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<Original Research Paper>

A Repair-Time Limit Replacement Model with Imperfect Repair

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불완전 수리에서의 수리시간한계를 가진 교체모형

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This article concerns a profit model in a repair limit replacement problem with imperfect repair. If a system fails, we should decide whether we repair the failed system (repair option) or replace it by new one (replacement option with a lead time). We assume that repair times are random variables and can be estimated before repair with estimation error. If the estimated repair time is less than the specified limit (repair time limit), the failed unit is repaired but the unit after repair is different from the new one (imperfect repair). Otherwise, we order a new unit to replace the failed unit. The long run average profit (expected profit rate) is used as an optimization criterion and the optimal repair time limit maximizes the expected profit rate. Some special cases are derived.

Keywords: Imperfect Repair, Repair Time Limit, Replacement, Profit Rate, Lead Time

1. Introduction

For repairable systems, the maintenance plan during life cycle is important and affects the life cycle cost. Usually we can repair the failed system or sometimes replace it by new one. The maintenance engineer estimates repair cost (time) and if the repair cost (time) is relatively cheap (short), the failed system is repaired, otherwise, replaced by new one. A lot of papers deal with replacement problems based on repair limit. In the existing literature, the proposed models may be classified into two main types: repair-cost limit and repair-time limit models. In the repair-cost limit models, when a unit fails, the repair cost is estimated and repair is undertaken if the estimated cost is less than a pre-specified cost limit. Otherwise, the unit is replaced (Wang and Pham, 2005). In the repair-time limit models, a unit is repaired at failure: if the repair is completed within a pre-specified time, it is put into operation again. Otherwise, it is replaced by a new one and the new one is used.

This paper concerns a repair-time limit replacement model. The repair-limit replacement problems are considered by Drinkwater and Hastings (1967). Hastings (1968, 1969, 1970), Lambe (1974) and White (1989) formulate the repair limit replacement problems by applying the dynamic programming. Love *et al*. (1982) and Love and Guo (1996) consider repair limit problems in vehicle replacement cases. Nakagawa and Osaki (1974), Kaio and Osaki (1981, 1982), Muth (1977), and Nguyen and Murthy (1980, 1981) derive the optimal repair-time limits minimizing the expected cost rates with and without discounting.

Dohi *et al*. (1995, 1996, 1997, 1998, 2000a, 2000b, 2001a, 2001b, 2003a, 2003b, 2006, 2007, 2010) deal with cost and profit models in repair time limit problems with and without discounting. In particular, Dohi *et al*. (1998, 2006, 2010) consider stochastic profit models recently under earning rate

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criterions. Dohi *et al*. (1995, 1996, 1997, 1998, 2000a) introduce the concept of subjective repair-time distribution and considered graphical optimization problems to minimize the expected cost in estimation of the optimal decision. Dohi *et al*. (1996, 2000a, 2001a, 2001b, 2003b) also study imperfect repair models. For an excellent survey of repair-time limit replacement problems, see Dohi and Kaio (2005). In the most models related to repair-time limit policies, the exact repair times are not known before completing the repair. Kim and Yun (2010) consider a repair time limit model with estimation error.

In this paper, a profit model in repair time limit problem with imperfect repair is studied. If the estimated repair time is less than the repair time limit, the failed unit is repaired but the unit after repair is different from a new one (imperfect repair). Otherwise, we order a new unit to replace the failed unit. The long run average profit (expected profit rate) is obtained and the optimal repair time limit maximizes the expected profit rate. Two special cases about repair time information are derived.

