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Research on Risk-Averse Newsboy under Supply Uncertainty

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위험회피성향을 고려한 공급 불확실성下 신문팔이소년 문제에 대한 고찰

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In this paper, the single-period inventory problem, what is called newsboy problem, has been revisited with two different conditions, uncertain supply and risk-averseness. Eeckhoudt et al. [5] investigated the effect of risk-averseness of a newsboy on the optimal order quantity in a stochastic demand setting. In contrary to Eeckhoudt et al. [5] this paper investigates the effect of risk-averseness in a stochastic supply setting. The findings from this investigation say that if α^* represents the optimal order quantity without risk-averseness then the risk-averse optimal order quantity can be greater than α^* and can be less than α^* as well.

Keywords : Single-period Inventory Problem, Newsboy, Risk-averseness

1. Introduction

In a classic newsboy problem the newsboy should decide how many newspapers to order for his daily business very early in the morning. Not only too many newspapers but also too few will incur him unnecessarily high costs. In other words, in case he orders too many he will struggle with left-over newspapers at the end of day, while he would have missed an opportunity for incremental profit in case he orders too few in the morning. The most important reason why this classical newsboy problem has been most popular research topic for decades is that it allows researchers to investigate a variety of single period inventory problems under so many different conditions and scenarios such as the max(or target) capacity constraint, target production levels, target profit levels, various pricing schemes, underlining detail inventory holding/ shortage cost structures and so on. The importance of the newsboy research problem can also be easily proved by counting the number of OM/OR research articles published so far. The number of articles is close to 400, which use the words ‘newsboy’ or ‘newsvendor’.

Similar to the structure of inventory problems the newsboy problem can have following structures where daily demand and daily supply can be either deterministic or stochastic.

In the original newsboy problem setting discussed in Morse and Kimball [12] stochastic demand with deterministic supply situation has been assumed. Morse and Kimball’s [12] research was followed by numerous researchers such as Hanssman [8], Porteus [13], Schwerizer and Cachon [15] to deal with more sophiscated and realistic problems while maintaining the basic framework of stochastic demand and deterministic supply.

Even though stochastic demand assumption with deterministic supply is adopted in most of the single-period inventory

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model, in many real-life situations, one can easily observe random gaps between the originally placed order quantity and the actually achieved quantity. Increased popularity of global sourcing is one big reason for generating less than perfect supply processes from suppliers to retailers. For example, to reduce purchase costs and attract a larger base of customers, retailers such as Wal-Mart, Home Depot and Dollar General are constantly seeking suppliers with lower prices and finding them at greater and greater distances from their distribution centers (DCs) and stores. Consequently, a significant proportion of shipped products from overseas suppliers is susceptible to defects. Reasons for defects include missing parts, misplaced products (at DCs, stores) or mistakes in orders and shipments. A similar example could be a typical production line where the production yield assumes less than 100% resulting in a different number of goods manufactured than originally planned. In these situations, the problem is how to choose the size of an order or how many parts to begin production to meet one time fixed demand. See Kim et al. [9] for precise problem formulation and solution analytics.

The basic newsboy research framework produced some other meaningful research topics incorporating uncertain supply situations. Newsboy problem research with uncertain supply framework was initiated by Silver [17] and followed by many other researchers. Silver [17] is one of the earliest papers on the uncertain supply under economic order quantity (EOQ) framework. He studied two cases, in the first case, the standard deviation of the amount received is independent of the lot size, while in second case the standard deviation is proportional to the lot size. One of the interesting results among his findings was that the optimal order quantity depends only on the mean and the standard deviation of the amount received. Yano and Lee [19] is the most popular reference.

Newsboy problems so far have not considered the newsboy's attitude towards various potential risks such as financial risk, meeting the target profit or cost etc. In other words researchers have assumed the newsboy's indifference on those risks and focused on developing risk neutral optimal solutions optimizing the expected profit or cost.

