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Antisymmetric-Symmetric Mode Conversion of Ultrasonic Lamb Waves and Negative Refraction on Thin Steel Plate

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Abstract In this study, focusing of ultrasonic Lamb wave by negative refraction with mode conversion from antisymmetric to symmetric mode was investigated. When a wave propagates backward by negative refraction, the energy flux is antiparallel to the phase velocity. Backward propagation of Lamb wave is quite well known, but the behavior of backward Lamb wave at an interface has rarely been investigated. A pin-type transducer is used to detect Lamb wave propagating on a steel plate with a step change in thickness. Conversion from forward to backward propagating mode leads to negative refraction and thus wave focusing. By comparing the amplitudes of received Lamb waves at a specific frequency measured at different distance between transmitter and interface, the focusing of Lamb wave due to negative refraction was confirmed.

Keywords: Lamb Wave, Mode Conversion, Backward Propagating Wave, Negative Refraction, Veselago Lens

1. Introduction

There has been increasing interest on materials with negative refractive index [1]. Having negative refractive index means that a wave's energy flux (or group velocity) is anti-parallel to its phase velocity in that material.

In these materials, interesting physical phenomena such as negative refraction appears [2]. After Horace Lamb provided theoretical background for the existence of negative-index materials[3], backward propagation of electro-magnetic and acoustic waves have been observed on many materials including metamaterials [4,5].

Backward propagation of Lamb wave, a type of plate wave, is well known both theoretically and experimentally. Lamb waves' modes are determined by the solutions of Rayleigh-Lamb dispersion equation [6]. There are symmetric and antisymmetric modes of Lamb waves; in specific modes group velocity is negative, showing backward wave propagation. However, properties of Lamb waves at the interface of a plate with step thickness change where forward propagation occurs on one side and backward propagation occurs on the other side was not well known until the recent research by Bramhavar et al. [7]. They observed that Lamb wave originated from a point source focuses in another media, which is called Velelago lens effect [8]. They used a plate with two parts with different thicknesses such that forward-propagating symmetric mode is converted to backward-propagating symmetric mode at the selected frequency.

Jeong Ki Lee et al. showed that Lamb wave generated by a point source perpendicular to a side of an isotropic plate includes A_1 mode, an antisymmetric mode, dominantly. So, negative refraction produced by antysimmetric incident wave would enhance efficiency of experiment and practicality of backward propagating waves.

On a plate with appropriate thickness, a forward-propagating antisymmetric Lamb wave would convert into a backward-propagating symmetric Lamb wave, and the symmetric wave

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would focus due to Veselago lens effect.

In this study, we have observed conversion of antisymmetric Lamb wave into symmetric Lamb wave with negative refraction resulting in Velelago lens effect on a steel plate with a step change in thickness.

2. Theoretical Background

2.1 Dispersion Curve of Lamb Wave on Infinite Isotropic Plane

When a longitudinal wave propagating in a solid media enters aslant to an interface with fluid or vacuum, partial conversion into transverse wave occurs, resulting in the propagation of Lamb wave, a combination of longitudinal and transverse waves. Lamb waves can be classified into two types in which the vertical displacement is either symmetric or antisymmetric with respect to the center of the plate's thickness. Symmetric and antisymmetric Lamb waves are further divided into various modes depending on wavelengths and frequencies.

Applying appropriate boundary conditions to the wave equation of elastic waves on infinite isotropic plate, the dispersion equation of Lamb wave is:

$$\left(\frac{\tan\pi\sqrt{1-s^2}}{\tan\pi\sqrt{n^2-s^2}}\right)^{\pm 1} = -\frac{4s^2\sqrt{1-s^2}\sqrt{n^2-s^2}}{(1-2s^2)^2} \qquad (1)$$

where the exponent is +1 for symmetric modes and -1 for antisymmetric modes. $n=c_t/c_l$, $\nu=\omega h/\pi c_t$, and $s=c_t/c_x=c_t k/\omega$ in which h is the thickness of the plate, c_l is the speed of longitudinal waves, ct is the speed of transverse waves, c_x is phase velocity of Lamb wave, k is wavenumber, and ω is the angular frequency.

