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Complexity Results for the Design Problem of Content Distribution Networks

Byung-Cheon Choi
Department of Business Administration, Chungnam National University
Jibok Chung*
Department of Business Administration, Daejeon University

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ABSTRACT

Content Delivery Network (CDN) has evolved to overcome a network bottleneck and improve user perceived Quality of Service (QoS). A CDN replicates contents from the origin server to replica servers to reduce the overload of the origin server. CDN providers would try to achieve an acceptable performance at the least cost including the storage space or processing power. In this paper, we introduce a new optimization model for the CDN design problem considering the user perceived QoS and single path (non-bifurcated) routing constraints and analyze the computational complexity for some special cases.

Keywords: CDN, Computational Complexity, Single Path

* Corresponding Author, E-mail: jibok5343@gmail.com

1. INTRODUCTION

An increasing large demand of traffic has brought a congestion and bottleneck of web services such as Skype, YouTube, Netflix and video data. Video-on-demand (VOD) traffic will nearly triple by 2017. The amount of VoD traffic in 2017 will be equivalent to 6 billion DVDs per month and CDN traffic will deliver almost two-thirds of all video traffic by 2017 (Cisco, 2012).

A CDN is a collection of network elements arranged for more effective delivery of contents to end-user. In the context of CDN, contents refer to any digital data resources (Pallis and Vakali, 2006). To improve the network performance, a CDN replicates the same contents or services over several replica servers strategically placed at various locations. If a requested content can be retrieved in a nearby replica server, the user will not need to contract the remote origin server to get the requested content.

CDN operators charge their customers according to the contents delivered to the end-users by their replica servers. It is known that the most influencing factors affecting the price of CDN services include bandwidth cost, variation of traffic distribution, and size of content replicated, number of replica servers, reliability and stability of delivery system (Pallis and Vakali, 2006). CDN operators would try to achieve an acceptable performance at the least cost including the storage space or processing power. Therefore, a key issue for the CDN operator is to determine the best CDN topology considering the potential server location, various cost and capacity of each server, user demand for contents and service performance (Nguyen *et al*., 2003).

Researches about the optimal location of replica server can be found in (Jamin *et al*., 2001; Kangasharju *et al*., 2001; Laoutaris *et al*., 2004; Venkataramani *et al*., 2001; Xu *et al*., 2002), while researches about the overall design of CDN can be found in (Bektas *et al*., 2007; Jason *et al*., 2012; Nguyen *et al*., 2003; Walkowiak, 2004). Jason *et al*. (2012) proposed a multi-objective formulation considering the content allocation and routing cost simultaneously. Walkowiak (2004) showed the complexity results for the CDN design problem under MPLS protocol.

Nguyen *et al*. (2003) proposed an optimization model for CDN design and some heuristic approaches. Multipath routing model of Nguyen *et al*. (2003) can increase the throughput for single-source, single-destination applications. However, CDNs' clients may access the contents from multiple servers, which makes single path routing model less effective considering the increasing dynamics of demand (Minlan *et al*., 2012).

The remainder of this paper is organized as follows: in Section 2, our problem is defined formally. In Section 3, we establish the computational complexity of our problem. In Section 4, we develop the polynomial-time algorithm or show the NP-hardness for two special cases. Finally, we provide the concluding remarks

2. NOTATIONS AND PROBLEM DEFINITION

In this section, we introduce the notations and the definition of our problem. The notations used in this paper are as follows:

m : number of candidate replica server;
n : number of customers;
l : number of the contents types;
D : total demand from all customers;
C_i : capacity of replica server i;
$\lambda_{j,k}$: amount of demand by customer j for content k;
T : threshold of user perceived QoS;
β_i : serving cost per unit at server i;
γ_i : setup cost for server i;
$d_{i,j}$: distance between nodes i and j

Let

$$x_{i,j,k} = \begin{cases} 1, \text{ if order } (j,k) \text{ is delivered from serve} \\ 0, \quad \text{otherwise} \end{cases}$$

and $y_i = \begin{cases} 1, \text{ if replica server } i \text{ is selected} \\ 0, \quad \text{otherwise} \end{cases}$

