논문 2013-50-11-21

스케일 텔레 로보틱스 시스템에 적용된 Routh-Hurwitz와 절대 안정도 기준의 비교

(Comparison of Routh-Hurwitz and Absolute Stability Criteria in Application to Scaled Telerobotics Systems)

이고르 가포노프*, 조 현 찬**, 전 홍 태***

(Igor Gaponov, Hyun Chan Cho^{\odot} , and Hong-Tae Jeon)

요 약

본 논문에서는 스케일 텔레 로보틱스 시스템에 Routh-Hurwitz와 Llewellyn의 절대 안정도 기준의 적용에 관한 비교 연구 결과를 보인다. 텔레 로보틱스 시스템의 동적 방정식이 주어지고, 시스템의 전달함수는 더 나은 안정도 해석을 얻게 된다. 제 어기 이득의 안정적인 마진은 두 가지의 안정도 해석 방법을 통하여 얻을 수 있으며, 본 논문에서 그 결과의 차이를 밝히고 설명했다. 본 논문은 안정도 분석결과를 수치 예제를 통해 증명하는 것으로 결론을 도출한다.

Abstract

This paper presents a comparative study on application of Routh-Hurwitz and Llewellyn absolute stability criteria to a scaled telerobotic system. The dynamic equations of the telerobotic system are given, and the transfer function of the system is obtained for further stability analysis. The stable margins of controller gains are obtained using both stability analysis methods, and the differences in the results are described and explained. The paper is concluded by a numerical example verifying performed stability analysis.

Keywords: telerobotic system, stability, Routh-Hurwitz criterion, Llewellyn absolute stability criterion.

I. Introduction

Manipulation of micro- and nanoscale objects is required in many applications nowadays, like intracytoplasmic sperm and DNA injection in living cells, molecular docking, minimally invasive and micro-surgery, and many others. The main reason of the emerging role of micro-telerobotic systems is that they provide the human operator with the feeling of interaction (haptic force) with the micro-environment, while granting the operator an opportunity to be at a remote site while performing required manipulations.

Stability is the fundamental requirement of every control system. In addition, a telerobotic system must provide the operator with an accurate feeling of the environment (be transparent), the condition that usually interferes with stability. To this day, many

^{*} 정회원, 한국기술교육대학교 기계공학부

⁽School of Mechanical Engineering, KoreaTech) ** 정회원, 한국기술교육대학교 전기전자통신공학부 (School of Electrical, Electronics and

Communication Engineering, KoreaTech) *** 정회원, 중앙대학교 전자전기공학부

⁽Dept. of Electrical and Electronics Engineering, Chung-Ang University)

[©]Corresponding Author(E-mail: cholab@koreatech.ac.kr)

접수일자 : 2013년7월25일, 수정완료일 : 2013년10월26일

bilateral control architectures have been developed and applied to telerobotic systems^[1-3]. However, there is still need in guidelines for engineers who perform stability analysis of the scaled telerobotic systems due to the complexity of their structure and uncertainties in human hand and environment parameters.

This paper presents a comparative study of application of Routh-Hurwitz and Llewellyn absolute stability analysis. In Section II, the architecture of the proposed control system is described, and the assumptions for further stability analysis are given. In Section III, a comparative investigation of system stability using Routh-Hurwitz and Llewellyn absolute stability criteria is given and verified by a numerical simulation. The paper is concluded by Section IV.

II. Control System Architecture

A general 6-channel architecture of a telerobotic control system is presented in Fig. 1. Proposed by Hannaford^[4], this architecture was originally 4-channelled (signals between the master and the slave manipulators flow through four controllers (C_1 ,



그림 1. 일반적 6-채널 스케일 텔레 로봇 시스템의 제 어구조

Fig. 1. General 6-channel scaled telerobotic system control architecture.

 C_2 , C_3 , C_4) and was later extended by Hastrudi–Zaad and Salcudean^[5] by adding master and slave local force feedback loops (controllers C_5 and C_6). It should be also noted that the signals exchanged between master and slave sides are scaled by respective scaling coefficients(α and β or α^{-1} and β^{-1}), which

표 1. 안정도 분석에 사용된 변수 리스트 Table 1. List of variables used in stability analysis.