Notation

t_0 : repair time limit
$F(t),\ \mu_f$: cdf and mean life of new units
$F_{ir}(t),\ \mu_{ir}$: cdf and mean life of units after imperfect repair
$G(t),\ g(t),\ r(t)$: cdf, pdf and failure rate function of repair time
k : penalty cost per unit time when the production machine is in down state
e_0 : earning rate per unit operation time
e_r : repair cost per unit time
c : fixed cost associated with the ordering of a new unit
L : lead time for delivery of a new unit
$\overline{\phi}(\cdot) : 1-\phi(\cdot)$

2. A Repair Time Limit Replacement Model

In this section, we consider a repair time limit replacement model with imperfect repair. Consider a simple system that is repairable and can be replaced by a new unit. Once the machine is failed, the maintenance engineers wish to decide whether to repair it or order a new unit for replacement. Basically, the repair times can be considered as a random variable and it is assumed that the unit after repair is different from the new one. The mean failure times after repair and replacement are μ_{ir} and μ_f respectively. If the new one unit is ordered, then it is delivered after the lead time L. it is assumed that the replacement time is negligible. In the repair time limit replacement model, we decide to repair or replace the unit (ordering a new unit) based on the estimated repair time.

We assume that we can estimate the actual repair time T_a before repair but there is the estimation error in estimation. Once the machine is failed, we estimate the repair time. If the estimated repair time is greater than a pre-specified limit t_0, then we order a new unit. After the new unit is delivered after the lead time $\boldsymbol{L}$, the failed unit is replaced by the new one and it is operated, where the replacement time can be negligible. Otherwise we repair the failed part. In case that the actual repair time is less than t_0 but the repair time is estimated greater than t_0, a replacement is carried out. In the reverse case, a repair is performed over t_0. It is assumed that the estimated repair time, $\boldsymbol{T_e}$ is a function of the actual repair time and estimation error, i.e. $\boldsymbol{T_e = T_a + \varepsilon}$. Given the actual repair time $\boldsymbol{T_a = t}$, the estimated repair time is $\boldsymbol{T_e | t}$, whose conditional probability density function is denoted as $\boldsymbol{h(u|t)}$.

In this paper, we consider the decision point of repair and replacement as the starting point of a cycle and the time to failure of units as the ending point of a cycle. We consider earning rate for operation period, ordering cost, repair cost and penalty cost for downtime. The length and profit of a cycle depend on our repair/replacement decision based on the estimated repair time.

To obtain the expected duration and profit of a renewal cycle, we first consider the conditional expected duration and profit, and then obtain the unconditional ones. If the actual repair time is given as $\boldsymbol{t}$, then the estimated value of repair time for a given actual repair time is a random variable with conditional distribution function, $H(y|t) = \int_0^y h(u|t)du = P(T_e \le y \,|\, T_a = t)$. Since the decision point about repair and replacement is regarded as a renewal point, the expected duration of a renewal cycle for a given actual repair time is given by

$$(t+\mu_{ir})H(t_0|t) + (L+\mu_f)\overline{H}(t_0|t)$$

Also, the expected profit of a renewal cycle for a given actual repair time becomes

$$[e_0\mu_{ir} - (e_r+k)t]H(t_0|t) + [e_0\mu_f - (kL+c)]\overline{H}(t_0|t)$$

Thus, the unconditioned expected duration and profit of a renewal cycle, which are denoted by $T_{L1}(t_0)$ and $V_{L1}(t_0)$, respectively, can be written as

$$\begin{aligned} T_{L1}(t_0) &= \int_0^\infty \big[(t+\mu_{ir})H(t_0|t) + (L+\mu_f)\overline{H}(t_0|t)\big]dG(t) \qquad (1) \\ &= \mu_f + L + \int_0^\infty (t+\mu_{ir} - L - \mu_f)H(t_0|t)dG(t) \\ V_{L1}(t_0) &= \int_0^\infty \big[[e_0\mu_{ir} - (e_r+k)t]H(t_0|t) + [e_0\mu_f - (kL+c)]\overline{H}(t_0|t)\big]dG(t) \end{aligned}$$

$$= (e_0\mu_f - kL - c) + \int_0^{\infty} [e_0\mu_{ir} - e_0\mu_f - (e_r+k)t + (kL+c)]H(t_0|t)dG(t)$$