Fig. 1 shows some solutions of the Rayleigh-Lamb dispersion equation (Eq. (1)) for steel plate, where longitudinal speed is 5690 m/s and transverse speed is 3240 m/s. Since phase velocity is calculated ω/k and group velocity is

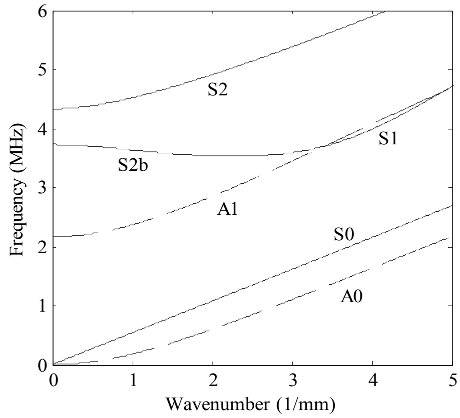

Fig. 1 Dispersion curve of some modes of Lamb wave on a steel plate 0.78 mm thick. Lamb wave propagates backwards in S_{2b} mode.

$d\omega/dk$, we can determine phase velocity from the slope of the line connecting the origin and the point on a dispersion curve, whereas group velocity is the slope of the tangent at that point from Fig. 1.

Using this method, we can see that for a part of S_1 mode, group velocity of Lamb wave has negative value. This mode is related to S_2 mode by an imaginary loop, so this is called S_{2b} mode[11]. In this mode, Lamb wave propagates backwards.

2.2 Negative Refraction and Veselago Lens

Refraction occurs when a wave enters a media with refractive index difference. Especially, if a wave enters a media with negative refractive index, negative refraction occurs, in which the incident wave and the refracted wave is on the same side of the normal line as in Fig. 2(a).

If the wavenumbers of incident and refracted waves are equal, incidence and refraction angles are equal by Snell's law. This is also true for negative refraction. Thus if the wavenumber does not change when negative refraction occurs over straight line, a wave generated from a

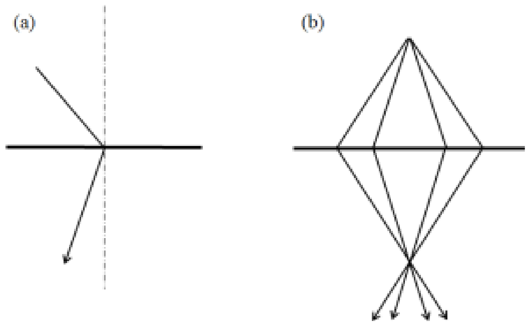

Fig. 2 (a) Incident and refracted waves are on the same side of the normal in negative refraction. (b) Negative refraction leads to Veselago lens effect, where waves refocus.

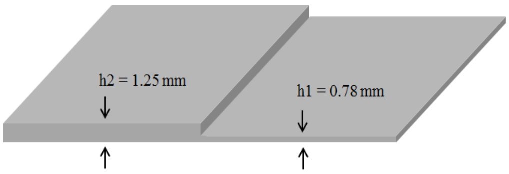

Fig. 3 Steel plate used in experiments, each part 5 centimeters long and wide.

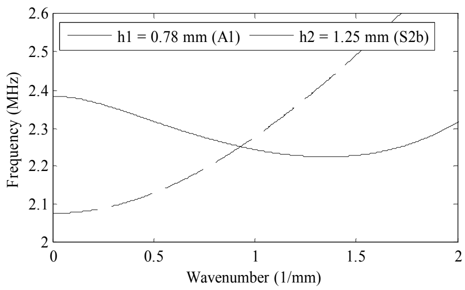

Fig. 4 Dispersion curve of parts overlapped, intersecting at 2.25 MHz frequency and 0.93 mm^{-1} wavenumber.

point source refocuses symmetrically on the other side of the interface as in Fig. 2(b). Since a wave is focused similarly to a lens, this effect is called Veselago lens effect after its proposer.