Variable	Description		
x_m	Master device position		
x_s	Slave device position		
$\dot{x_m} = v_m$	Master device velocity		
$\dot{x_s} = v_s$	Slave device velocity		
${f}_m$	Master device force		
f_s	Slave device force		
f_h^*	Exogenous force of operator		

표 2. General 6-channel 텔레 로보틱스 구조의 전달 함수

Variable	Description		
Z_m	Master device impedance		
Z_s	Slave device impedance		
Z_e	Impedance of environment		
Z_h	Impedance of human operator		
C_m	Master device controller		
C_{s}	Slave device controller		
C_1	Master position feedforward control		
C_2	Slave force feedforward control		
C_3	Master force feedforward control		
C_4	Slave position feedforward control		
C_5	Slave local force feedforward		
C_6	Master local force feedforward		
α	Position scaling coefficient		
β	Force scaling coefficient		

Table 2 Transfer functions of a general 6-channelled Telerobotic architecture.

표 3. 전달함수 내용 Table 3. Descriptions of some transfer functions.

Transfer Function	Form
Z_m	$m_m s + b_m + k_m/s$
Z_s	$m_s s + b_s + k_s/s$
Z_e	$m_es + b_e + k_e/s$
Z_h	$m_h s + b_h + k_h/s$
C_m	$k_{mv} + k_{mp}/s$
C_s	$k_{sv} + k_{sp}/s$

was beyond the scope of the above mentioned works.

In Table 3, parameters m, b, k denote mass, viscosity, and stiffness coefficients of the respective transfer functions, k_{mv} and k_{mp} denote the velocity and position gains of master PD-controller, and k_{sv} and k_{sp} stand for the velocity and position gains of slave device PD-controller.

In general, the impedance of the operator Z_h varies from person to person, and the environment, especially micro-environment, is hard to model exactly in most cases. However, since both operator and environment are passive systems, it is enough to analyze the stability of the two-port network model of the teleoperator only, which includes master device, slave device, and the communication block in Fig. 1.

The hybrid matrix is defined as^{$[4 \sim 5]}$ </sup>

$$\begin{bmatrix} f_m \\ \vdots \\ -x_s \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \vdots \\ x_m \\ f_s \end{bmatrix}$$

It can be shown that the elements of the hybrid matrix h_{ij} , where i, j = 1, 2, can be found to be^[5]

$$h_{11} = \frac{Z_{c+s}Z_{c+m} + C_1C_4}{(1+C_6)Z_{c+s} - \alpha^{-1}\beta^{-1}C_3C_4}$$
(1)

$$h_{12} = \frac{\beta C_2 Z_{c+s} - \alpha^{-1} C_4 (1 + C_5)}{(1 + C_6) Z_{c+s} - \alpha^{-1} \beta^{-1} C_3 C_4}$$
(2)

$$h_{21} = \frac{-\beta C_3 Z_{c+m} - \alpha C_1 (1 + C_6)}{(1 + C_6) Z_{c+s} - \alpha^{-1} \beta^{-1} C_3 C_4}$$
(3)

$$h_{22} = \frac{(1+C_5)(1+C_6) - C_2 C_3}{(1+C_6)Z_{c+s} - \alpha^{-1}\beta^{-1}C_3 C_4}$$
(4)

where $Z_{c+m} = C_m + Z_m, Z_{c+s} = C_s + Z_s..$

Adopting force-position (force reflecting) architecture yields $C_3 = C_4 = C_5 = C_6 = 0$. Choosing the remaining controllers to be $C_1 = C_{1v} + C_{1p}/s$ and $C_2 = 1$, the transfer functions F_h/F_e and F^*_h/F_e can be obtained as follows:

$$\frac{F_h}{F_e} = \frac{1}{Z_{cm} + Z_h} \bullet \frac{C_1 C_2 Z_e}{Z_{cs} + Z_e} \equiv Z_{he}$$

$$\frac{F_h^*}{F_e} = \frac{Z_{he}}{1 + Z_{he}}$$
(5)

The characteristic equation of the transfer function (5) can be expressed in the following form:

$$(Z_{cm} + Z_h) \bullet (Z_{cs} + Z_e) + C_1 C_2 Z_e = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0$$
 (6)

The following assumptions are made in order to simplify the further analysis:

$$k_{mv}, k_{sv} \gg b_m, b_h, b_e, b_s$$

 $k_{mv}, k_{sv} \gg m_m, m_h, m_e, m_s$
 $k_{mp} \gg b_m, b_h, b_e$
 $k_{sp} \gg k_e$
 $m_e \ll m_h, m_m, m_s$