In our problem, unlike the previous research, the constraint for the QoS (Quality of Service) threshold is defined as follows:

$$\frac{\sum_{i=1}^{m}\sum_{k=1}^{l}\lambda_{j,k}d_{i,j}x_{i,j,k}}{\sum_{k=1}^{l}\lambda_{j,k}} \le T, \quad \forall j.$$

Our problem can be formulated as follows:

$$Min\ z(x,y) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\lambda_{j,k}\beta_i x_{i,j,k} + \sum_{i=1}^{m}\gamma_i y_i$$

$$s.t.\ \sum_{j=1}^{n}\sum_{k=1}^{l}\lambda_{j,k}x_{i,j,k} \le C_i,\ \forall i$$

$$\frac{\sum_{i=1}^{m}\sum_{k=1}^{l}\lambda_{j,k}d_{i,j}x_{i,j,k}}{\sum_{k=1}^{l}\lambda_{j,k}} \le T,\ \forall j$$

$$\sum_{j=1}^{m} x_{i,j,k} = 1,\ \forall j, k$$

$$\sum_{i=1}^{m} C_i y_i \ge D$$

$$x_{i,j,k} \le y_i,\ \forall i, j, k$$

Let the problem above be referred to as *Problem P*. The relationships between potential replica servers and customers can be described as the graph below, denoted $G(M, N)$ and, furthermore, one between C_i and $\lambda_{j,k}$ as follows:

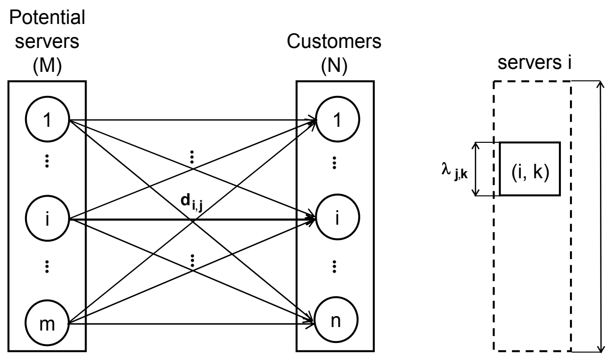

Figure 1. $G(M, N)$ and the Relationship with Demand (j, k) and Server i

3. COMPUTATIONAL COMPLEXITY OF PROBLEM P

In this section, we prove that the Problem P is strongly NP-hard.

Theorem 1: *The decision version of Problem P is NP-complete in the strong sense, even if*
(i) the number of contents type is one, that is, $i = 1$;
(ii) the capacity of each server is identical, that is, $C_i = C$;
(iii) the amount of request per unit time for customer j is identical, that is, $\lambda_{j,k} = \lambda$.

Proof: It can be proved by the reduction from the exact cover by 3-sets (X3C) problem, which is proven to be NP-complete in the strong sense (Garey and Johnson, 1979).

The X3C problem can be stated as follows: Given a set $X = \{1, 2, \cdots, 2q\}$ and a collection $\boldsymbol{S} = \{\boldsymbol{S}_1, \boldsymbol{S}_2, \cdots, \boldsymbol{S}_g\}$ of 3-element subset of X, is there a subcollection $\boldsymbol{S}' \subseteq \boldsymbol{S}$ such that each element of X occurs in exactly one subset in $\boldsymbol{S}'$?

For simplicity, let $\boldsymbol{S}_i = \{\pi_i(1), \pi_i(2), \pi_i(3)\}$, $i = 1, 2, \cdots, g$. Note that $|\boldsymbol{S}'| = q$ and $\{\pi_1(1), \pi_1(2), \pi_1(3)\} \subset X$, $i = 1, 2, \cdots, g$.

The decision version of Problem P can be stated as follows: Given a threshold Z^*, is there a feasible solution (x, y) with $z(x, y) \le z^*$?