To overcome this 'flaw of average' in solving the single-period inventory problem many researchers studied the behavior of Risk Averse Newsboy. These include Spulber [18], Bouakiz and Sobel [2], Eeckhoudt et al. [5], Agrawal and Seshadri [1], Chen and Federgruen [3], Seifert et al. [16], Chen et al. [4], Haksoz and Seshadri [7]. Lau [11], Gan et al. [6] examined newsvendor solutions which maximize expected utility. Gan et al. [6] also investigated the new objective function of maximizing the probability of achieving a budgeted profit. Eeckhoudt et al. [5] examined the risk and risk aversion in a single-period inventory problem where demand is stochastic while supply is deterministic. They show that the optimal order quantity decreases as decision maker's risk-aversion increases because a lower order amount definitely reduces the inherent risks of the outcome. In Bouakiz and Sobel [2], they explored the newsvendor problem with the exponential utility and showed that a base- stock policy is optimal when a multi-period newsvendor problem is optimized with an exponential utility criterion. Agrawal and Seshadri [1] also investigated the newsvendor problem with the objective being maximizing the expected utility. In their problem setting, both price and order quantity are decision variables for the risk-averse retailer.

In this paper, the research output of Eeckhoudt et al. [5] is revisited to the case of uncertain supply situation. In Eeckhoudt et al. [5] they started with the basic newsboy problem, where the demand is stochastic while supply is deterministic, then deployed the utility functions over the newsboy's expected profit function to embed his risk attitude into the problem. From their research they derived and summarized comparative statics of the risk-averse newsboy as the changes of the optimal order quantity. To our best knowledge and literature review results, no article on the risk-averse newsboy problem under uncertain supply problem has been published.

The rest of this paper is organized as following : Section 2 reminds of the basic newsboy problem framework with uncertain supply setting and the risk neutral optimal order quantity as well. In Section 3, we introduce the risk-averse newsboy problem framework followed by a derivation of characteristics of the optimal order quantity. In Section 4, we present a brief numerical study to demonstrate the result from Section 3. Finally, in Section 5, we conclude this paper by summarizing the findings and insights throughout our journey.

2. Previous Model : Risk-Neutral Newsboy Problem with Uncertain Supply

In this section we consider a newsboy problem with uncertain supply instead of uncertain demand in the classical newsboy problem. And we assume θ, the variable representing daily demand, is fixed and known. This assumption might be unrealistic but in our search the main purpose is to verify the impact of risk-aversion on the optimal order quantity under unreliable supply condition. To focus on the research purpose we sacrifice the uncertain demand assumption. Under this assumption, when the newsboy's order quantity is α, the number of arrived newspapers to the newsboy is Y α, where Y represents random yield proportion of α with distribution function $G(y)$ (p.d.f. $g(y)$). Let's define p as retail price and c as wholesale price. And all unsold newspapers are returned to the distributor office at salvage price v. Finally the newsboy is allowed to obtain additional newspapers if demand is greater than what he has on hand, but at a higher cost, $c^{\wedge}$. As addressed in Eeckhoudt et al. [5] a natural assumption is that $0 \leq v < c < c^{\wedge} \leq p$ Then, the function, $Z(Y,\ \alpha)$, represents the newsboy's total revenue at the end of each day. The newsboy, facing an uncertain supply via random yield process, has to determine α, the size of his original newspaper order early in the morning.

$$\begin{aligned} Z(Y,\ \alpha) = {} & P \cdot \min(\theta,\ Y\alpha) \\ & - cY\alpha + v \cdot \max(0,\ Y\alpha - \theta) \\ & - c^{\wedge} \cdot \max(0,\ \theta - Y\alpha) \end{aligned} \qquad (1)$$

or equivalently, using the two mutually exclusive ranges, $\theta \leq Y\alpha$, $\theta > Y\alpha$, $Z(Y,\ \alpha)$ can be rewritten as following

$$Z_1(Y, \alpha) = (p-v)\theta - (c-v)Y\alpha$$

$$\text{if } \theta \leq Y\alpha,$$

$$Z_2(Y, \alpha) = (p + c^{\wedge} - c)Y\alpha - c^{\wedge}\theta,$$

otherwise.

Using the above equations, the expected revenue function can be expressed as :

$$\begin{aligned} E[Z(Y,\ \alpha)] &= \int_{\frac{\theta}{\alpha}}^{1} Z_1(Y,\ \alpha)\,dG + \int_0^{\frac{\theta}{\alpha}} Z_2(Y,\ \alpha)\,dG \\ &= \int_{\frac{\theta}{\alpha}}^{1} [(p-v)\theta - (c-v)Y\alpha]\,dG \\ &\quad + \int_0^{\frac{\theta}{\alpha}} [(p + c^{\wedge} - c)Y\alpha - c^{\wedge}\theta]\,dG \end{aligned} \qquad (2)$$

where $E[\cdot]$ denotes the expectation operator.

