

# Robust CFO Acquisition in PN-Padded OFDM Systems

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Guanghui Liu, Liaoyuan Zeng, Hongliang Li, Linfeng Xu, and Zhengning Wang

*As an alternative to the traditional pilot-aided orthogonal frequency division multiplexing (OFDM), the time-domain pseudonoise (PN)-padded OFDM provides a higher spectral efficiency. However, the carrier frequency offset (CFO) attenuates peaks of the conventional PN correlation output, which limits the CFO estimation range of the OFDM synchronizer. An improved correlation is proposed in this letter to remove the CFO-induced amplitude attenuation of correlation peaks. For a synchronizer adopting the designed correlator, a larger range of CFO acquisition is obtained through using wider correlation windows with a smaller interval between them. The proposed method of CFO acquisition is verified in a digital terrestrial multimedia broadcast receiver, in which the synchronizer is able to acquire CFOs up to  $\pm 320$  kHz in the DVB-T F1 channel. Furthermore, the acquisition range can be expanded in more favorable channels.*

**Keywords:** Orthogonal frequency division multiplexing (OFDM), carrier frequency offset (CFO), composite PN correlation (CPC), DTMB.

## I. Introduction

Pseudonoise (PN)-sequence-padded orthogonal frequency division multiplexing (OFDM), also known as time-domain synchronous OFDM (TDS-OFDM) [1], [2], removes pilot overhead from the transmitted spectrum, resulting in a higher

spectral efficiency than that of the conventional cyclic prefix (CP) OFDM [3]. PN-padded OFDM is now considered a key modulation technology in the digital terrestrial multimedia broadcast (DTMB) standard [4].

In the PN-padded OFDM, the PN sequence replaces the CP as the time guard interval (TGI). Due to the no-pilot signaling, the receiver parameters regarding channel state and synchronization are estimated only by making full use of the inserted PN. A majority of PN correlation-based estimation schemes can be found in [5]-[10] for synchronization or in [11], [12] for channel estimation. However, the carrier frequency offset (CFO) attenuates correlative gain, which leads to a poor accuracy of parametric estimation, degrading the performance of the PN-padded OFDM receiver.

The acquisition of a large CFO is a real challenge for the TDS-OFDM synchronizer. In [6], [7], some synchronization recovery methods based on a long PN correlation were examined, while the CFO endurance in the synchronizers is limited to a couple of subcarrier intervals [8]. A scheme of three-level estimation was proposed in [9] to deal with an initial CFO within a range of about  $\pm 10$  kHz, that is, a few subcarrier intervals. In [10], a noncoherent summation of segmented PN correlation was introduced to acquire the frame timing in the presence of about  $\pm 50$ -kHz CFO. A method of composite PN correlation (CPC) was constructed in [5] to estimate a large CFO up to  $\pm 160$  kHz, which significantly enforces the robustness against CFO compared with the conventional methods. In the CPC-based scheme, however, there exist parametric constraints preventing the synchronizer from further expanding the CFO estimation range. In this letter, a novel correlation method is proposed to remove the constraints so that the improved CFO estimator can maximize the acquisition range in a flat fading channel. Even in a hostile channel, the

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proposed method doubles the CFO estimation range compared with the CPC-based scheme.

## II. Proposed CFO Estimation Scheme

### 1. CPC in PN-Padded OFDM

In the  $l$ -th OFDM symbol with  $N_c$  subcarriers, the inverse fast Fourier transform implements the modulation of frequency domain data and yields the time-domain signal  $x(n)$ . Variable  $L_{PN}$  denotes the PN length. The  $L_{PN}$  modulated complex sequence  $\{PN(n)\}$  is inserted at the start of  $\{x(n)\}$ , producing

$$s_l(n) = \begin{cases} PN(n), & n \in [0, L_{PN}], \\ s_l(n), & n \in [L_{PN}, N], \end{cases} \quad (1)$$

where  $N=N_c+L_{PN}$ . For simplicity, we only consider the effects of the CFO in the received signal; thus,

$$r_l(n) = s_l(n)e^{j\frac{2\pi}{N_c}(IN+n)\epsilon_c}, \quad (2)$$

where  $j^2=-1$  and  $\epsilon_c$  is the CFO normalized to the tone spacing  $\Delta f$ . The CPC is defined in [5] as follows:

$$R(n) = \underbrace{(r_l(n) \otimes PN(n+k_0))}_{\text{the left correlation window}}^* \underbrace{(r_l(n+P) \otimes PN(n+k_0+P))}_{\text{the right correlation window}}, \quad (3)$$