Choosing the controllers C_1 and C_2 to be $C_{1p} = k_e$ + k_{sp} , $C_2 = (\alpha\beta)^{-1}$, the coefficients of the polynomial described by (6) can be defined as follows:

$$a_0 = k_{mp} \bullet k_{sp} \tag{7}$$

$$a_1 = k_{mv} \bullet k_{sp} + k_{sv} \bullet k_{mp} \tag{8}$$

$$a_2 = k_{mv} \bullet k_{sv} \tag{9}$$

$$a_3 = (m_m + m_h) \bullet k_{sv} + m_s \bullet k_{mv}$$
(10)

$$a_4 = (m_m + m_h) \bullet m_s \tag{11}$$

III. Stability Analysis

Stability is a critical issue in telerobotics, especially in the presence of time delays. The following techniques are most commonly used to estimate the stability of telerobotic systems:

- Routh-Hurwitz criterion
- Lyapunov functions-based approach
- · Nyquist criterion
- Llewellyn's(absolute) stability criterion
- · Passivity analysis

the above mentioned methods All of are well-known in control theory and have their flaws and benefits. For instance, the Routh-Hurwitz criterion is the simplest and the least computational-demanding technique, however, it does not reflect the nonlinearities existing in the system. Lyapunov stability approach guarantees the asymptotical stability of the system; however, its applications are restricted due to the difficulties in finding satisfactory Lyapunov function candidates for every particular system. Passivity theory is relatively simple in implementation, but provides conservative. conditions^[6]. Absolute stability criterion (Llewellyn's criterion) is a less conservative condition compared to passivity, and has gained more popularity among control engineers in recent years.

In this research, two stability criteria, namely the Routh-Hurwitz criterion and Llewellyn's stability criterion, are applied to analyze the stability of a general scaled telerobotic system. The qualitative comparison between those two criteria is performed, restrictions and their are discussed. The Routh-Hurwitz criterion was chosen due to its simplicity, and the absolute stability criterion was selected as one of the most advanced stability analysis methods that can provide stable margins for controller gains while taking into the account the frequency of the input signal.

1. Routh-Hurwitz Criterion

For a fourth-order polynomial, the Routh-Hurwitz criterion is formulated as follows:

- $a_n > 0$;
- $a_3a_2 > a_4a_1;$
- $a_3a_2a_1 > a_4a_1^2 + a_3^2a_0$.

In order for the first condition to hold, it is necessary that all the parameters used in the mathematical model presented in Section II, namely controllers' gains and physical properties of the human operator, master and slave devices, and environment, are positive. It is assumed that only the positive values of the above mentioned parameters are used in the further analysis.

The second condition of the Routh-Hurtwitz criterion $(a_3a_2 > a_4a_1)$ provides the following stable margins for the regulator gains:

$$k_{mp} < \frac{1}{m_s} \bullet k_{sv} \bullet k_{mv} - \frac{1}{k_{sv}} \bullet k_{mv} \bullet k_{sp} + \frac{1}{m_m + m_h} \bullet k_{mv}^2 , \qquad (12)$$

$$k_{sp} < \frac{1}{m_s} \cdot k_{sv}^2 - \frac{1}{k_{mv}} \cdot k_{sv} \cdot k_{mp} + \frac{1}{m_m + m_h} \cdot k_{sv} \cdot k_{mv}$$
(13)

$$0 < k_{sv}^2 - m_s \cdot k_{sp} + (k_{mv} \cdot \frac{m_s}{m_m + m_h} - m_s \cdot \frac{k_{mp}}{k_{mv}}) \cdot k_{sv}$$
(14)

$$0 < k_{mv}^{2} - (m_{m} + m_{h}) \cdot k_{mp} + \frac{(k_{mv}^{2} - m_{s} \cdot k_{sp}) \cdot (m_{m} + m_{h})}{m_{s} \cdot k_{sv}} \cdot k_{mv}$$
(15)

Now it is possible to find the stable margins of the controller gains k_{mv} , k_{mp} , k_{sv} , k_{sp} . Using the approximate values of the coefficients of the characteristic equation, the expression (12) can be written as follows:

$$m_s^2 k_{mp} k_{mv} > -[(m_m + m_h)k_{sv} + m_s \bullet k_{mv}]k_{mv} k_{sv} \quad (16)$$

Since the values of all the regulator gains and

physical parameters in expression (16) are non-negative, it is possible to conclude that this inequality will hold for all of the non-negative values of k_{mp} . Therefore, we can state that the choice of the value of k_{mp} gain does not affect the stability of the system according to the Routh-Hurwitz criterion. Similarly, it can be shown that selecting a non-negative value of k_{sp} will not result in the deterioration of system stability, as it follows from (13).

