathematics instruction become the main concerns.

LITERATURE REVIEW

Effective Classroom Mathematics Instruction

Many researchers have attempted to identify important features of effective classroom instruction (e.g. Confrey, 1990; Noddings, 1990; Porter & Brophy 1988; Shulman, 1986a; 1986b). Noddings (1990) and Confrey (1990) both stated that the mathematics teacher's main function is to establish mathematical learning environments that encourage students to explore and raise questions as they learn. They claim that in so doing, teachers' pedagogical content knowledge (PCK), which includes the questions they ask, the activities they design, the teaching aids they use, and the student's suggestions they follow, is based in and modified from their mathematics subject content knowledge (SCK). Thus they felt that an important initial step in improving mathematics teaching should be better SCK preparation for mathematics teachers. Moreover, many researchers (Ball, 1991; Confrey, 1990; Noddings, 1990; Shulman, 1986a) have assumed that SCK and PCK are positively correlated with teaching effectiveness in mathematics, and that PCK is influenced by SCK. Wong (2002) also found that mathematics teachers' instruction performance is correlated with teachers' SCK, PCK, and their attitude toward mathematics (ATM). Figure 1 explains the relationships between SCK, PCK, and ATM and teachers' teaching performance.

Besides exploring the nature of a good mathematics teacher, some researchers also found that a high quality of classroom mathematics instruction requires a careful analysis of the mathematics content students learn in the classroom (Cai, 2005; Bransford et al., 2000). Cai (2005, p. 52) also stated that

> "by conceptualizing the effectiveness of mathematics instruction, we need to focus not only on students' learning outcomes, but also on the processes that lead to desirable learning outcomes"

Figure 2 shows a framework, adapted from Cai (2005), for examining the effectiveness of mathematics instruction.

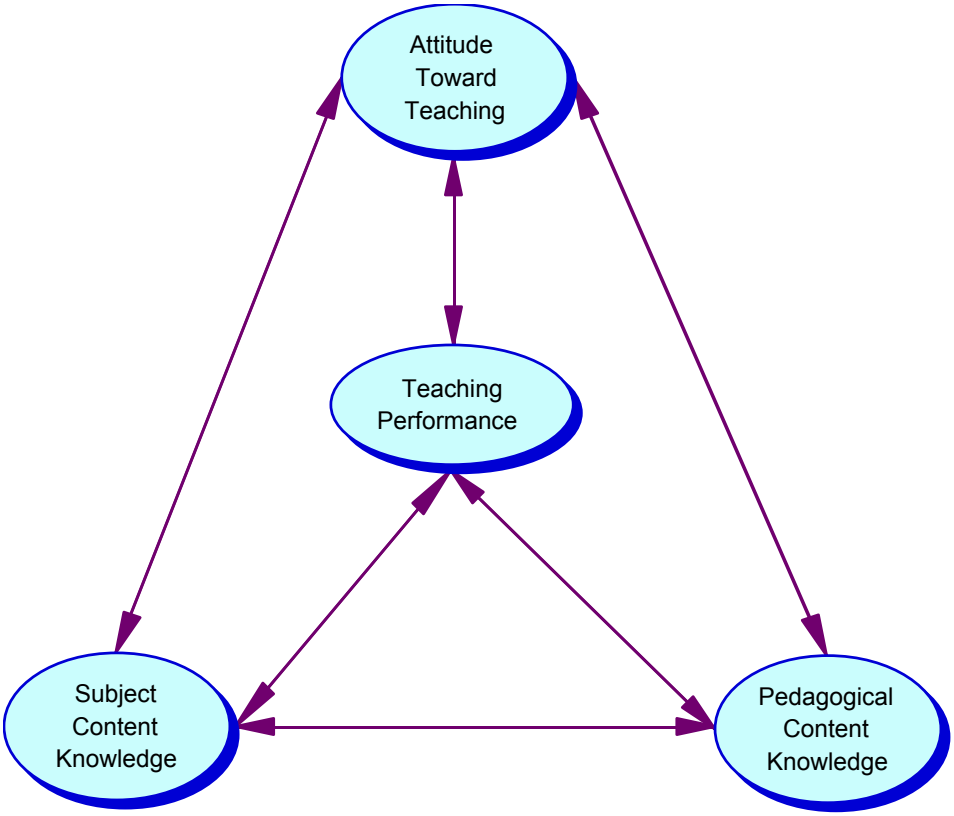


Figure 1. The relationships among the factors of SCK, PCK and AMT with teachers' teaching performance