where  $\otimes$  stands for the linear correlation and  $k_0$  and  $P$  are the correlation offset and the interval of the right to the left correlation window, respectively. The CPC as (3) calculates the point-wise conjugate product of two phase-shifted PN correlations of  $L$ . Using (2) as a substitute in (3), we obtain the correlation peak at  $n=k_0$  as

$$R(k_0) = \left( \sigma_{pn}^2 L \operatorname{sinc}\left(\frac{\epsilon_c L}{N_c}\right) \right)^2 e^{j\frac{2\pi}{N_c} P \epsilon_c}, \quad (4)$$

where  $\sigma_{pn}^2$  is the averaged PN power and the SINC function is  $\operatorname{sinc}(x)=\sin(\pi x)/(\pi x)$ . The CFO is estimated as

$$\hat{\epsilon}_c = \operatorname{angle}(R(k_0)) \frac{N_c}{2\pi P}, \quad (5)$$

where  $\operatorname{angle}(x)$  denotes the phase angle of the complex argument  $x$ . In (4), the amplitude of  $R(k_0)$  depends not only on the correlation  $L$  but also on the CFO  $\epsilon_c$ . The interval  $P$  between two correlation windows determines the CFO estimation range,  $\pm N_c/2P$ , which can be expanded by decreasing  $P$ . However, the amplitude attenuation caused by  $\epsilon_c$  necessitates the condition  $L \approx P$  to maximize the correlative gain in the CFO acquisition stage [5]. Consequently, sufficient correlative gain requires an appropriate correlation  $L$ , which

restricts the CFO acquisition range.

### 2. Proposed Approach of CFO Acquisition

Theoretically, the estimation range of the CPC-based method reaches  $\pm N_c/2$  when  $P=1$ . According to the constraint  $L \approx P$ , a small  $L$  is chosen; therefore, the correlation peak is “submerged.” Consequently, the CFO estimation cannot be implemented. To expand the acquisition range, we must remove the peak amplitude’s dependence on the CFO. Here, an improved CPC (ICPC) is defined as

$$R_l(n) = \sum_{i=0}^{L-1} \left[ \left( PN^*(i+k_0)r_l(n+i) \right)^* \cdot \left( PN^*(i+k_0+P)r_l(n+P+i) \right) \right]. \quad (6)$$

The correlation peak can be searched at  $n=k_0$ , thus

$$R_l(k_0) = \sum_{i=0}^{L-1} \left[ \left( PN^*(i+k_0)PN(k_0+i)e^{j\frac{2\pi}{N_c}(IN+k_0+i)\epsilon_c} \right)^* \cdot \left( PN^*(i+k_0+P)PN(k_0+P+i)e^{j\frac{2\pi}{N_c}(IN+k_0+P+i)\epsilon_c} \right) \right] = \sum_{i=0}^{L-1} \left( \sigma_{pn}^2 \right)^2 e^{j\frac{2\pi}{N_c} P \epsilon_c} = L \left( \sigma_{pn}^2 \right)^2 e^{j\frac{2\pi}{N_c} P \epsilon_c}. \quad (7)$$

Through searching peaks of the ICPC, the CFO is acquired by

$$\hat{\epsilon}_c = \operatorname{angle}(R_l(k_0)) \frac{N_c}{2\pi P}. \quad (8)$$

In (7), the correlative gain is determined only by  $L$ , which removes the dependence of the gain on the CFO. To compare the gain of the ICPC with that of the CPC, we choose  $L_1$  and  $L_2$  as the correlation  $L$  of the CPC and the ICPC, respectively. Thus, the gain ratio of the ICPC to the CPC is expressed as

$$\gamma = \frac{L_2}{L_1^2 \operatorname{sinc}^2(\epsilon_c L_1 / N_c)}. \quad (9)$$

Considering the maximized CFO as  $\epsilon_c = \pm N_c/2P$ , we have

$$\gamma = \pi^2 L_2 (2P)^{-2} \sin^{-2}(\pi L_1 / (2P)). \quad (10)$$

In (10), the gain ratio is related to  $L_1$  as a sinusoidal law. We obtain the minimized ratio as (11) by choosing  $L_1=P$ .

$$\gamma_{\min} = \pi^2 L_2 (2P)^{-2}. \quad (11)$$

According to (11), the ICPC provides a higher correlative gain only if  $\gamma_{\min} \geq 1$ , that is,  $P \leq \pi \sqrt{L_2}/2$ , indicating that the ICPC is a more appropriate scheme than the CPC for the CFO acquisition when  $|\epsilon_c| \geq N_c / (\pi \sqrt{L_2})$ . Therefore, the proposed method makes it possible to break through the maximum range limit of the CFO estimation in the CPC-based synchronizer.