It is clear that the decision version of Problem P is in NP. Given an instance of the X3C problem, we can construct the instance as follows: Let $m = g$, $n = 3$ and $l = 1$. For $i = 1, 2, \cdots, g$, let $\beta_i = 0, \gamma_i = 1, C = 3,$

$$d_{i,j} = \begin{cases} 1, \text{ if } j \in \{\pi_i(1), \pi_i(2), \pi_i(3)\} \\ 3, \quad \text{otherwise} \end{cases}$$

Note that for $i = 1, 2, \cdots, g$, $\boldsymbol{S}_i$ and $\boldsymbol{S}_i = \{\pi_i(1), \pi_i(2), \pi_i(3)\}$ are corresponding to server i and three customers in $\{1, 2, \cdots, 3q\}$ respectively. Note that since $l = 1$, we can drop an index k. For $j = 1, 2, \cdots, 2q$, let $\lambda_j = 1$. Let $T = 2, D = 3q$, and $z^* = q$. Henceforth, we show that there exists a feasible solution (x, y) with $z(x, y) \le z^*$ if and only if there exists a feasible set to the X3C problem.

Suppose that there exists a feasible subcollection $\bar{\boldsymbol{S}}' = \{\bar{\boldsymbol{S}}_1', \bar{\boldsymbol{S}}_2', \cdots, \bar{\boldsymbol{S}}_q'\}$ for the X3C problem. Let $\bar{\boldsymbol{M}}$ be the set of indices of the servers corresponding to $\bar{\boldsymbol{S}}'$. Let $(\bar{x}, \bar{y})$ be such that, for $i = 1, 2, \cdots, g$,

$$\bar{x}_{i,j} = \begin{cases} 1 \text{ if } i \in \bar{\boldsymbol{M}} \text{ and } j \in \{\pi_i(1), \pi_i(2), \pi_i(3)\} \\ 0 \quad \text{otherwise} \end{cases}$$

and $\bar{y}_i = \begin{cases} 1 & \text{if } i \in \bar{\boldsymbol{M}} \\ 0 & \text{otherwise} \end{cases}$

Then, it is observed that our constraints are satisfied. Thus, $(\bar{x}, \bar{y})$ is a feasible solution. Then, since $|\bar{\boldsymbol{S}}'| = q$, $z(\bar{x}, \bar{y}) = \sum_{i=1}^{g} \bar{y}_i = q$.

Suppose that there exists a feasible solution $(\hat{x}, \hat{y})$. such that $z(\hat{x}, \hat{y}) \le q$. Note that server i' cannot serve customer j' if $j' \notin \{\pi_{i'}(1), \pi_{i'}(2), \pi_{i'}(3)\}$, because $d_{i',j'} > T$. Furthermore, each server node can serve at most three customers because of $C = 3$. Let $\hat{\boldsymbol{M}}$ be the set of the indices of the server selected by $(\hat{x}, \hat{y})$. Since each server can serve at most three customer, the number of the selected server should be at least q, and thus,

$$z(\hat{x}, \hat{y}) = \sum_{i=1}^{g} \hat{y}_i \ge q.$$

Since $z(\hat{x}, \hat{y}) \le q$, $z(\hat{x}, \hat{y}) = q$.

Let $\hat{\boldsymbol{S}}'$ be the subcollection corresponding to $\hat{\boldsymbol{M}}$. Then, $\hat{\boldsymbol{S}}'$ becomes the feasible solution to the X3C problem. ■

4. TWO SPECIAL CASES

In this section, we consider some special cases of Problem P. Since Problem P is strongly NP-hard as we have analyzed in section 3, we consider some special cases of Problem P and investigate when Problem P is polynomially solvable or not.

4.1 When $l = 1$ and $\lambda_{j,k} = \lambda$

In this case, if $d_{i,j} > T$, then server i cannot deliver contents to customer j. Thus, edge (i, j) can be deleted from $G(M, N)$. For simplicity, furthermore, since the number of types of contents is one, an index k can be deleted.