From the renewal reward argument, the expected profit rate under imperfect repair which is denoted by $TP_{p1}(t_0)$ can be written as $\boldsymbol{TP_{p1}(t_0) = V_{L1}(t_0)/T_{L1}(t_0)}$. The optimal repair-time limit, t_{01}^* is the value maximizing the expected profit rate and the optimal profit is given by

$$TP_{p1}(t_{01}^*) = \max_{0 \le t_0 < \infty} TP_{p1}(t_0) = \frac{V_{L1}(t_0)}{T_{L1}(t_0)} \quad (2)$$

In general case, it is difficult to obtain analytically the optimal solution of the Equation (2). We consider a special case where the expected profit rate over an infinite time horizon can be derived as a closed form. The estimated repair time, $\boldsymbol{T_e}$ is a function of the actual repair time and estimation error, i.e. $\boldsymbol{T_e = T_a + \varepsilon}$ where the estimation error is assumed to be an independent and normally distributed random variable with mean 0 and variance $\boldsymbol{\sigma_e^2}$. We also assume that given the actual repair time $\boldsymbol{T_a = t}$, the estimated repair time $\boldsymbol{T_e|t}$, has an independent and identical normal distribution with mean $\boldsymbol{t}$ and variance $\boldsymbol{\sigma_e^2}$, i.e. $\boldsymbol{T_e|t \sim N(t, \sigma_e^2)}$. Secondly, the actual repair time, $\boldsymbol{T_a}$ is assumed to follow a normal distribution with mean, $\boldsymbol{\mu_a}$ and variance, $\boldsymbol{\sigma_a^2}$. Generally, normal distribution has the following results (Kim and Yun, 2010);

1) $\int_{-\infty}^{\infty} H(t_0|t)dG(t) = \Phi\left(\frac{t_0-\mu_a}{\sqrt{\sigma_a^2+\sigma_e^2}}\right)$

2) $\int_{-\infty}^{\infty} tH(t_0|t)dG(t) = \mu_a\Phi\left(\frac{t_0-\mu_a}{\sqrt{\sigma_a^2+\sigma_e^2}}\right) - \frac{\sigma_a^2}{\sqrt{\sigma_a^2+\sigma_e^2}}\phi\left(\frac{t_0-\mu_a}{\sqrt{\sigma_a^2+\sigma_e^2}}\right)$

Using the previous results, the expected duration and profit of a renewal cycle are given by

$$\begin{aligned} T_{L1}(t_0) &= \mu_f + L + (\mu_a + \mu_{ir} - L - \mu_f) \\ &\quad \times \Phi\left(\frac{t_0-\mu_a}{\sqrt{\sigma_a^2+\sigma_e^2}}\right) - \frac{\sigma_a^2}{\sqrt{\sigma_a^2+\sigma_e^2}}\phi\left(\frac{t_0-\mu_a}{\sqrt{\sigma_a^2+\sigma_e^2}}\right) \\ V_{L1}(t_0) &= (e_0\mu_f - kL - c) \\ &\quad + (e_0\mu_{ir} - e_0\mu_f + kL + c - (e_r+k)\mu_a) \\ &\quad \times \Phi\left(\frac{t_0-\mu_a}{\sqrt{\sigma_a^2+\sigma_e^2}}\right) + \frac{(e_r+k)\sigma_a^2}{\sqrt{\sigma_a^2+\sigma_e^2}}\phi\left(\frac{t_0-\mu_a}{\sqrt{\sigma_a^2+\sigma_e^2}}\right) \end{aligned} \quad (3)$$

For normally distributed repair times, the closed form of the expected profit rate is obtained. It is difficult to obtain the optimal repair time limit to maximize the expected profit rate analytically but we can find the approximate optimal solutions numerically from Equation (3).