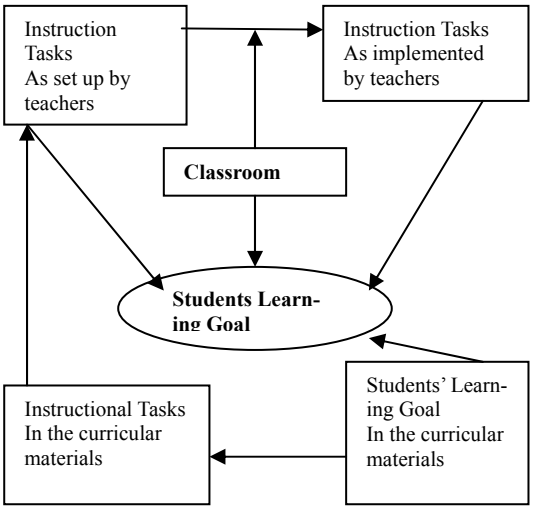


Figure 2. The framework of examining effectiveness of mathematics instruction

In this framework, the classroom instruction is considered a vital element in helping students to achieve the planned learning objectives. Besides the curricular materials from textbooks and the guidance of the syllabus, teachers also need to provide their own supplementary teaching materials. In the teaching-learning process, it is critical for teachers to decide which critical feature of the teaching content should be focused on. They also need to organize and arrange the students' learning process. For example, teachers should think about what questions should be asked in their teachings, what teaching aids should be employed, and what variations should be adopted in their teaching for the purpose of helping students to learn mathematical concepts or skills effectively. Learning Study fully understands that the factors mentioned above can directly affect the effectiveness of students' learning and teachers' teaching. Thus all these factors will be discussed in the Learning Study meetings.

Learning Study

In Hong Kong, "Leaning Study" is recognized as a powerful means of improving teaching and learning in the classroom as well as staff development. What is "Learning Study"? Learning Study is the Hong Kong version of "Lesson Study" (Lo, 2004), which is a kind of action research developed in Japan over a period of 40 years and introduced to the world by Stigler & Hiebert (1999) several years ago. It then received a lot of attention and is now commonly thought of as a powerful tool for improving classroom practice. Meanwhile, a number of researchers have implemented Lesson Study programs in different countries (Fernandez, 2002; Lewis, 2002; Watanabe, 2002; White & Southwell, 2003). Thus Learning Study can be treated as an educational action research. Its participants all come from educational institutions and their studies nearly all focus on the development of teaching materials, teaching strategies, and professional development. It is often the case that staff from the Hong Kong Institute of Education act as academic and teaching development consultants, that and primary and secondary teachers act as both committee members and lesson presenters in school-based projects. In Hong Kong, Learning Study was first introduced in two primary schools in 2002; by 2004, more than 100 Learning Studies had been developed in both primary and secondary schools (Lo, 2004).

RESEARCH METHODOLOGY AND RESULTS OF FINDINGS

In Hong Kong, Learning Studies mainly conduct as practical cases in either primary or secondary schools. In this paper, a primary school case was used to explore whether new teacher roles exist, and to evaluate and analyze what the new roles are. Thus this study is

a case study research project. It emphasizes the qualitative analysis of a limited number of Learning Study cases to bring us to an understanding of how the current roles of teachers have changed and also show how Learning Study projects can serve to improve teachers' PCK, SCK, and ATM. The following three steps are used in this case study.

Step 1: The researchers define and determine the research questions

Research Questions: The main aim of this study is to explore whether Learning Study has a positive effect on improving teachers' PCK, SCK, and ATM. Hence the questions, does the traditional teacher's role change through the practice of Learning Study on teaching mathematics? If so, what are the new teacher roles?

Step 2: Select the case

The researchers focus on the research questions and ideas to analyze (review) the procedure and the outputs of a Primary Four Learning Study case. This was one of the cases in a Hong Kong government-funded project on the promotion of teaching and learning outcomes, based on the Learning Study model designed by the Learning Centre of the Hong Kong Institute of Education. One of the authors was the researcher in charge of this Learning Study case. The research lesson was a Primary Four lesson on comparing the size of fractions which was taught to three classes, 118 pupils, in a well-known Hong Kong primary school. The research lesson committee consisted of one vice-principal, one mathematics panel teacher and three grade four mathematics teachers, one research center TDC (Teaching Development Consultant), one academic (from the Mathematics Department of the Hong Kong Institute of Education), and one DEO (District Education Officer of the Education Department).