Frequency and wavenumber of Lamb wave depends on the thickness of a plate. Thus by changing the thickness, mode conversion can occur without change in frequency.

We tried to confirm negative refraction indirectly by observing Veselago lens effect while A_1 mode Lamb wave generated at 2.25 MHz is converted into S_{2b} mode. If the thickness of steel plate is h_1 = 0.78 mm on one side and h_2 = 1.25 mm on the other side (Fig. 3), A_1 mode and S_{2b} mode intersects at 2.25 MHz frequency and 0.93 mm^{-1} wavenumber(Fig. 4). Antisymmetrical interface as in Fig. 3 was intentionally designed to reduce reflection at the interface, since Lamb wave is converted from antisymmetric mode to symmetric mode.

3. Experiment

We manufactured a steel plate with dimensions 100×200×0.78/1.25. On the 0.78 mm thick side, we generated Lamb wave pulses with 2.25 MHz ultrasonic transducer (diameter 12.7 mm) connected to a pulser. To generate A_1 mode Lamb wave selectively using oblique incidence method, we put an acrylic wedge with 10.8° incidence angle between the transducer and the plate. On the 1.25 mm thick side, we obtained refracted signal with pin-type transducer held perpendicular to the plate by a specially designed equipment, and connected to a digital oscilloscope. During the experiment, the flat side of the steel plate was faced downwards.

To prevent changes in the contact state of the pin-type transducer, we moved the transmitter as well as the acrylic wedge instead of the receiver during each experiment set. In all cases, the transmitter and the receiver was located on the axis of symmetry of the plate. There were five experiment sets, in which the receiver was located 30, 35, 40, 45, and 50 mm from the interface. For each experiment set, we moved the transmitter 20 mm to 80 mm from the interface with 5 mm step and saved waveforms.

We tried to generate A_1 mode with oblique incidence method, but since conversion into modes other than S_{2b} can occur during the refraction. Thus we performed FFT (fast Fourier transform) on the saved waveform to select 2.25 MHz component only.

It is possible that waves reflected at the ends of the plate are detected. We have experimentally determined that first signal arrives 15 μs after the trigger and the reflected wave arrives 115 μs after the trigger at the receiver. We performed FFT for signal between 15 μs and 115 μs after the trigger, and analyzed the magnitude of the largest amplitude around 2.25 MHz.

4. Result and Discussion

Fig. 5(a) shows original waveforms received by the pin-type transducer 30 mm from the interface. Although we used oblique transmission method, Lamb wave dispersed into various modes as it passed through interface. Theoretical calculation indicates that S_{2b} mode wave arrives 60 μs after pulsation. In fact, we observe largest amplitude of Lamb wave at 60 μs. S_{2b} mode has slower group velocity compared to A_0, S_0, and A_1 modes, so they arrive faster, before 60 μs. In Fig 5(a), they arrive around 30 μs.

From Fig. 5(a), we see that Lamb waves including S_{2b} mode arrives later as the receiver moves farther away from the interface. However, Lamb wave received at 60 μs can include reflected modes other than S_{2b} mode, so we extracted 2.25 MHz component using FFT.

Intensity of a wave generated from a point source decreases inversely proportional to the distance from the source, so as the transmitter moves away from the interface, received Lamb wave must decrease in amplitude. However, if Veselago lens effect occurs due to negative refraction, the intensity of the signal should either increase or mildly decrease at a special configuration. This occurs when the transmitter and the receiver are located the same distance away from the interface due to the design of the plate.

In Fig. 5(b), we see that the intensity of the 2.25 MHz Lamb wave component is larger when the transmitter is 30 mm away compared to 15 mm away from the interface. For this case, we can confirm that as A_1 mode Lamb wave was converted into S_{2b} mode as it propagated into the part of the plate with differing thickness, negative refraction occurred, resulting in Veselago lens effect.