3. Special Cases

In this section, we consider two special cases in the repair time limit model with imperfect repair and all model assumptions are same as in section 2.

Firstly, we assume that we have no information about repair time at decision points for repair and replacement but know only general information (distribution of repair time). Thus, it is impossible to decide to repair or replace the failed unit based on the estimated repair time. In this model, we start to repair the unit immediately when it is failed. If the repair is completed up to the time limit t_0, then we operate the unit again. Otherwise, we scrapped the unit and order a new unit, and the new unit is delivered after the lead time L.

In this model, the total expected profit for a cycle consists of the expected repair cost and total earning amount. The expected cost is same as

$$\begin{aligned} &(k+e_r)\int_0^{t_0} t\,dG(t) + (c + kL + (k+e_r)t_0)\overline{G}(t_0) \\ &= (k+e_r)\int_0^{t_0} \overline{G}(t) + (c+kL)\overline{G}(t_0) \end{aligned}$$

The total earning amount for a cycle is $e_0(\mu_{ir}G(t_0) + \mu_f\overline{G}(t_0))$. The expected duration of one cycle is same as

$$\mu_{ir} + \int_0^{t_0} \overline{G}(t)dt + (L + \mu_f - \mu_{ir})\overline{G}(t_0).$$

The expected profit rate under no information which is denoted by $TP_{p2}(t_0)$ can be written as

$$TP_{p2}(t_0) = \frac{e_0\mu_{ir} - (k+e_r)\int_0^{t_0}\overline{G}(t)dt + (e_0(\mu_f - \mu_{ir}) - (c+kL))\overline{G}(t_0)}{\mu_{ir} + \int_0^{t_0}\overline{G}(t)dt + (L+\mu_f-\mu_{ir})\overline{G}(t_0)} \quad (4)$$

- **Theorem 1 :**

(1) If the repair-time distribution $G(t)$ is IHR (increasing hazard rate), the optimal repair time limit is 0 (always repair case) or infinite (always ordering case).

(2) If the repair time distribution $G(t)$ is strictly DHR (decreasing hazard rate), there exists a finite and unique optimal repair time limit $t_{02}^*(0 < t_{02}^* < \infty)$ under some conditions (Kaio and Dohi, 2005) and the corresponding minimum expected profit rate is given by

$$TP_{p2}(t_{02}^*) = \frac{-(e_r+k) - (e_0(\mu_f-\mu_{ir}) - (kL+c))r(t_{02}^*)}{1-(L+\mu_f-\mu_{ir})r(t_{02}^*)} \quad (5)$$

Secondly, we assume that we can estimate the repair time perfectly. When the system is failed, the repair time can be estimated and known. This special case is equal to the model in Dohi *et al*. (2006). If the estimated repair time is less than t_0, we start to repair the failed system. Otherwise, we order a new unit and replace the system after the unit is delivered after a lead time. Then, in this model, the total profit for a cycle consists of the expected repair cost and total earning amount. The expected cost is same as

$$(e_r+k)\int_0^{t_0} tdG(t)+(c+kL)\overline{G}(t_0)$$

The total earning amount for a cycle is $e_0(\mu_{ir}G(t_0)+\mu_f \overline{G}(t_0))$.

The expected duration of one cycle is same as

$$\mu_{ir}+\int_0^{t_0} tdG(t)+(L+\mu_f-\mu_{ir})\overline{G}(t_0).$$

The expected profit rate under perfectly estimated repair time which is denoted by $TP_{p3}(t_0)$ can be written as

$$TP_{p3}(t_0)=\frac{e_0\mu_{ir}-(k+e_r)\int_0^{t_0} tdG(t)+(e_0(\mu_f-\mu_{ir})-(c+kL))\overline{G}(t_0)}{\mu_{ir}+\int_0^{t_0} tdG(t)+(L+\mu_f-\mu_{ir})\overline{G}(t_0)} \qquad (6)$$

- **Theorem 2 :**

(1) If the repair-time distribution $G(t)$ is IHR (increasing hazard rate), the optimal repair time limit is 0 (always repair case) or infinite (always ordering case).