Now let us find the stable margins for the k_{sv} gain. Putting the inequality (14) in the form x^{2+} bx – c > 0 and solving it, we obtain the following boundaries of the k_{sv} gain:

$$k_{sv} < -\frac{b}{2} - \frac{\sqrt{b^2 + 4c}}{2} \text{ or } k_{sv} > -\frac{b}{2} + \frac{\sqrt{b^2 + 4c}}{2}$$

where $b = m_s [k_{mv}/(m_m + m_h) - k_{mp}/k_{mv}]$, $c = m_s k_{sp}$. Since we are looking only for the positive value of

표 4. 텔레 로보틱스 파라메터의 대략적 승수

Table 4.	Approximate	orders	of	telerobotic	system
	parameters.				

Parameter	Value	Physical meaning	
i urumeter	range		
m_m, m_s, m_h	10^{-1} 1	Masses of robots and	
	$10 \sim 1$	human hand	
m_e	$10^{-6} \sim 10^{-4}$	Mass of the object	
b_m, b_s, b_h, b_e	$10^{-1} \sim 1$	Viscous friction	
		coefficients	
$k_{h,}k_{e}$	1	Stiffness coefficients	
k_{mv}, k_{mp}	$10^{3} \sim 10^{4}$	Master device	
		controller gains	
k_{sv},k_{sp}	$10^{3} \sim 10^{4}$	Slave device controller	
		gains	
C_{1v}, C_{1p}	103 104	Scaling controller C1	
	$10^{\circ} \sim 10^{10}$	gains	
α	$10^{-3} \sim 10^{-4}$	Position scaling	
		coefficient	
β	$10^3 \sim 10^4$	Force scaling	
		coefficient	
C_2	$10^{-1} \sim 10$	Scaling controller C2	
		gain	

the k_{sv} gain, we are interested only in the second inequality among those two, or

$$k_{sv} > -\frac{b}{2} + \frac{\sqrt{b^2 + 4c}}{2} \tag{17}$$

From Table 4, we find that the parameters b and c have the order of 10^3-10^4 ; therefore, it can be conjured that $b^2 >> 4c$. Hence, the minimum stable margin of the k_{sv} gain, according to inequality (17), would be a positive value close to zero. Accordingly, we can find that the same conditions apply to the stable range of the k_{mv} gain. Therefore, it can be stated that choosing the controllers gains k_{mv} and k_{sv} to be positive values of the third or fourth order will not lead to the instability of the control system.

2. Llewellyn's Criterion

Llewellyn's criterion for absolute stability is expressed in terms of immitance matrix parameters as follows:

1. z_{11} and z_{22} have no poles in the right half plane.

2. Any poles of z_{11} and z_{22} on the imaginary axis are simple with real and positive residues. These two conditions can be written in mathematical form as follows:

$$\mathbb{R}(z_{11}) \ge 0 \tag{18}$$

$$\mathbb{R}(z_{22}) \ge 0 \tag{19}$$

3. The third condition can be written in several ways:

• Llewellyn (1952)

$$4 [\mathbb{R}(z_{11})\mathbb{R}(z_{22}) + \mathbb{I}(z_{12})\mathbb{I}(z_{21})]
\cdot [\mathbb{R}(z_{11})\mathbb{R}(z_{22}) - \mathbb{R}(z_{12})\mathbb{R}(z_{21})]
- [\mathbb{R}(z_{12})\mathbb{I}(z_{21}) - \mathbb{R}(z_{21})\mathbb{I}(z_{12})]^2 > 0$$
(20, a)

• Hashtrudi-Zaad and Salcudean (2001)

$$2\frac{\mathbb{R}(z_{11})\mathbb{R}(z_{22})}{|z_{12}z_{21}|} - \frac{\mathbb{R}(z_{12}z_{21})}{|z_{12}z_{21}|} \ge 1$$
(20, b)

• Adams and Hannaford (1999); Son et al. (2009)

$$2\mathbb{R}(z_{11})\mathbb{R}(z_{22}) - \mathbb{R}(z_{12}z_{21}) - |z_{12}z_{21}| \ge 0 \qquad (20, c)$$

In the expressions above, the terms $\mathbb{R}(x)$ and $\mathbb{I}(x)$ denote real and imaginary part of expression *x*.