Step 3: Collect and analyze data

In order to collect data relevant to answering the research questions above, we first need to understand the conceptual framework of Learning Study.

Lo (2004) has pointed out that the conceptual framework of Learning Study is based on variation, and that this is the main feature differentiating Learning Study from the Japanese Lesson Study. In planning each research lesson, teachers need to consider three types of variation:

V1. Variation in terms of students' understanding of what is taught,

V2. Variation in teachers' ways of dealing with particular topics, and

V3. Variation as a pedagogical tool.

Within this conceptual framework, the teacher should be actively constructing learning experiences for the students so that they can experience appropriate variations in the object of learning to bring about the intended discernment and learning (Ko, 2004). Simply speaking, Learning Study is a cyclical development process, each cycle comprising four main steps: plan, act, observe, and reflect. Figure 3 shows the framework of a Learning Study.

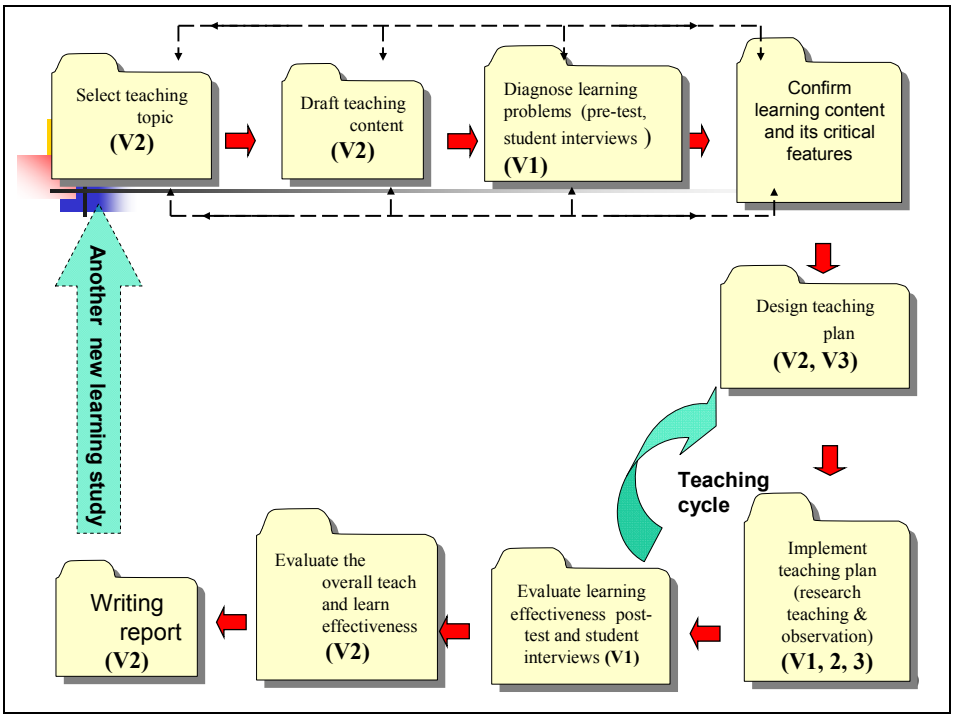


Figure 3. The framework of a Learning Study

In general, Learning Study emphasizes the effectiveness of using three kinds of variations in lesson planning. It is a cycle of professional development focused on:

1. Students' problems (V1),
2. Teachers' planning (V2, V3),
3. Observing and revising " research lessons"(V1, V2, V3) ,and also
4. Assessing and analyzing students' learning outcomes (V1).

In the case of teaching fractions, during the first committee meeting, teachers were asked to review and discuss students' different views and understanding of the learning content. They needed to ensure that students understood the concepts of comparing frac-

tions with different denominators. They also needed to identify any difficulties students would likely encounter in comparing fractions with different denominators. Hence, the teachers needed to produce a pre-test (see Appendix 1 for the pre-test paper).

They found that students had the following problems comparing fractions with different denominators:

i) Fractions with the same nominator but different denominators such as 10/11 and 10/13,

ii) Fractions with the same difference of 1, where the nominator is 1 less than the denominator, such as 12/13 and 15/16,

iii) a common difference between nominator and denominator, such as 7/10 and 11/14 (common difference of 3), and

iv) Fractions with different denominators, but no common pattern, such as

(a) 5/7 with 9/14,

(b) 3/5 with 5/9, and

(c) 5/8 with 4/ 6.