(2) If the repair time distribution $G(t)$ is strictly DHR (decreasing hazard rate), there exists a finite and unique optimal repair time limit $t_{03}^*(0<t_{03}^*<\infty)$ under some conditions (Kaio and Dohi, 2005) and the corresponding minimum expected profit rate is given by

$$TP_{p3}(t_{03}^*)=\frac{(e_0(\mu_f-\mu_{ir})-(kL+c))+(e_r+k)t_{03}^*}{(L+\mu_r-\mu_{ir})-t_{03}^*} \qquad (7)$$

We derived the expected profit rates of two special cases. The difference between three cases is the information amount about repair times at failures. Thus, we can evaluate the value of information from three cases, for example, the expected value of perfect information per unit time (EVPI). From Equation (4) and Equation (6), we find the optimal repair time limits and EVPI is given by

$$\begin{aligned} EVPI &= TC_{p3}(t_{03}^*)-TC_{p2}(t_{02}^*) \qquad (8)\\ &= \frac{(e_0(\mu_f-\mu_{ir})-(kL+c))+(e_r+k)t_{03}^*}{(L+\mu_r-\mu_{ir})-t_{03}^*} \\ &\quad - \frac{-(e_r+k)-(e_0(\mu_f-\mu_{ir})-(kL+c))r(t_{02}^*)}{1-(L+\mu_f-\mu_{ir})r(t_{02}^*)} \end{aligned}$$

Thus, the cost to get the perfect information should be less than EVPI to use the perfect information. Using similar way, we can evaluate the value of partial information.

4. Illustrative Examples

In this section, we present numerical examples to illustrate the analysis and obtain the optimal repair time limits. Suppose that the actual repair time distribution follows a normal distribution with mean $\mu_a=5$ and standard deviation $\sigma_a=1$. The standard values of other cost and model parameters are given as $\mu_f=100$, $\mu_{ir}=80$, $L=5$, $e_0=2$, $e_r=1$, $c=3$, and $k=2$. We consider some cases with different values of σ_e, k, and find some trends of optimal repair time limits.

Using Equation (3)~Equation (6), we calculate the expected duration, profit of a renewal cycle and the long run

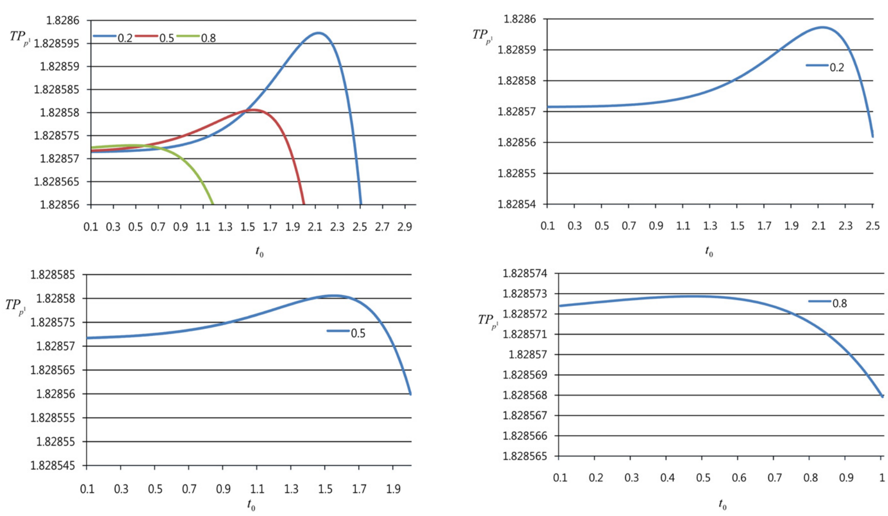

Figure 1. Long-run average profit rates per unit time under various values of σ_e

average profit per unit time. We used Matlab 7.0 to calculate these equations and find the optimal t_0 to maximize the long run average profit rate. <Figure 1> shows the long run average profit rates with different values of σ_e and k = 2. From <Figure 1>, we can know that the long run average profit rate and the optimal repair time limit decrease as the variance of estimation error increases. It means that if it is not easy to estimate the repair time precisely, we prefer ordering the new one to repairing.