For the selected control system, assuming that the time delay is negligible, the following relationships hold:

$$\dot{x}_m = (f_m - C_2 \beta f_s - C_m \dot{x}_m) Z_m^{-1}$$
 (21)

$$\dot{x}_s = (\alpha C_1 \dot{x}_m - f_s - C_s \dot{x}_s) Z_s^{-1}$$
 (22)

From (21)-(22), it follows that

$$f_m = (Z_{c+m})\dot{x}_m + \beta C_2 f_s \tag{23}$$

$$\dot{x} = \frac{\alpha C_1}{Z_{c+s}} \dot{x}_m - \frac{1}{Z_{c+s}} f_s \tag{24}$$

where $Z_{C+m} = C_m + Z_m, Z_{C+s} = C_s + Z_s$.

Assuming that the order of the environment transfer function Z_e is known and $f_s = v_s Z_e$, we can rewrite (23) as follows:

$$\begin{aligned}
x_s \dot{Z}_{cse} &= \alpha C_1 \dot{x}_m \\
\dot{x}_s &= \frac{\alpha C_1}{Z_{cse}} \dot{x}_m
\end{aligned}$$
(25)

where $Z_{cse} = C_s + Z_s + Z_e$. As can be seen from (25), we can minimize the positioning error by setting C_1 = $C_s >> Z_e$, Z_s , which results in $v_s = v_m$. In addition, knowing that $f_m = Z_{c+m}v_m + \beta C_s f_s$, we set $C_2 = 1$ for transparency purposes in order to make $f_m = \beta f_s$ at the settled state.

The impedance matrix can be written as follows:

$$Z = \begin{bmatrix} Z_{c+m} + \alpha\beta C_1 C_2 & \beta C_2 Z_{c+s} \\ \alpha C_1 & Z_{c+s} \end{bmatrix}$$
(26)

Applying Llewellyn's criterion to analyze stability, we need to prove that three conditions required for the system to be stable hold.

In order to prove that the first and second conditions hold, we first derive

$$z_{11} = Z_{c+m} + \alpha \beta C_1 C_2 = m_m s + (b_m + k_{mv}) + (k_m + k_{mp})/s + \alpha \beta (k_{sv} + k_{sp}/s)$$
(27)

$$z_{22} = m_s s + (b_s + k_{sv}) + (k_s + k_{sp})/s$$
(28)

From (27)–(28), it follows that z_{11} and z_{22} have poles only at s = 0, which satisfies the first two conditions of Llewellyn's stability criterion. Further investigation of criteria (18)–(19) reveals that from (27)–(28) one can find

$$\mathbb{R}(z_{11}) = b_m + k_{mv} + \alpha \beta k_{sv} \ge 0$$
(29)

$$\mathbb{R}(z_{22}) = b_s + k_{sv} \ge 0 \tag{30}$$

Since the viscosity coefficients of master and slave b_m and b_s are positive, two conditions above impose the following restrictions on the controller gains k_{mv} and k_{sv} :

$$k_{sv} \ge -b_s \tag{31}$$



그림 2. 입력신호 주파수와 이득 ksv의 변화 값의 함수로써 기준 (20, a)-(20, c) 값의 변화

Fig. 2. The values of criteria (20, a)-(20, c) as a function of input signal frequency and various values of the ksv gain.

$$k_{mv} \ge -\alpha\beta b_s - b_m \tag{32}$$

For the sake of convenience of notation, let us introduce the following variables:

$$b_{m+c} = b_m + k_{mv} \tag{33}$$

$$k_{m+c} = k_m + k_{mp} \tag{34}$$

$$b_{s+c} = b_s + k_{sv} \tag{35}$$

$$k_{s+c} = k_s + k_{sp} \tag{36}$$

Hence, expressions (29)–(30) will take the following form:

$$\mathbb{R}(z_{11}) = b_{m+c} + \alpha \beta k_{sv} \ge 0 \tag{37}$$

$$\mathbb{R}(z_{22}) = b_{s+c} \ge 0 \tag{38}$$

The third criterion of Llewellyn's stability can be investigated by dividing one of the three conditions (20, a-c) into two parts. For instance, for condition (20, c) this may look as follows^[7, 8]:

$$2\mathbb{R}(z_{11})\mathbb{R}(z_{22}) - \mathbb{R}(z_{12}z_{21}) \ge 0$$
(39)

$$\left[2\mathbb{R}(z_{11})\mathbb{R}(z_{22}) - \mathbb{R}(z_{12}z_{21})\right]^2 \ge |z_{12}z_{21}|^2$$
(40)