Diagrams 1-3 below show an example of the pilot/pre-test findings and also examples of analysis of the "paper and pen" and "student interview" results.

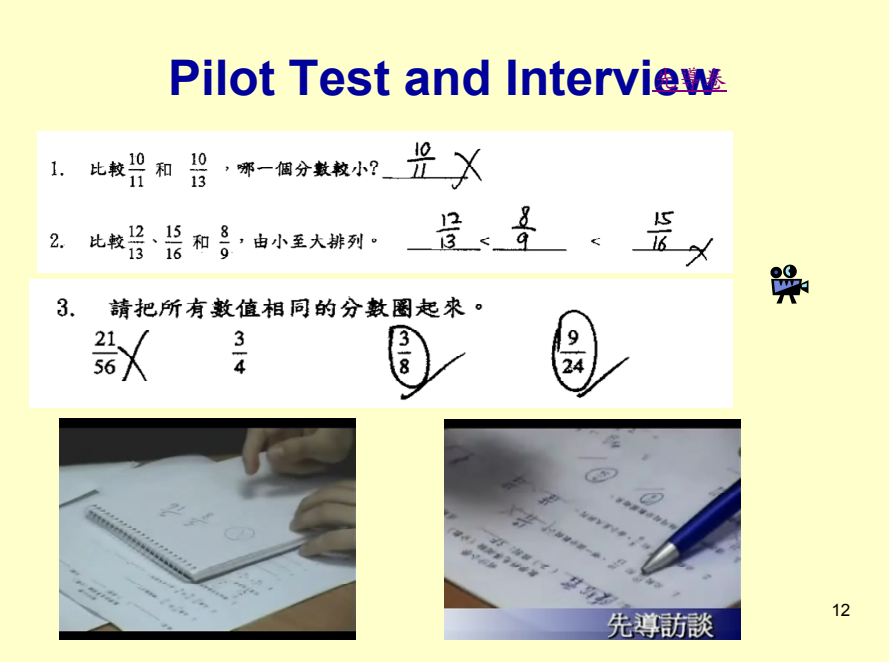


Diagram 1. Example of pre-test findings

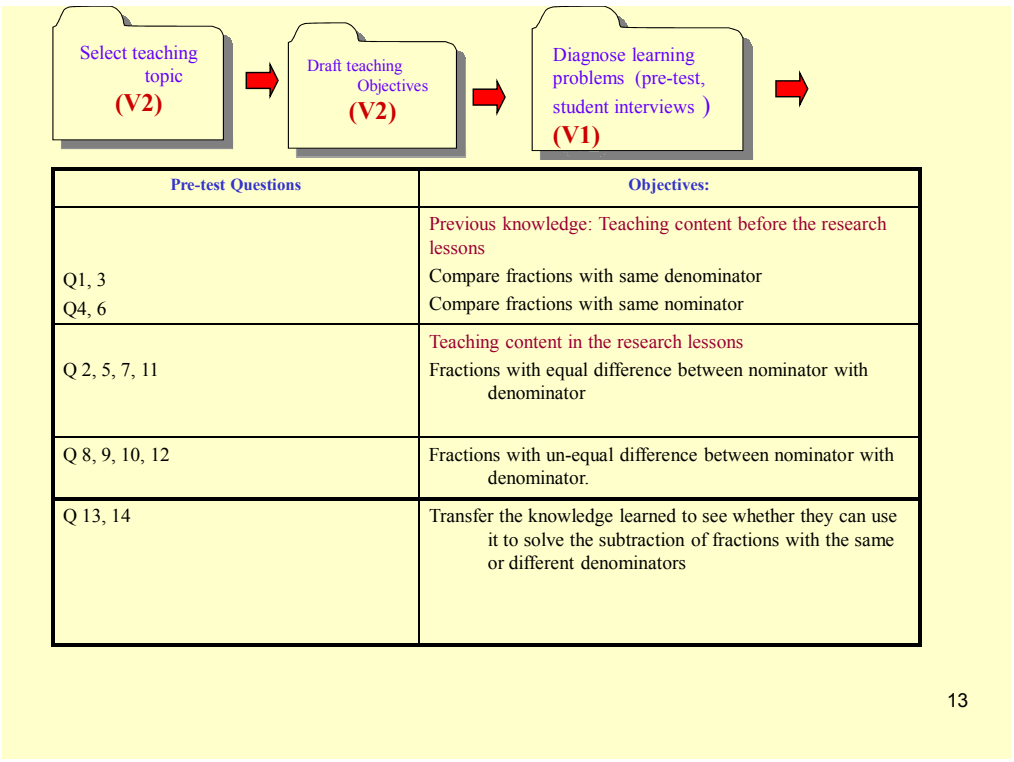


Diagram 2. Example of pre-test analysis findings

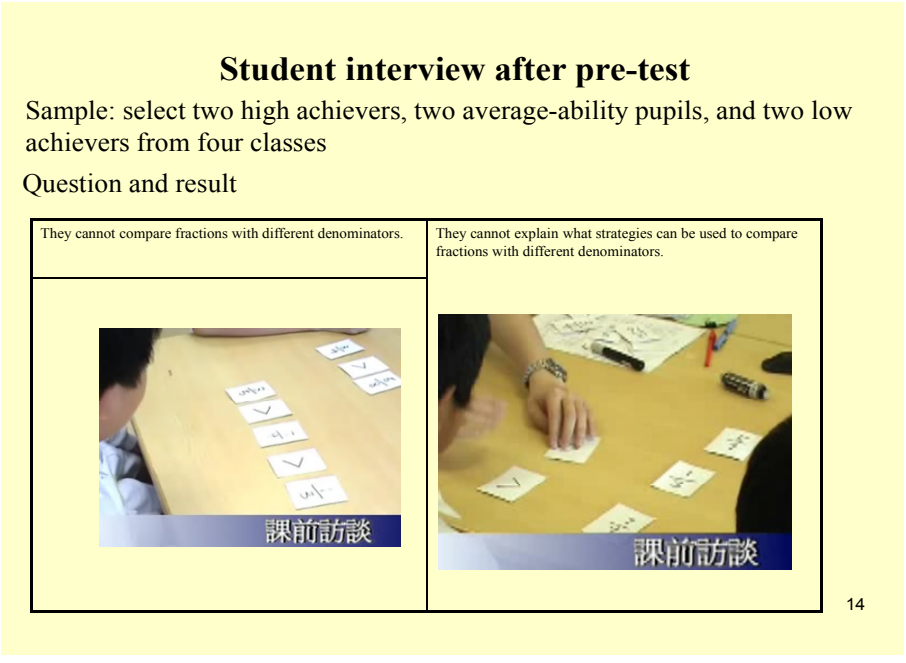


Diagram 3. Example of the result from a student interview after the pre-test

Based on these findings, teachers confirmed that the teaching content of the research lesson is "Use different strategies to compare fractions with different denominators". Then they went on to share, analyze, and confirm the critical features of the research lesson content (V2), which are as follows:

i) Fractions with an equal difference between nominator and denominator:

   Where the pattern exists, the fraction with the larger denominator is larger than that with smaller denominator (e.g., 7/9 vs 11/13 and 5/9 vs 9/13).

ii) Fractions with an unequal difference between nominator and denominator:

   Analyze their denominators; find an easy way to give them the same denominator, e.g.

3/4	vs	5/8	becomes	6/8	vs	5/8
3/5	vs	5/8	becomes	24/40	vs	25/ 40
5/8	vs	7/32	becomes	15/24	vs	14/24