To investigate the effect of penalty cost of machine breakdown and the degree of imperfect repair, we consider some cases with various values of the degree of imperfect information ($\sigma_e = 0, 0.2, 0.5, 0.8$). <Figure 2> and <Figure 3> show that the optimal repair time limit increases if the penalty cost k and the mean life after repair increase. It implies that the repair is preferred to replacement as the penalty cost becomes relatively large or the mean life after repair is long. In addition, we can know from <Figure 4> and <Figure 5> that the optimal repair time limit increases as the lead time increases. In particular, the difference between the optimal repair time limits decreases as the lead time increases and the optimal repair time limit eventually converge to the same value.

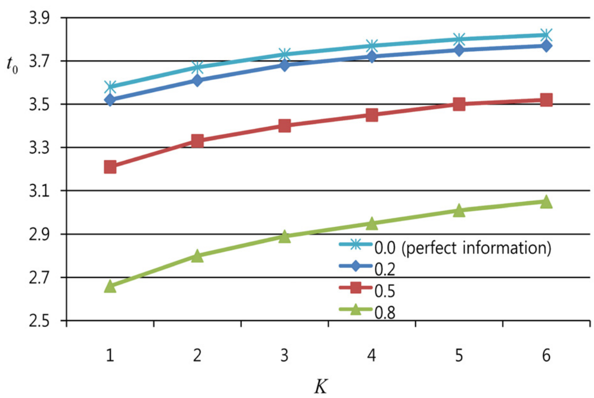

Figure 2. Optimal t_0 under various values of σ_e and K

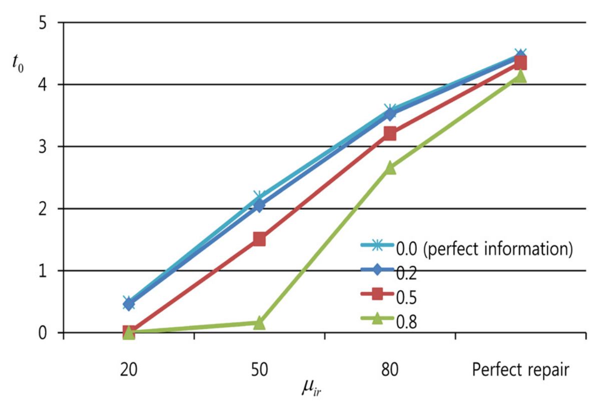

Figure 3. Optimal t_0 under various values of σ_e and μ_{ir}

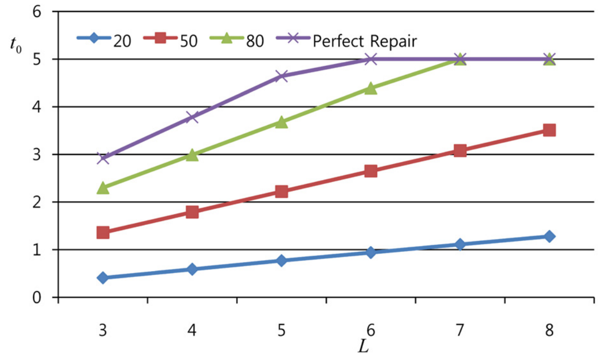

Figure 4. Optimal t_0 under various values of L and μ_{ir}

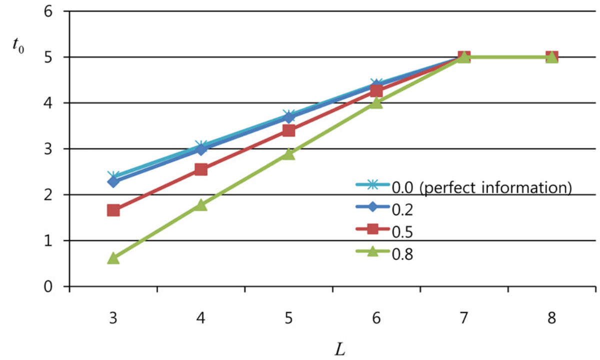

Figure 5. Optimal t_0 under various values of σ_e and L

5. Conclusions

In this paper, we considered a repair-time limit replacement model with imperfect repair and estimation error. The repair time is assumed to be a random variable but the replacement time with a new unit is negligible. In order to replace the failed unit, the new unit should be ordered and is delivered after a lead time L. If a system fails, we should decide to start to repair or to order a new unit. Based on the available information for repair time, we can choose one among repair and replacement options and we estimate the repair time to recover the system failure. If the estimated repair time is less than the pre-specified limit, then we start to repair the failed system. Otherwise we order the new item and finally replace the system. We studied a profit model with partial information about repair time and imperfect repair. The expected profit rate was derived and some special cases have been studied.