The expected revenue function is concave in the order quantity (or batch size), α. Readers can refer to Kim et al. [10] for rigorous proof of concavity. And the concavity of the expected revenue function allows us to rely on the first order condition to find the optimal batch size which maximizes the expected revenue. From the simplification of the first order condition, it can be shown that the risk-neutral solution α^* should satisfy the following equation :

$$\int_0^{\frac{\theta}{\alpha^*}} Y\,dG = \frac{(c-v)}{(p + c^{\wedge} - v)} E[Y] \qquad (3)$$

For interested readers the derivation of above expression (3) can be found in Kim et al. [10].

Once the distribution of Y and the demand level, θ are specified, the corresponding solution can be computed using the above equation.

3. Risk-Averse Newsboy Problem with Uncertain Supply

Eeckhoudt et al. [5] showed that the risk-averse newsboy always would order fewer newspapers and they proved that this quantity would decrease as the newsboy's risk-aversion increases. But, under random supply situation, this is not quite true and we will discuss about it in this section.

Similar to our research, Kim et al. [10] considered the impact of the risk aversion on the optimal order quantity under supply uncertainty. They introduced downside-risk constraint to reflect the newsboy's risk attitude on his expected revenue at the end of a business day. The downside-risk constraint enables the newsboy to constrain the probability of meeting his target revenue to a desirable level. Readers can think of the following form of downside-risk constraint for intuitive understanding : $P(Z(Y,\ \alpha) \leq \tau_1) \leq \omega_1$.

In addition, the downside-risk parameter pair(τ_1, ω_1) exhibits higher risk aversion than another pair(τ_2, ω_2) whenever ($\tau_1 \geq \tau_2$) and ($\omega_1 \leq \omega_2$). Suppose that ω_1 and ω_2 are same at 95% level and $\tau_1 = 100$, $\tau_2 = 75$, respectively. Then the corresponding downside-risk constraint with (τ_1, ω_1) implies that the probability of the newsboy's payoff being less than or equal to 100 should be no greater than 95% while constraint with (τ_2, ω_2) implies the payoff being no greater than 75 should be 95% or smaller. In this way, whenever $\tau_1 \geq \tau_2$ downside-risk constraint with the parameter with τ_1 represents more risk-averse attitude of the newsboy than that with the parameter τ_2. In their article, Kim et al. [10] presented numerical examples to show that as the risk aversion increases the size of newspaper order placed by the newsboy also increases when the supply probability is not certain. In other words, the more the newsboy exhibits risk aversion, the more he orders the newspaper to reflect his risk attitudes. But in this paper we show that for the same problem setting the more the newsboy exhibits risk aversion does not guarantee the increase in the newspaper order size. We adopted concave transformation of the utility function to see the effect of newsboy's increased risk-aversion in our search rather than adopting the downside-risk aversion constraint as in Kim et al. [10].

In our paper, similar to Eeckhoudt et at. [5], the newsboy's preference over the final wealth is assumed to be of the expected-utility type where $u(\cdot)$ is the corresponding utility function. Then, the risk-averseness of the newsboy can be considered by choosing $u(\cdot)$ to be increasing and concave. Then the resulting problem can be formulated as a maximization problem where the objective function represents the expected-utility of the newsboy :

$$\max_{\alpha \geq 0} H(\alpha) = E[u(Z(Y, \alpha)] \tag{4}$$

(3.1)

For any function $q(\alpha)$, which is second order differentiable in α if we let $\frac{\partial q}{\partial \alpha} = q'$, $\frac{\partial^2 q}{\partial \alpha^2} = q''$ respectively, then the first order condition for (4) is

$$\left.\frac{\partial H}{\partial \alpha}\right|_{\alpha^*} = -(c-v)\int_{\frac{\theta}{\alpha^*}}^{1} Yu'(Z_1)dG + (p+c^{\wedge}-c)\int_0^{\frac{\theta}{\alpha^*}} Yu'(Z_2)\, dG = 0 \tag{5}$$

where α^* is the optimal order quantity which maximizes the expected revenue function of the newsboy.