Since

$$\mathbb{R}(z_{11}z_{22}) = \alpha\beta\left(\frac{k_{sv}b_{s+c}\omega^2 + k_{sp}m_s\omega^2 - k_{sp}k_{s+c}}{\omega^2}\right)$$
(41)

substituting equations (26), (33)–(36), and (41) into (39) and taking values from the Table 3 yields

$$2\mathbb{R}(z_{11})\mathbb{R}(z_{22}) - \mathbb{R}(z_{12}z_{21}) = (2b_{m+c}b_{s+c} + \alpha\beta(k_{sv}b_{s+c} - k_{sp}m_s))\omega^2 + \alpha\beta k_{sp}k_{s+c} \ge 0$$
(42)

Having $b_{s+c} \ge 0$ from (38) and assuming b_{m+c} , $k_{sp} \ge 0$, we can then prove that the inequality (42) is satisfied for all frequencies of the input signal ω .

Since both of the viscosity gains of master and slave devices (b_m and b_s), along with slave mass m_s and stiffness coefficient k_s , are always non-negative values, we can prove that for the control system to be stable for all real values of ω , it is required that

$$k_{sp} \ge 0 \tag{43}$$

$$k_{sp}m_s \le \frac{2b_{m+c}b_{s+c}}{\alpha\beta} + k_{sv}b_{s+c} \tag{44}$$

Hence, in order to satisfy the absolute stability criteria, the controller gains should not violate conditions (37)–(38) and (43–44). Similar reasoning can be applied to any of the criteria (20, a) – (20, c), and they will produce the same stable margins of controller gains. Figure 2 presents the values of conditions (20, a) – (20, c) as a function of input signal frequency ω and controller gain k_{sv} , and Figure 3 shows the values of the gain k_{sv} at which the criteria intersect horizontal axis for input signal frequencies ω ranging between 0 and 100 rad/s. It can be noted from Figure 3 that the results obtained using different criteria remain equivalent.

We may conclude application of Llewelyn's absolute stability criteria imposes less conservative stability conditions on controllers gains in comparison



그림 3. C₂ = 1일때 기준 (20,a) - (20,c)로 얻은 이득 *k*_{sv}의 안정된 값들의 범위

Fig. 3. Range of stable values of gain k_{SV} calculated with criteria (20,a) - (20,c) for C_2 = 1.

to those obtained using the Routh-Hurwitz criterion. These margins are also *frequency-dependent* in case of absolute stability criteria.

3. Numerical Example

A numerical simulation has been conducted in order to verify the stability margins obtained above. The exogenous input force was the unit step with the magnitude of 1 N. The following system parameters have been chosen:

- human operator: $m_h = 1$ kg, $b_h = 0.1$ Ns/m, $k_{h}= 0$ N/m;
- master device: $m_m = 0.7$ kg, $b_m = 0.1$ Ns/m;
- slave device: m_s =1 kg, b_s =1 Ns/m;
- environment: $m_e=10^{-6}$ kg, $b_e=0.1$ Ns/m,

$k_e = 10 \text{N/m};$

• master controller: k_{mv} =2000 Ns/m, k_{mp} =10000 N/m;

- slave controller: k_{sv} =5000 Ns/m, k_{sp} =15000 N/m;
- scaling coefficients: $\alpha = 10^{-3}$, $\beta = 2000$;
- position and force feedforward regulations:

 C_{1v} =5000 Ns/m, C_{1p} =45000 N/m, C_2 =1.