For further studies, repair time limit models with preventive maintenance, repair time limit models with finite time horizon, and repair time limit optimization problems with finite or random time horizon (Nakagawa and Mizutani, 2009) may be promising topics.

References

Dohi, T. and Kaio, N. (2005), *Repair-limit replacement program in industrial maintenance-renewal reward process modeling*, Applied Economic informatics and Systems Sciences edited by T. Tokimasa, S. Hiraki and N. Kaio, 157-172, Kyushu University Press, Fukuoka.

Dohi, T. Aoki, T. Kaio, N., and Osaki, S. (1998), Non-parametric preventive maintenance optimization models under earning rate criteria, *IIE Trans*., **30**, 1099-1108.

Dohi, T. Aoki, T. Kaio, N., and Osaki, S. (2001), Optimization the repair time limit replacement schedule with discounting and imperfect repair, *Journal of Quality in Maintenance Engineering*, **7**, 71-84.

Dohi, T. Aoki, T. Kaio, N., and Osaki, S. (2003), The optimal repair-time limit replacement policy with imperfect repair : Lorenz transform approach, *Mathematical and Computer Modellin*, **38**, 1169-1176.

Dohi, T. Aoki, T. Kaio, N., and Osaki, S. (2006), Statistical estimation algorithms for some repair-limit replacement scheduling problems under earning rate criteria, *Computers and Mathematics with Applications*, **51**, 345-356.

Dohi, T. Kaio, N., and Osaki, S. (2007), *Stochastic profit models under repair-limit replacement program*, Proceedings of International Workshop on Recent Advances in Stochastic Operations Research II, 27-34.

Dohi, T. Kaio, N., and Osaki, S. (2010), *A stochastic profit model under repair-limit replacement program with imperfect repair*, Proceedings of 4th Asia-Pacific International Symposium (APARM 2010), 153-160.

Dohi, T., Kaio, N., and Osaki, S. (1995), Solution procedure for a repair limit problem using the TTT concept, *IMA Journal of Mathematics Applied in Business and Industry*, **6**, 101-111.

Dohi, T., Kaio, N., and Osaki, S. (2000), A graphical method to repair cost limit replacement policies with imperfect repair, *Mathematical and Computer Modelling*, **31**, 99-106.

Dohi, T., Kaio, N., and Osaki, S. (2001), Determination of optimal repair-cost limit on the Lorenz curve, *Journal of the Operations Research Society of Japan*, 44, 207-219.