As discussed in Eeckhoudt et al. [5], the optimal order quantity, α^*, divides the random yield proportion variable into ranges where an increased order provides a cost or benefit. If we assume $y_2 < y_0 = \frac{\theta}{a^*} < y_1$ and a strict concave utility function, $u(\cdot)$, we have

$$u[Z(y_0, \alpha^*)] > \max[u(Z_1(y_1, \alpha^*)), u(Z_2(y_2, \alpha^*))] \tag{6}$$

(3.3)

and we have

$$u'[Z(y_0, \alpha^*)] < \min\left[u'(Z_1(y_1, \alpha^*)), u'(Z_2(y_2, \alpha^*))\right] \tag{7}$$

To see the effect of the increased risk-aversion on the optimal order quantity, we utilize the concave transformation of the utility function. In general, the increment of the risk aversion is equivalent to a concave transformation as explained in Pratt [14]. Let us assume a concave function $k(\cdot)$ with (3.4) $k'(\cdot) > 0$ and $k''(\cdot) < 0$. Then $E[k(u(Z(Y, \alpha)))]$ is the objective function of a newsboy who exhibits more risk-averseness compared to the newsboy with an objective function as shown in equation (4). In addition, with the strict concavity condition of $k(\cdot)$, the following inequality also holds

$$k'[u(Z_0)] < \min\{k'[u(Z_1)], k'[u(Z_2)]\} \tag{8}$$

where $u(Z_0) = u[Z(y_0, \alpha^*)]$, $u(Z_1) = u[Z_1(y_1, a^*)]$, $u(Z_2) = u[Z_2(y_2, \alpha^*)]$, respectively. (3.5)

Denote by $H_2(\alpha) = E[k(u(Z(Y, \alpha)))]$, the first derivative of $H_2(\alpha)$ at $\alpha = \alpha^*$ can be shown as following :

$$\left.\frac{\partial H_2}{\partial \alpha}\right|_{\alpha^*} = -(c-v)\int_{\frac{\theta}{\alpha^*}}^{1} Yk'(u(Z_1)) \cdot u'(Z_1)dG \quad (3.6)$$

$$+(p+c^{\wedge}-c)\int_0^{\frac{y}{\alpha^*}} Yk'(u(Z_2)) \cdot u'(Z_2)dG \quad (3.6) \tag{9}$$

Now, using (5) and (8) the following two inequalities can be derived :

$$\left.\frac{\partial H_2}{\partial \alpha}\right|_{\alpha^*} < (p+c^{\wedge}-c)\int_0^{\frac{\theta}{\alpha^*}} Y(k'(u(Z_2)) - k'(u(Z)))u'(Z_2)dG \quad (3.7) \tag{10}$$

$$\left.\frac{\partial H_2}{\partial \alpha}\right|_{a^*} > (c-v)\int_{\frac{\theta}{\alpha^*}}^{1} Y(k'(u(Z)) - k'(u(Z_1)))u'(Z_1)dG \quad (3.8) \tag{11}$$

Denote by β_1, β_2 the right-hand-side of (10), (11) respectively, β_1 is always positive ($\beta_1 > 0$) and β_2 is always negative ($\beta_2 > 0$). Please refer the Appendix for the proofs of (10) and (11). From these we have the following condition

$$\beta_2 < \left.\frac{\partial H_2}{\partial \alpha}\right|_{a^*} < \beta_1 \tag{12}$$

According to the inequality (12), (3.9) $\left.\frac{\partial H_2}{\partial \alpha}\right|_{a^*}$ can take values between a negative number and a positive number. This implies that the optimal order quantity of more risk-averse newsboy under uncertain supply environment can be either less of more at the same time when compared to the optimal order quantity of the less risk-averse newsboy.

4. Numerical Study

The purpose of our numerical study is to verify the results from the previous section. For this purpose we assume the information shown in <Table 1>. The uniform distribution assumption of $G(y)$ might not be realistic. But with this assumption we still manage to show the validity of our important findings from Section 3.

<Table 1> Parameter Assumptions

Parameter	Values/Assumptions
θ	100(Qty/day)
p	28($)
c	20($)
c^	24($)
v	0($)
$G(y)$	Uniform Distribution in(0, 1)

Under assumptions in <Table 1> if we deploy the equation (3) we can easily show that the optimal number of newspaper which maximizes $E(Z(\alpha))$ to be $\alpha^* = 161$.

Let us now assume that the strict concave utility function $u(x)$ to be $-\exp(-rx)$. Furthermore let $k(x)$, the concave transformation function for artificially adding the risk aversion, to be $-\frac{1}{x}$.

<Figure 1> shows the optimal order quantities which maximizes $E[u(Z(Y,\ \alpha))]$ at various levels of r, the risk aversion parameter.