In later meetings, teachers shared their views on designing teaching plans and strategies for the selected teaching content (V2) and then, using the collective talents and experience of each member in the research team, the planned teaching strategy was developed for use in the research lessons (V3). Many teaching aids are used in the research lesson, such as paper folding, pictures, information technology, and – the most important – questioning techniques. All the SCK and PCK used in the lesson were the outputs of a collaboratively devised teaching design based on the committee's different views and teaching strategies for the learning objectives selected (an example is shown in Table 4).

After the research lesson, we conducted student interviews and research conferencing, so that research members could share their own comments and make suggestions on how to improve teaching effectiveness (V2).

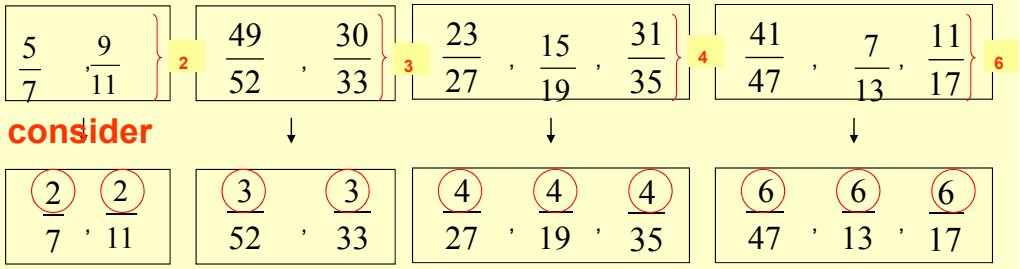
Finally, students were asked to do the post-test paper, allowing us to compare pre- and post-tests results in order to determine whether the Learning Study had improved teaching and learning effectiveness. The table below is an analysis of the pre- and post-test results which shows clearly that Learning Study is a good way of improving teaching and learning effectiveness.