Let $G(M, N)$ be referred to as the graph with *interval properties* if the following conditions are satisfied: Let $i < i'$ and $j < j'$

(i) if node i in M is connected to nodes j and j' in N, then node i is connected to nodes in $\{j+1, j+2, \cdots, j'-1\}$;

(ii) if nodes i and i' in M is connected to nodes j' and j in N, respectively, then nodes i and i' are connected to nodes in $\{j+1, j+2, \cdots, j'-1\}$.

For instance, the graph below has the interval properties.

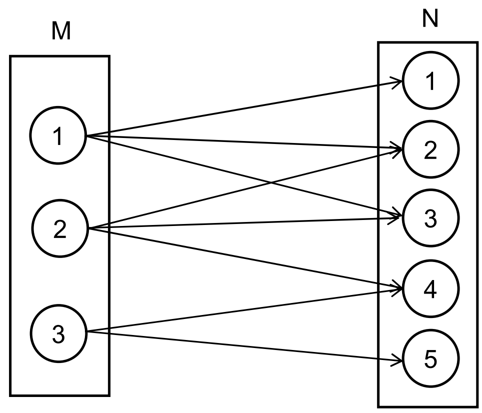

Figure 2. Example of $G(M, N)$ with the interval properties

Let $G(M, N, x)$ be the graph constructed by connecting node i in M to node j in N if $x_{i,j} = 1$ in x.

Lemma 1: *If $G(M, N)$ has the interval properties, then there exists an optimal solution (x^*, y^*) such that $G(M, N, x^*)$ has the interval properties.*

Proof: Let $i < i'$ and $j < j'$. Suppose that $x^*_{i,j'} = 1$ and $x^*_{i',j} = 1$. Then, we can construct a new solution $(\bar{x}, y^*)$ by letting $\bar{x}_{i,j} = 1$, $\bar{x}_{i',j} = 0$, $\bar{x}_{i,j'} = 0$ and $\bar{x}_{i',j'} = 1$. Then, it is observed that $z(\bar{x}, y^*) = z(x^*, y^*)$. By applying this argument repeatedly, we can obtain Lemma 1. ■

Let n_i be the maximum number of orders allocated to server i, which can be calculated as follows: For $i = 1, 2, \cdots, m$,

$$n_i = \min\left\{\delta_i, \frac{C_i}{\lambda}\right\}.$$

where δ_i is the number of egdes coming from server i.

Based on Lemma 1, henceforth, we reduce Problem P to the shortest path problem. Let $N(g, n)$ denote the node that represents the following:
(i) server g is selected for the delivery of contents;
(ii) servers in $\{1, 2, \cdots, g-1\}$ have been considered as the nodes delivering the contents. Note that some nodes in $\{1, 2, \cdots, g-1\}$ deliver the contents while the others do not deliver them;
(iii) customers in $\{1, 2, \cdots, n\}$ receive the contents from some severs in $\{1, 2, \cdots, g-1\}$ including server g.

Let $N(0, 0)$ and $N(end, end)$ be the source and the sink nodes, respectively. For $g \neq m$ and $h \neq n$, let $N(g, h)$ be connected to $N(g', h')$ with length $\left(\lambda \beta_{g'}(h'-h)+\gamma_{g'}\right)$ for $g' = g+1, g+2, \cdots, m$ and $h' = h+1, h+2, \cdots, \min\left\{n, h+n_{g'}\right\}$. This edge means that
(i) server g' is selected for the delivery of contents;
(ii) servers in $\{g+1, g+2, \cdots, g'-1\}$ are not selected for the delivery of contents;
(iii) customers in $\left\{h+1, h+2, \cdots, \min\left\{n, h+n_{g'}\right\}\right\}$ 5receive the contents from sever g', which requires the cost, calculated as $\left(\lambda\beta_{g'}(h'-h)+\gamma_{g'}\right)$.

For $g = m$ and $h \neq n$, let $N(g, h)$ be connected to $N(end, end)$ with length ∞. This edge means that since all servers have been considered while the demand for customers has not yet met, the feasible solution for Problem P cannot be constructed from current status.

For $h = n$, let $N(g, h)$ be connected to $N(end, end)$ with length zero. This edge means that since the demand for customers has already been met, the feasible solution for Problem P has been constructed, and, furthermore, the additional cost is not required.