<Table 2> summarizes how the optimal order quantity varies as the risk-aversion parameter r increases. In here the optimal order quantity maximizes the expected utility function, $E[u(Z(Y,\ \alpha))]$. In contrast to the result of Eeckhoudt et al. [5] the optimal quantities have been increased when the degree of risk-aversion increased.

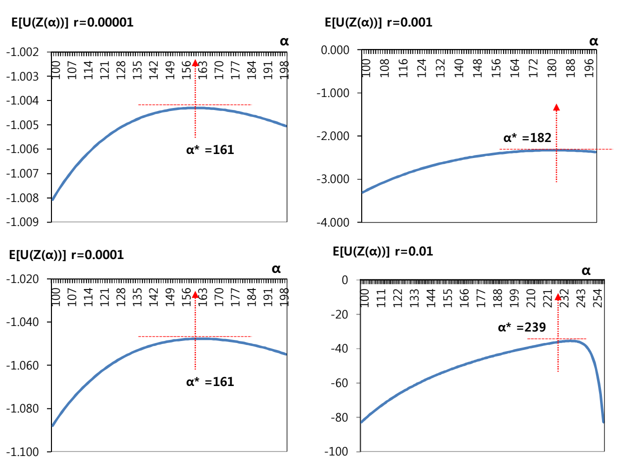
<Table 2> Optimal Order Quantities for $E[u(Z(Y,\ \alpha))]$

Risk Aversion Parameter	Optimal Order Quantity
$r = 0.00001$	161
$r = 0.0001$	161
$r = 0.001$	182
$r = 0.01$	239

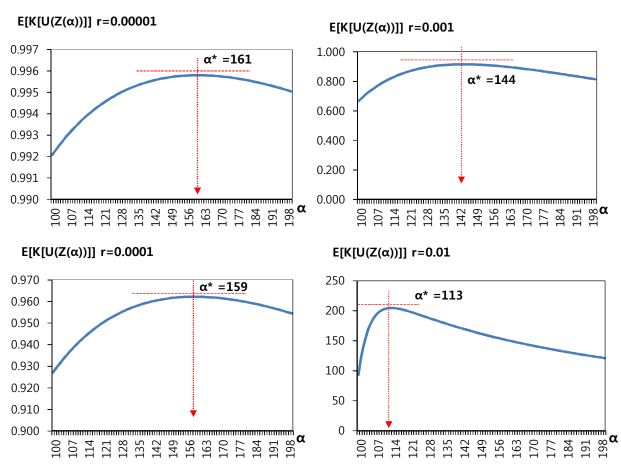
When the level of risk-aversion parameter is small enough ($r \le 0.0001$) the optimal order size is same with the risk-neutral solution which maximizes $E[Z(\alpha)]$. But as the risk-aversion characteristic increased (i.e., r increased) the optimal order size rapidly increased to achieve the maximum expected utility. But when the risk-aversion parameter is at its highest level (i.e., $r = 0.01$), it seems that ordering beyond the computed optimal order size could rapidly ruin the newsboy's expected utility values.

<Figure 3> illustrates the optimal order quantities maximizing $E[k(u(Z(Y,\ a)))]$ where a different kind of utility function, $k(u)$, is introduced. The resulting utility function, $k(u(x))$, intrinsically exhibit additional risk-aversion level than the previous utility function $u(x) = -\exp(-rx)$ via the concave transformation effects.

Surprisingly enough the optimal order size from added risk-aversion, via the concave transformation, is smaller when compared at the same level of risk-aversion parameter values. When $r = 0.00001$, the optimal order size of our new problem starts at 161 and this is identical with previous optimal order size using $u(x) = -\exp(-0.00001x)$ regardless of the concave transformation. But as r increases the optimal order size decreases down to 113. Again at the highest level of risk aversion parameter (i.e., $r = 0.01$) it seems that ordering below the computed optimal order size could ruin the newsboy's



<Figure 1> Optimal Order Quantities for $E[u(Z(Y,\ \alpha))]$



<Figure 2>Optimal Order Quantities for $E[k(u(Z(Y,\ a)))]$.

expected utility values. This is the opposite result compared to the previous case when we do not introduce the concave transformation for introducing the additional risk level or the added risk.

<Table 3> summarizes optimal order quantities from using objective function $E[k(u(Z(Y, a)))]$.