Fig. 6 shows amplitude of 2.25 MHz component of received Lamb wave at various configurations of transducers. The shapes of the curves are generally U-shaped, but there are bumps sticking up when the transmitter is 30-50 mm away from the interface, when the

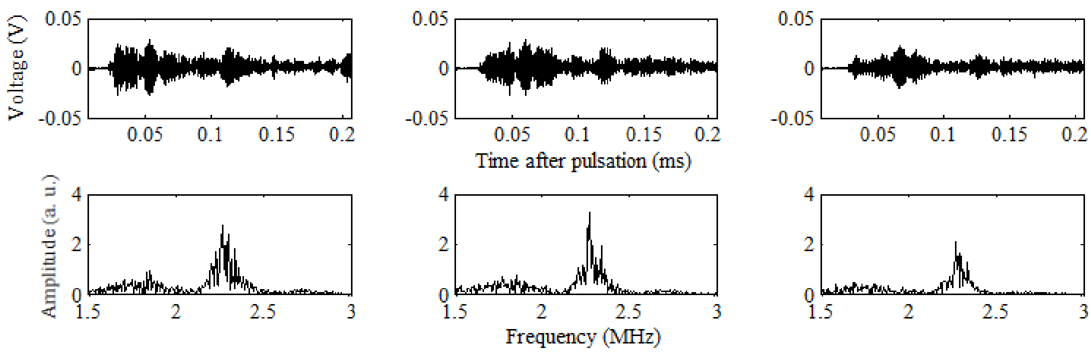

Fig. 5 (a) Original waveform of Lamb wave received. (b) FFT results over entire waveforms. The distance of transmitter from interface is 15 mm (left), 30 mm (center), 45 mm (right). The distance of receiver from interface is 3 cm for all three experiments.

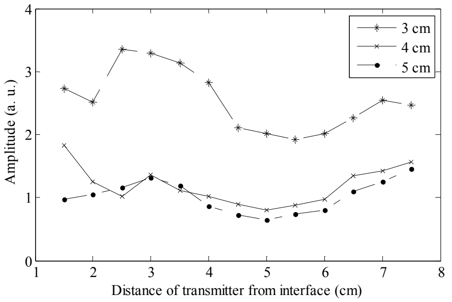

Fig. 6 Peak values of FFT results classified by distance of receiver from interface.

transmitter and the receiver are located symmetrically with respect to the interface of the plate. These bumps are evidences of negative refraction and Veselago lens effect.

As the transmitter moves away from the receiver, received signal should decrease, but Fig. 6 shows that the intensity of the signal increases as the transmitter reaches the end of the plate. This phenomena can be explained by constructive interference of the wave reflected right after the pulsation and the original wave.

The distance of the transmitter and the receiver does not agree exactly in Fig. 6 because the transmitter is not a point source and there may be error during plate manufacture. Moreover, measurement precision was quite large due to limits in the physical size of experimental equipment, so it was difficult to locate focusing location precisely. Nevertheless, we were able to observe qualitatively that negative refraction occurred along with mode conversion from antisymmetric to symmetric mode.

5. Conclusion

Since changing the thickness of a plate changes the frequency and the wavenumber of a Lamb wave mode, we calculated an appropriate thickness such that dispersion curves of A_1 and S_{2b} modes intersect at 2.25 MHz. Based on the calculation, we manufactured a steel plate with which we observed evidences negative refraction as the Lamb wave enters the part of the plate with different thickness. Negative refraction causes Veselago lens effect, which we observed by fixing the location of the receiving transducer and changing the location of the transmitting transducer and finding that there is a point where refracted wave focuses.

This result is an improvement from previous researches that we observed negative refraction occurs along with conversion from antisymmetric to symmetric mode. Antisymmetric mode is dominant when a guided wave is generated from an impact on one side of the plate, but it would be possible to replace a symmetric-mode-dominant Lamb wave generated from a point source by an antisymmetric-mode-dominant wave using Veselago lens effect that appears along negative refraction.

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