However, in pursuing a large acquisition range, the CFO estimation precision inevitably worsens. From (8), it follows that the variance of the CFO estimate is proportional to  $1/P^2$  if we assume a certain variance of phase-shift estimation. A smaller  $P$  means a larger acquisition range but more noticeable CFO estimation errors. Therefore, a robust CFO estimator cannot be realized only by decreasing the correlation interval  $P$ . The tradeoff between the range and the precision demands a multistage scheme to realize a robust estimator with a large acquisition range and high-level precision.

For the small-scale acquisition of the CFO, that is,  $|\mathcal{E}_c| < N_c / (\pi\sqrt{L_2})$ , the CPC can yield higher correlative gain than the ICPC. Hence, the multistage scheme can be designed as the combination of two correlation methods, that is, the initial CFO acquisition by the ICPC combined with the small-scale CFO tracking by the CPC. The ICPC is used to acquire the CFO in the initial stage, in which what really counts is the estimation range. When the initial CFO is reduced to a degree that the CPC-based method can handle, the initial stage comes to an end. In the following stages, the CPC-based CFO estimation is invoked. Regarding the configuration of the estimator, refer to [5]. In the CFO tracking stage, the CPC with  $L=L_m$  and  $P=N$ , which can be shared with the channel estimator in [12], is adopted to provide accurate CFO tracking. Note,  $L_m$  is the  $m$ -sequence length in the middle of the TGI.

### III. Numerical Results

We set up a PN-padded OFDM system compatible with the DTMB standard to evaluate the ICPC-based CFO acquisition. As detailed in [5], the main parameters are set as follows:  $N_c=3,780$ ,  $L_m=255$ ,  $L_{PN}=420$ ,  $\Delta f=2$  kHz, and the constellation type is 64-QAM. Since the ICPC-based acquisition is combined with the CPC-based tracking with  $L=255$  and  $P=L_{PN}+N_c$ , the open-loop performance is simulated to examine the acquisition range of the CFO, and estimates are averaged over 100 trials. Regarding the accuracy of CFO tracking, refer to [5].

Due to the more hostile time dispersion in the DVB-T F1 channel, the synchronizer performs worse than in other tested channel models [5]. We hence choose DVB-T F1 as our main channel to test the worst case. Figure 1 gives S-curves of the ICPC-based acquisition with different values of  $P$ , and the result in [5] is listed as a reference. Parameter  $L$  is limited to 20 for the CPC-based curve but set as 230 for the ICPC-based curves when  $P \leq \pi\sqrt{L_2}/2 = \pi\sqrt{230}/2 \approx 24$  to obtain higher correlative gains. Therefore, the ICPC-based method can work in the case of smaller correlation intervals. Given  $P=10$ , the CFO acquisition range is doubled compared with

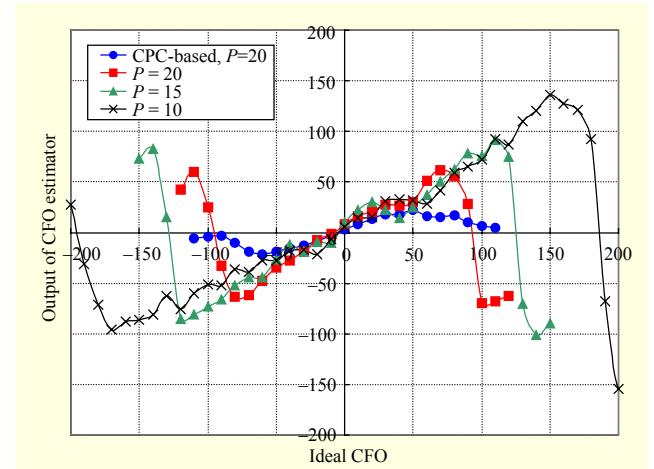


Fig. 1. S-curve of CFO estimator in DVB-T F1 channel with SNR = 0 dB.