<Table 3> Optimal Order Quantities for $E[k(u(Z(Y, a)))]$.

Risk Aversion Parameter	Optimal Order Quantity
$r = 0.00001$	161
$r = 0.0001$	159
$r = 0.001$	144
$r = 0.01$	113

Results presented in <Table 3> together with those in <Table 2> imply many things. Firstly, the added risk aversion under the uncertain supply condition plays a role to increase the size of the optimal order quantity as shown in the second column of <Table 2>. But the added risk aversion sometimes plays a role to decrease the size of the optimal order quantity as in the second column of <Table 3>. In summary when fore-mentioned two kinds of risk-aversion factors are applied together the optimal order size can possibly be greater or smaller than that of low risk-aversion newsboy problem.

5. Conclusion and Future Research

In this paper we have revisited the famous newsboy problem to develop meaningful insights while the newsboy's daily newspaper supply is not fixed due to the various reasons. These can include that too many newsboys want to sell newspapers (this situation induces competition on daily newspaper supply) or unexpected poor quality newspapers which can't be sold to customers. In addition to the supply uncertainty framework we add the risk-aversion into this basic newsboy with uncertain supply to analyze the change of optimal order quantity along the degree of newsboy's risk-aversion. For setting up the different degree of risk-aversion we introduced utility functions over the revenue function as used in Eeckhoudt et al. [5] and adopted concave transformation on the standard revenue function to produce non-zero positive risk-aversion. From this effort we have found that regardless of the degree of risk aversion the corresponding optimal order quantity is not always greater than that of the risk-neutral newsboy or not always smaller. This implies that the degree of risk-aversion alone cannot make the decision-maker decide decisive actions. In other words the risk-averse newsboy should simultaneously consider other factors such as his retail price, whole price, salvage value, opportunity cost to determine the optimal order quantity.

Dealing with the same topic under relaxed the known fixed demand assumption can be a challenging future research problem. The added demand uncertainty might result in different insight from our findings so far.

It will also be an interesting future research problem to investigate the exact conditions under which the optimal order quantity of a more risk-averse newsboy is less than that of a less risk-averse one or vice versa.

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<Appendix>

A.1 Proof of (10)

Equation (9) can be rewritten as following

$$\left.\frac{\partial H_2}{\partial \alpha}\right|_{\alpha^*} = (p + c^\wedge - c)\int_0^{\frac{\theta}{\alpha^*}} Y(k'(u(Z_2)) - k'(u(Z))) \quad (\mathrm{A.1})$$
$$\cdot\, u'(Z_2)dG + (p + c^\wedge - c)\int_0^{\frac{\theta}{\alpha^*}} Yk'(u(Z)) \quad (\mathrm{A.1})$$
$$\cdot\, u'(Z_2)dG - (c - v)\int_{\frac{\theta}{\alpha^*}}^1 Yk'(u(Z_1)) \quad (\mathrm{A.1})$$
$$\cdot\, u'(Z_1)dG$$

Now, to prove (9) it suffices to show that

$$(p + c^\wedge - c)\int_0^{\frac{\theta}{\alpha^*}} Yk'(u(Z))$$
$$\cdot\, u'(Z_2)dG - (c - v)\int_{\frac{\theta}{\alpha^*}}^1 Yk'(u(Z_1))$$
$$\cdot\, u'(Z_1)dG < 0$$

Firstly, from the first integral we can apply the result (7), $u'(z_2) > u'(Z)$, to have

$$-c\int_0^{\frac{\theta}{\alpha^*}} Yk'(u(Z)) \cdot u'(Z_2)dG$$
$$< -c\int_0^{\frac{\theta}{\alpha^*}} Yk'(u(Z)) \cdot u'(Z)dG$$

secondly, from the second integral we can apply the result of (7) and (8) to have

$$k'(u(Z_1)) \cdot u'(Z_1) > k'(u(Z)) \cdot u'(Z)$$

Now if we combine above two results we have

$$(p + c^\wedge - c)\int_0^{\frac{\theta}{\alpha^*}} Yk'(u(Z)) \cdot u'(Z_2)dG$$
$$-(c - v)\int_{\frac{\theta}{\alpha^*}}^1 Yk'(u(Z_1)) \cdot u'(Z_1)dG$$
$$-(c - v)\int_{\frac{\theta}{\alpha^*}}^1 Yk'(u(Z)) \cdot u'(Z)dG < 0$$

Similarly, we can prove that (11) holds.