Dohi, T., Kaio, N., and Osaki, S. (2003), A new graphical method to estimate the optimal repair-time limit with incomplete repair and discounting, *Computers and Mathematics with Applications*, **46**, 999-1007.

Dohi, T., Koshimae, H., Kaio, N., and Osaki, S. (1997), Geometrical interpretations of repair cost limit replacement policies, *International Journal of Reliability*, *Quality and Safety Engineering*, **4**, 309-333.

Dohi, T., Matsushima, N., Kaio, N., and Osaki, S. (1996), Nonparametric repair limit replacement policies with imperfect repair, *European Journal of Operational Research*, **96**, 260-273.

Dohi, T., Takeita, K., and Osaki, S. (2000), Graphical methods for determining/estimating optimal repair-limit replacement policies, *International Journal of Reliability, Quality and Safety Engineering*, **7**, 43-60.

Drinkwater, R. W. and Hastings, N. A. J. (1967), An economical replacement model, *Operational Research Quarterly*, **18**, 121-138.

Hastings, N. A. J. (1968), Some notes on dynamic programming and replacement, *Operational Research Quarterly*, **19**, 453-464.

Hastings, N. A. J. (1969), The repair limit replacement method, *Operational Research Quarterly*, **20**, 337-349.

Hastings, N. A. J. (1970), Equipment replacement and the repair limit method, In *Operational Research in Maintenance*, edited by Jardine, A. K. S., 100-118, Manchester University Press, Barnes and Noble Inc., New York.

Kaio, N. and Osaki, S. (1981), Optimum repair limit policies with cost constraint, *Microelectronics and Reliability*, **21**, 597-599.

Kaio, N. and Osaki, S. (1982), Optimum repair limit policies with a time constraint. *International Journal of Systems Science*, **13**, 1345-1350.

Kaio, N., Dohi, T. and Osaki, S. (2002), Classical replacement models, In *Stochastic Models in Reliability and Maintenance*, edited by Osaki, S., 65-87, Springer-Verlag, Berlin.

Kim, H. G. and Yun, W. Y. (2010), A repair time limit replacement policy with estimation error, *Communications in Statistics-Theory and Methods*, **39**, 1-14.

Koshimae, H., Dohi, T., Kaio, N. and Osaki, S. (1996), Graphical/statistical approach to repair limit replacement problem, *Journal of the Operations Research Society of Japan*, **39**, 230-246.

Lambe, T. A. (1974), The decision to repair or scrap a machine, *Operational Research Quarterly*, **25**, 99-110.

Love, C. E., Rodger, R., and Blazenko, G. (1982), Repair limit policies for vehicle replacement, *INFOR*, **20**, 226-237.

Love, C. E. and Guo, R. (1996), Utilizing Weibull failure rates in repair limit analysis for equipment replacement/preventive maintenance decisions, *Journal of Operational Research Society*, **47**, 1366-1376.

Muth, E. J. (1977), An optimal decision rule for repair vs replacement, *IEEE Transactions on Reliability*, **R-26**, 179-181.

Nakagawa, T. (2006), Maintenance theory of reliability, Springer.

Nakagawa, T. and Mizutani, S. (2009), A summary of maintenance policies for a finite interval, *Reliability Engineering and System Safety*, **94**, 89-96.

Nakagawa, T. and Osaki, S. (1974), The optimum repair limit replacement policies, *Operational Research Quarterly*, **25**, 311-317.

Nguyen, D. G. and Murthy, D. N. P. (1980), A note on the repair limit replacement policy. *Journal of Operational Research Society*, **31**, 1103-1104.

Nguyen, D. G. and Murthy, D. N. P. (1981), Optimal repair limit replacement policies with imperfect repair, *Journal of Operational Research Society*, **32**, 409-416.

Wang, H. and Pham, H. (2005), *Reliability and optimal maintenance*, Springer-Verlag, Lodon.

White, D. J. (1989), Repair limit replacement, *OR Spectrum*, **11**, 143-149.