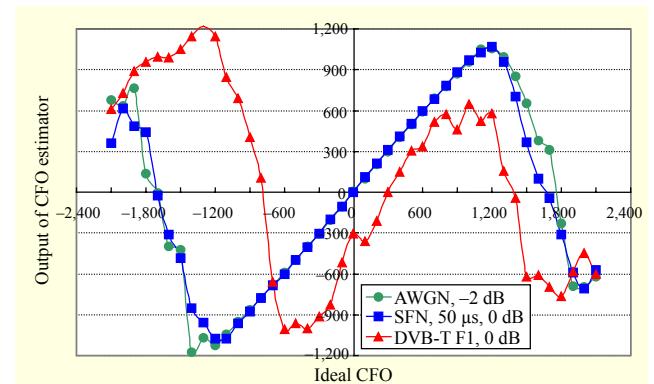


Fig. 2. S-curve in case of  $P=1$  over different channels.

the result in [5]. We also examine the low bound  $P=1$ ; as shown in Fig. 2, the estimator cannot converge in the DVB-T F1 channel. For more favorable channels, such as the tested additive white Gaussian noise (AWGN) and single frequency network (SFN) channels, CFOs within  $\pm N_c/2$  can be acquired, which indicates that the acquisition range of the ICPC method is close to the theoretical limit of CFO estimation, that is, the transmission bandwidth.

Figure 3 shows S-curves of  $P=10$  with different values of  $L$ . The stability of the CFO estimator becomes worse as  $L$  decreases. The CFO acquisition becomes invalid when  $L=120$ . Accordingly, if the  $L_{PN}$  permits, we choose the longest possible  $L$  to maximize correlative gains. In the DTMB system, the ICPC-based synchronizer of the PN595 mode or the PN945 mode outperforms that of the PN420 mode since  $L$  can be increased further.

The verification of the closed-loop CFO tracking is illustrated in Fig. 4. The ideal CFO is set to  $160 \times 2 = 320$  kHz. The three-stage CFO estimation scheme is adopted as follows:

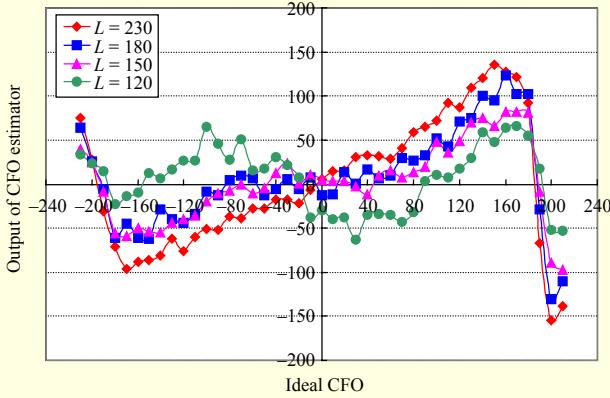


Fig. 3. S-curve of CFO estimator with different correlation widths in DVB-T F1 channel with SNR = 0 dB.

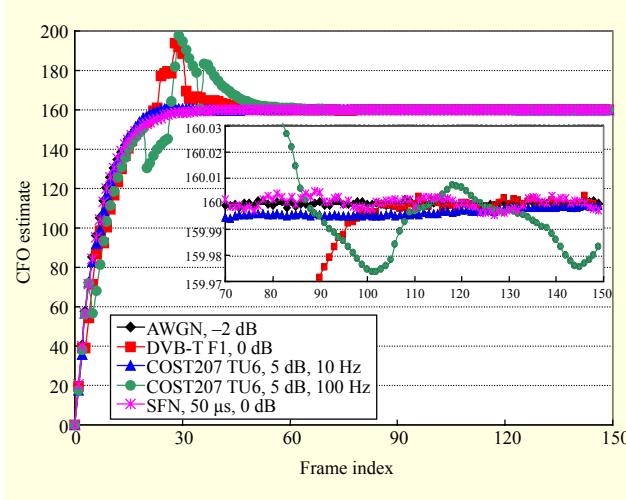


Fig. 4. Closed-loop tracking of CFO.

The first stage is the ICPC with  $L=230$  and  $P=10$ ; The second stage is the CPC with  $L=230$  and  $P=250$ ; The third stage is the CPC with  $L=255$  and  $P=4,200$ . The closed-loop tracking of the CFO is examined in five channel models, as was in [5]. As shown in Fig. 4, the estimator converges very rapidly. The precision is ideal for OFDM demodulation [5], even though an obvious fluctuation is observed in the fast fading channel.

#### IV. Conclusion

To expand the acquisition range of a CFO estimator, an improved CPC was presented to obtain correlation peaks, of which the magnitude is independent of the CFO. By removing the parameter's dependence between  $P$  and  $L$ , a significant CFO acquisition range was achieved by an ICPC-based CFO estimator with a small  $P$  and a large  $L$ . Simulation results show that the improved scheme is quite effective and overcomes the

shortcomings of the CPC-based method.

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