Table 4. Example of a teaching design for an activity worksheet that involves the use of the theory of variations

Teaching design and use of theory of variations

Activity：Worksheet

comparing fractions with the equal different between denominator and nominator



Teaching	Judge the critical feature	Change	No Change
Use one example to guide pupils discover the pattern, hence use more examples let them generize the comparing method	While they have the same nominator, we can compare their sizes via the size of their denominators	Different numbers used as nominators and denominators	Fractions with the same different between denominator and nominator

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From this case we can conclude that Learning Study has a positive effect on improving teachers' PCK, SCK and ATM as, during the meetings, teachers had many opportunities to share their views – on subject matter, teaching suggestions, teaching aids, designs, and even ways to cater to learning diversity – with their colleagues and the professionals from Hong Kong Institute of Education and Hong Kong Education Department.

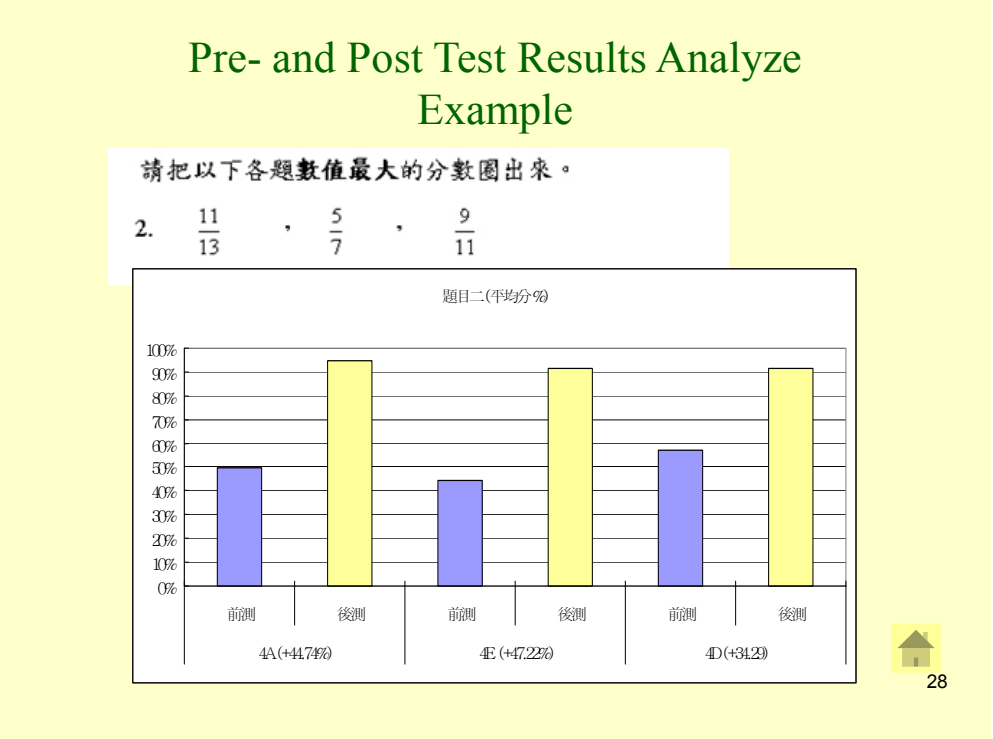
Thus, reviewing the whole practice of Learning Study, does the traditional teacher's role change? If so, what are the new teacher roles? The section below presents our findings.

The Role of Teachers in Learning Study

In traditional teaching, teachers act as knowledge transmitters. Their teachings are always textbook oriented, which means that the teaching contents and teaching methods are always guided or suggested by authors who are uninvolved in the teaching practice. Thus we can assume that there are very few school-based curricula, and no worksheets, no creative teaching strategies offering different content for different students. Teachers al-

ways point out that they do not have the expertise or experience to develop school-based teaching materials and teaching plans.