The objective is to find the shortest path from the source to the sink nodes. Note that the server corresponding to the first index in the node in the shortest path are selected for the delivery of contents, and from the second index in its immediately previous node and its second index, it is possible to find which customers should receive the contents from that server. This can be due to Lemma 1.

Theorem 2: *Problem P with the interval properties is polynomially solvable if*
(i) the number of contents type is one, that is, $l = 1$;
(ii) the amount of request per unit time for customer j is identical, that is, $\lambda_{j,k} = \lambda$.

Proof: The number of nodes in the reduced graph is at most $O(mn)$. Since the number of edges emanating from each nodes in M is at most $O(n)$, the total number of edges is at most $O(mn^2)$. Note that the above reduction can be done in polynomial time. Since the reduced graph is acyclic, this shortest path problem can be solved in $O(mn^2)$ by the algorithm in (Ahuja *et al*., 1990). The proof is complete. ■

4.2 When $C_i = 1$, $n = 1$ and $\lambda_{j,k} = 1$

Theorem 3: *Problem P is NP-hard, even if*
(i) each server can have exactly one content, that is, $C_i = 1$;
(ii) the number of customers is one, that is, $n = 1$;
(iii) the amount of demand for content k is identical, that is, $\lambda_{i,k} = 1$.

Proof: It can be proved by the reduction from the equal cardinality partition (ECP) problem, which is proven to be NP-complete (Garey and Johnson, 1979).

The ECP problem can be stated as follows: Given 2g positive integers $b_1, b_2, \cdots, b_{2g}$ such that $\sum_{j=1}^{2g} b_j = 2B$, is there a set $S \subseteq \{1, 2, \cdots, 2g\}$ such that $|S| = g$ and $\sum_{j\in S} b_j = B$?

The decision version of Problem P can be stated as follows: Given a threshold z^*, is there a feasible solution (x, y) with $z(x, y) \leq z^*$?

It is clear that the decision version of Problem P is in NP. Given an instance of the ECP problem, we can construct the instance as follows: Let $m = 2g$, $n = 1$ and $l = g$. For $k = 1, 2, \cdots, g$, let $\lambda_k = 1$. For $i = 1, 2, \cdots, 2g$, let $\beta_i = b_i/2$, $\gamma_i = b_i/2$, $C_i = 1$ and $d_i = M - b_i$, where $M > 0$ is a sufficiently large value. Note that since the number of customers is one, we use d_i and λ_k instead of $d_{i,j}$ and $\lambda_{j,k}$, respectively. Let $T = M - B/g$, $D = g$ and $z^* = B$. Henceforth, we show that there exists a feasible solution (x, y) with $z(x, y) \leq z^*$ if and only if there exists a feasible set to the ECP problem. Note that this instance is formulated as follows:

$$
\begin{aligned}
Min\ z(x, y) &= \sum_{i=1}^{2g}\sum_{k=1}^{g}\frac{b_i}{2}x_{i,k} + \sum_{i=1}^{2g}\frac{b_i}{2}y_i \\
s.t. &\sum_{k=1}^{g} x_{i,k} \leq 1, \quad \forall i \\
&\sum_{i=1}^{2g}\sum_{k=1}^{g}(M-b_i)x_{i,k} \leq gM - B \\
&\sum_{i=1}^{2g} x_{i,k} = 1, \quad \forall k \\
&\sum_{i=1}^{2g} y_i \geq g \\
&x_{i,k} \leq y_i, \qquad \forall i,k
\end{aligned}
$$