The transient responses of the studied control



그림 4. 서로 다른 이득 k_{sv}에서의 텔레 로보틱스 시스 템의 계단응답

Fig. 4. Step responses of the telerobotic system under different values of k_{sv} gains.

system for the different values of k_{sv} gain are shown in Fig. 4. One can ascertain that as soon as the marginal condition $k_{sv}=-b_s$ is met, the system shows oscillatory convergence, which means that it reaches the boundary of stability region (the bottom plot in Fig. 4), and once the condition (29) does not hold, the system diverges. Similar plots confirming the stability analysis conducted above can be plotted for all of the controller gains. It can be noted that the system remains stable even for some negative values of k_{sv} , which is confirmed by the absolute stability criteria contradicts the Routh-Hurwitz but criterion. Therefore, it can be concluded that the Llewelyn's stability criterion provides less conservative stability margins.

4. Conclusion

In this paper, the outlines on application of Routh-Hurwitz and Llewellyn stability criteria to a scaled Telerobotic system stability analysis are discussed. The dynamics of the scaled teleoperated system is studied, and the closed-loop transfer function of the system is derived. Next, the Routh-Hurwitz and Llewellyn stability criteria are applied in order to find stable margins of the controller gains of master and devices. It is shown that slave although Routh-Hurwitz criterion is more simple in its application to an actual control system, it gives more conservative conditions on controller gains. In addition, it is possible to obtain less conservative stable gain margins using the Llewellyn absolute stability criteria if the frequency of the input signal is known (lies in a given range). The conducted analysis is confirmed by a numerical simulation.

REFERENCES

- B. Hannaford, "A design framework for teleoperators with kinesthetic feedback," IEEE Tran. on Robotics and Automation, Vol. 5, no. 4, pp. 426 - 434, 1989.
- [2] Y. Yokokohji and T. Yoshikawa, Bilateral

control of master-slave manipulators for ideal kinematic coupling Formulation and experiment. IEEE Trans. on Robotics and Automation, pp. 605 - 620, 1994.

- [3] S. E. Salcudean, "Control for teleoperation and haptic interfaces", in Control Problems in Robotics and Automation, LNCIS-230, ed. B. Siciliano and K. P. Valavanis, pp. 51 - 66. New York: Springer-Verlag, 1998.
- [4] B. Hannaford, "A design framework for teleoperators with kinesthetic feedback," IEEE Tran. on Robotics and Automation, vol. 5, no. 4, pp. 426 - 434, 1989.
- [5] K. Hastrudi-Zaad, S.E. Salcudean, "On the use of local force feedback for transparent teleoperation", IEEE Proc. Int. Conf. Robotics and Automation, vol. 3, pp. 1863–1869, 1999.

- [6] Jazayeri, A.; Tavakoli, M., "A Passivity Criterion for Sampled-Data Bilateral Teleoperation Systems", Haptics, IEEE Transactions on, vol. 6, no. 3, pp. 363–369, July–Sept. 2013
- [7] H. I. Son, T. Bhattacharjee, H. Hashimoto, "Enhancement in operator's perception of soft tissues and its experimental validation for scaled teleoperation systems", IEEE/ASME Trans. on Mechatronics, vol. 16, issue 6, pp. 1096–1109, 2011.
- [8] H. I. Son, T. Bhattacharjee, D.Y. Lee, "Control design based on analytical stability criteria for optimized kinesthetic perception in scaled teleoperation", ICROS-SICE Int. Joint Conf., pp. 3365–3370, 2009.



──── 저 자 소 개 이고르 가포노프(정회원)

- 2006년 러시아 쿠르스크 대학교 메카트로닉스 및 자동화 공학과 공학사 2008년 한국기술교육대학교 기계
- 공학과 공학석사 2011년 한국기술교육대학교 기계 공학과 공학박사

2011년~현재 한국기술교육대학교 기계공학부 조교수

<주관심분야 : 로보틱스, 제어이론, 텔레로봇시스 템>



전 홍 태(정회원) 1976년 서울대학교 전자공학과 1986년 뉴욕 주립대학 전기전자공 학과(공학박사) 1986년 9월~현재 중앙대학교 전자전기공학부 교수

<주관심분야 : 지능시스템, 로보틱스, 시스템 제 어>

 조 현 찬(정회원)

 1983년 광운대학교 전자공학과 공학사

 1985년 중앙대학교 전자공학과 공학석사

 1991년 중앙대학교 전자공학과 공학박사

1991년~현재 한국기술교육대학교 전기전자통신 공학부 교수

<주관심분야 : 지능알고리즘 및 시스템제어, 바이 오-전자공학, 로보틱스>