Table 5. Example of a an analysis of the pre- and post-test result



Through the practice of Learning Study, teachers can develop their school-based teaching content with the help of academic and teaching development consultants. They are able to create high-quality teaching materials and ideal learning environments as well as equip themselves with effective strategies and teaching aids. In addition, since each teaching cycle includes four steps (plan, act, observe, and reflect), teachers have to assess and analyze students' learning outcomes. Those findings should be interpreted in light of how successful the teaching has been. Thus, if problems still exist in the learning-teaching process, the plan-act-observe-reflect process begins another cycle. This process continues until all research lessons are taught. Thus, through the analysis of the teachers' involvement in the Learning Study project, we can affirm that teachers no longer play the traditional role of knowledge transmitter. In fact, teachers in the Learning Study team play many roles which are new and exciting. It may be necessary to adopt several of these different roles at various stages of the Learning Study process:

Researcher	Currriculum planner
Problem solver	Mentor/Mentee

Listener	Observer
Synthesizer	Decision implementer

Some examples applied to mathematics teaching are listed below to illustrate teachers' roles in the Learning Study process.

Teacher as researcher

Lesson Study is action research by nature. In each study, teachers are teaching the same subject at the same level and also collaborate with researchers from universities to plan and develop research lessons. All participants in the Learning Study project, including teachers, are thus co-researchers. Based on the principle of collaborative study, each person's ideas are equally valid as potential resources for creating interpretive categories of analysis, to be negotiated among the participants. Very often, teachers provide valuable teaching insights. In this regard, teachers involved will gain concrete experience of dealing with problems encountered through action research and also provide valuable ideas. In other words, teachers are also researchers in school as they will inevitably be involved in this kind of scholarly activity when conducting a Learning Study.

Teacher as curriculum planner

The teachers involved in Learning Study are required to explore the object of learning in each lesson, which means that they should carefully consider what their pupils are supposed to learn. Before they make any decision, they must answer the following:

Is the selected topic worth enough to study? Please explain and provide evidence.

Teachers thus need to identify the topic's learning objectives. They are also required to identify the critical features in understanding the object of learning. In other words, teachers should examine what is critical for the students to be able to learn the object of learning. Thus, each teacher in the Learning Study team should know the subject curriculum well and understand how the topics should be sequenced and taught to their students. Teachers are assumed to be expert in curriculum design and implementation and must function as curriculum planners.

Teacher as problem solver

Action research is more of a holistic approach to problem-solving, rather than a single method for collecting and analyzing data. Therefore, once teachers identify problems in the teaching-learning process, they must solve them using the plan-act-observe-reflect cycle. Thus, as NCTM (1991) stated, the role of teachers is to select and develop tasks that are likely to foster the development of students' abilities to solve problems and rea-

son and communicate mathematically. Thus there is no doubt that teachers in the Learning Study project can be viewed as problem solvers.

Teacher as mentor/mentee

In each study, teachers are expected to share their experience in dealing with particular objects of learning. It was found that new teachers could learn much in the way of teaching strategies from their more experienced counterparts through these interactive discussions. Experienced teachers and new teachers will always form an informal mentor-mentee relationship. This also explains why Learning Study is recognized as a powerful tool for teacher's professional development.

Teacher as listener, observer, synthesizer

Learning Study embodies a range of viewpoints, commentaries, and critiques, leading to multiple possible actions and interpretations. This complex constitution of inquiry requires teachers to be more patient, tolerant, and open-minded. They should give and receive advice to and from other members in order to reach a consensus on planning lessons and to enable them to teach with understanding and clarity.

Teacher as collective decision implementer

For Learning Study researchers, planning informs practice and practice refines planning. It is a continuous transformation, a teaching cycle. In research lessons, teachers' teachings are based on implicitly held assumptions and hypotheses and, with every observed result, the expected achievement is improved. After each research lesson, it is important to have a post-lesson conference for the purpose of evaluating the lesson and reflecting on its effectiveness. As Lo (2004, p. 8) has stated,

> the focus of the evaluation is on the lesson rather than on the teacher who taught it. Since the lesson is a product of the joint efforts of everyone in the study, they are in fact evaluating themselves. Thus, instead of focusing on the personal evaluation of a teacher, the whole focus of the activity shifts toward lesson improvement.

Following the post-lesson conference, teachers should revise their lesson plan based on the evaluation.

CONCLUSION

Regardless of what education reform we undertake, what educational action research we conduct, we should bear in mind that our ultimate aim is to improve students' learning outcomes. In this paper, we discussed a research method for improving teaching effec-

tiveness which assumed that high-quality mathematics teaching should be developed through team discussion rather than individual planning or textbook oriented teaching. We also emphasized that observing classroom teaching, evaluating the lesson, and reflecting on its effect are important steps for improving teaching effectiveness. How do teachers rate the Learning Study? To answer this question, we would like to list teachers' comments below:

T1: Learning Study values us as professionals and allows us to use our collective talents and experiences to increase student achievement.