Suppose that there exists a set $\bar{S} \subseteq \{1, 2, \cdots, 2g\}$ such that $|\bar{S}| = g$ and $\sum_{j\in\bar{S}} b_j B$. For simplicity, let $\bar{S} = \{\bar{\pi}(1), \bar{\pi}(2), \cdots, \bar{\pi}(g)\}$. Then, we can construct a feasible solution $(\bar{x}, \bar{y})$ to our problem such that

$$
\bar{x}_{i,k} = \begin{cases} 1 & if\ (i,k) = (\bar{\pi}(h), h)\ for\ h = 1, 2, \cdots, g \\ 0 & otherwise \end{cases}
$$

and $$y_i = \begin{cases} 1 & if\ i \in \bar{S} \\ 0 & otherwise \end{cases}$$

Then, it is easy to see by the reduction scheme that constraints except the second are satisfied by $(\bar{x}, \bar{y})$ and for the second constraint, by $\sum_{k=1}^{g}\overline{x}_{i,k}=\overline{y}_i$, $i=1,2,\cdots,g$, $\sum_{i=1}^{2g}b_i\overline{y}_i=B$ and $\sum_{i=1}^{2g}\overline{y}_i=g$,

$$\sum_{i=1}^{2g}\sum_{k=1}^{g}(M-b_i)\overline{x}_{i,k}=\sum_{i=1}^{2g}(M-b_i)\overline{y}_i=gM-B$$

Thus, $(\overline{x},\overline{y})$ is a feasible solution to Problem P. Furthermore,

$$z(\overline{x},\overline{y})=\sum_{i=1}^{2g}\sum_{k=1}^{g}\frac{b_i}{2}\overline{x}_{i,k}+\sum_{i=1}^{2g}\frac{b_i}{2}\overline{y}_i=\sum_{i=1}^{2g}\overline{y}_i=B$$

Suppose that there exists a feasible solution $(\hat{x},\hat{y})$ with $z(\hat{x},\hat{y})\le z^*$. Under $(\hat{x},\hat{y})$, set $\{1,2,\cdots,2g\}$ can be partitioned as follows:

$$S_0=\left\{i\,\middle|\,\sum_{k=1}^{g}\hat{x}_{i,k}=\hat{y}_i=0\right\}$$
$$S_1=\left\{i\,\middle|\,\sum_{k=1}^{g}\hat{x}_{i,k}=0\ \ and\ \ \hat{y}_i=1\right\}$$
$$S_2=\left\{i\,\middle|\,\sum_{k=1}^{g}\hat{x}_{i,k}=\hat{y}_i=1\right\},$$

Note that by the fifth constraint of the formulation above,

$$\left\{i\,\middle|\,\sum_{k=1}^{g}\hat{x}_{i,k}=1\ \ and\ \ \hat{y}_i=0\right\}=\varnothing$$

Since the number of customers is one and the capacity of each potential server is one, it is observed that at most g servers to deliver the contents should be selected in $(\hat{x},\hat{y})$, that is,

$$|S_2|\ge g \tag{1}$$

$z(\hat{x},\hat{y})\le z^*$ and the second constraint can be rewritten as follows:

$$\sum_{i\in S_2}b_i+\sum_{i\in S_1}\frac{b_i}{2}\le B\ \ and\ \ \sum_{i\in S_2}(M-b_i)\le gM-B \tag{2}$$

If $S_1=\varnothing$, then by inequalities above, $|S_2|<g$. This is a contradiction. Furthermore, if $|S_2|<g$, then the second constraint is not satisfied by $(\hat{x},\hat{y})$, because $M>0$ is a sufficiently large value. This is a contradiction to the feasibility of $(\hat{x},\hat{y})$. Thus, it is observed from these contradictions and inequality (1) that in $(\hat{x},\hat{y})$,

$$S_1=\varnothing\ \ and\ \ |S_2|<g \tag{3}$$

By relations (3), inequalities (2) can be rewritten as follows:

$$\sum_{i\in S_2}b_i\le B\ \ and\ \ \sum_{i\in S_2}b_i\ge B$$

Thus S_2 is the solution to the ECP problem. ■

5. CONCLUDING REMARKS

We consider a new optimization model with the user-oriented QoS, which is defined newly in this paper. We prove its strong NP-hardness. Thus, we consider two special cases such that

(i) in the first case, $l=1$ and $\lambda_{j,k}=\lambda$;
(ii) in the second case, $C_i=1$, $n=1$ and $\lambda_{i,k}=\lambda$.

We show that the first case can be solved in polynomial time, and the second case is NP-hard.

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