T2: It also increases our subject matter knowledge and pedagogical content knowledge.

T3: Academic and teaching development consultants have supported us in constructing deep and clear mathematical concepts. They also helped us learn more about the method of conducting action research.

T4: Under their guidance, we have changed the way we talk about teaching and learning. Learning Study also makes our attitude toward mathematics teaching more positive.

However, we want to emphasize that teachers' SCK is not unimportant; teachers should spend more time expanding their mathematics subject knowledge to be better mathematics teachers. It should also be noted that teaching hardware, such as computers, overhead projectors, other teaching aids, and class size, are also important, as they are essential in providing a high-quality teaching materials and ideal learning environments for teachers to teach and students to learn.

REFERENCES

Ball, D. L. (1991). Research on teaching mathematics: Making subject matter knowledge part of the equation. In: J. Brophy (Ed.), *Advances in research on teaching* Vol. 2 (pp. 1–48). Greenwich, CT: JAI Press.

Bransford, J. D.; Brown, A. L. & Cooking, R. R. (2000). *How people learn: Brain, mind, experience, and school* (expanded edition). Washington, DC: National Research Council. ERIC ED481522

Cai, J. (2005). Critical features of effective mathematics instruction: issues and practices. In: Lam, L. (Ed.), *Recent Advances in Mathematics Education Research and Teaching Practices (Procedings of the International Conference on Mathematics Education, Hong Kong, China; July 6–8, 2005)* (pp. 51–70). Hong Kong: Hong Kong Institute of Education.

Confrey, J. (1990). What constructivism implies for teaching. In: R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist Views on the Teaching and Learning of Mathematics* (pp. 107–122). Reston, VA: National Council of Teachers of Mathematics. ME **1992f.**01043

Fernandez, C. (2002). Learning from Japanese approaches to professional development: The case of Lesson Study. *Journal of Teacher Education* **53(5)**, 393–405. ERIC EJ667199

Ko, P. Y. (2004). *The interface between teacher act and the improvement of students' learning.* .Symposium presentation, Australian Association for Research in Education 2004 International Education Research Conference. Australia, Nov. 28–Dec. 2, 2004.

Lewis, C. (2002). Does Lesson Study Have a Future in the United States? *Nagoya Journal of Education & Human Development* **1**, 1−23. Available from: http://www.lessonresearch.net/nagoyalsrev.pdf

Lo, M. L. (2004). *The development of Learning Study in Hong Kong as a powerful tool for teachers' professional learning and factors that lead to its success*. Symposium presentation, Australian Association for Research in Education 2004 International Education Research Conference. Australia, Nov. 28–Dec. 2, 2004.

National Council of Teachers of Mathematics (1991). Professional standards for teaching mathematics. Reston, VA: Author. ME **1991e.**00332

Noddings, N. (1990). Constructivism in mathematics education. In: R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist Views on the Teaching and Learning of Mathematics* (pp.7–18). Reston.VA: National Council of Teachers of Mathematics. ME **1992f.**01043

Porter, A. C., & Rousseau, E. W. (1988). Synthesis of research on good teaching: Insights from the work of the institute for research on teaching. *Educational Leadership* **45(8)**, 74–85. ERIC EJ373391

Shulman, L. S. (1986a). Those who understand: Knowledge growth in teaching. *Educational Researcher* **15(2)**, 4–14. ERIC EJ330821

_____ (1986b). Pardigms and research programmes in the study of teaching: A contemporary perspective. In: M. C. Wittrock (Ed.), *Handbook of research on teaching* (3rd ed.), pp.3–36). New York: Macmillan.

Stigler, J. W. & Hiebert, J. (1999). *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom.* New York: Free Press. ERIC ED434102

Watanabe, T. (2002). Learning from Japanese Lesson Study. *Educational Leadership*. **59(6)**, 36–39. ERIC EJ640977

White, A. L. & Southwell, B. (2003). *Lesson study project: Evaluation report*. Sydney, NSW: Department of Education and Training.

Wong, T. W. (2003). Competency in Mathematics Teaching: Subject Content knowledge, Pedagogical Content Knowledge and Attitudes toward Teaching Mathematics. *Doctoral dissertation*, Leicester, England: University of Leicester.