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렌탈 운영에서 용량 유연성 확보가 기업의 수익성에 미치는 영향

김 은 갑[†]
이화여자대학교 경영대학

The Impact of Capacity Flexibility in a Rental Operation on the Financial Performance

Eungab Kim
College of Business Administration, Ewha Womans University

■ Abstract ■

We present a new framework for rental capacity management in which rental capacity is dynamically managed by means of temporary inventory addition/return. While serving customers with its own (native) capacity, the rental firm rents additional rental capacity from an upper echelon rental company so that it can avoid lost sales which may occur when stock is not sufficient, and returns it when stock becomes sufficiently large enough to cope with demands. Formulating the model as a Markov decision process, we investigate a flexible capacity addition/return policy that maximizes the firm's profit with respect to system costs. Numerical study indicates that rental operation with capacity addition/return can be economically favorable over rental operation without capacity expansion/return and can contribute the reduction in the size of native rental capacity.

Keywords : Rental Operation, Flexible Capacity Management, Reverse Logistics, Capacity Expansion/Reduction, Inventory management

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† 교신저자, evanston@ewha.ac.kr

1. 서 론

Knowing how much rental inventory to maintain is critical to any rental business operation. In the context of a rental business capacity, which is the maximum amount of serviced fulfillment that can be attained, is equated with the amount of rental inventory. Too much rental capacity can reduce the cash position of a rental firm while too little rental capacity may turn away customers to its competitors. Since demand and rental processes are typically uncertain over any period of time, it is essential to develop effective policies for controlling the rental capacity. This paper presents a new framework for rental capacity management in which rental capacity is dynamically adjusted by means of temporary capacity addition/return. While serving customers with its own capacity, the rental company rents additional capacity from the upper echelon rental firm so that it can avoid lost sales which may occur when stock is insufficient, and returns it when stock becomes sufficiently large enough to cope with demands.

The capacity expansion and management literature is relevant to the model presented in this paper. Because we consider both capacity expansion and return, however, our model differs from capacity expansion models with growing demands over time [2, 4, 14, 15, 17]. Another research area relevant to our work is the capacity operation problem. Rocklin et al. [16] studied the problem of capacity expansion/contraction in a production/service facility with stochastic demands and showed that the optimal policy is characterized by two threshold values. Rajagopalan and Soteriou [13] considered a firm producing multiple items in a multi-period environment and explored the interaction between production planning and capacity acquisition decisions. They develop an integer programming model and an effective solution approach to determine the optimal capacity acquisition, production and inventory decisions over time. So and Tang [18] considered a problem of managing congestion in two types of service systems and investigated a policy that dynamically adjusts operating capacity according to the system state using queueing models. Angelus and Porteus [1] studied the issue of determining capacity size and production planning in a produce-to-stock facility. Under instantaneous capacity additions and reductions, they showed that a target interval policy is optimal for capacity management, provided that demands stochastically increase up to a peak and then decrease,

Inventory management with product returns is the other important area of research relevant to this problem. The reader is referred to Fleischmann et al. [7] and Fleischmann and Kuik [8] for the detailed literature review in this area. Our model differs from product recovery models in two aspects. First, all products issued in our model return (because they are rented), which means that the return rate is affected by the number of products issued. In contrast, product return models have partial returns from the products issued and it is often assumed that the demand and return processes are independent. Second, we consider both capacity (inventory) augmentation and reduction while product return models consider capacity (inventory) augmentation only by means of stock replenishment and remanufacturing.

The capacity management model presented in this paper and the cash flow management models [6, 11, 12] have some similar features. However, the return and demand processes in our model are correlated whereas cash inflow (return process) and cash outflow (demand process) in the cash flow management models are not directly correlated.

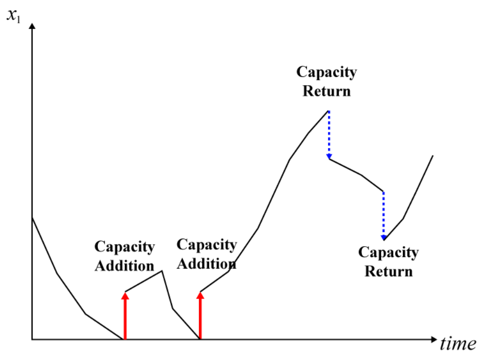
Finally, Kim and Byun [9] considered a special case of our model. They assume that a firm cannot have multiple batches of capacity expansion simultaneously. We extend their model in the sense that a firm can rent additional batch as long as it is economically favorable. Hence, the structure of capacity management policy becomes much more complicated.

The paper is organized as follows. In the next section, we provide a formulation of our model. Numerical analysis of the optimal capacity management policy is given in Section 3. Section 4 presents the performance evaluation of rental operations with and without flexibility. Finally we state our conclusions in the last section.

2. Problem Formulation

In this paper, we raise important strategic issues related to rental capacity management with dynamic capacity adjustment. More specifically, we address the following research questions : (i) When should the rental firm schedule its capacity addition via renting capacity from the upper echelon rental firm? (ii) While operating the expanded capacity, when should the rental firm return it to the upper echelon rental firm? (iii) How do changes in problem parameters affect the optimal capacity addition/return decision? (iv) What is the economic value of rental operation with capacity addition/return? We deal with these issues via a Markov decision process (MDP) model. Even though the MDP model may be restrictive for high-fidelity modeling of real world rental problems, it is a powerful vehicle for generating strategic-level insights into the effective rental capacity management.

We consider a rental firm, denoted by Company 1, that operates N rental items and decides over time when to acquire and return additional items to accommodate fluctuations in rental demand. We can say that it has N units of native capacity but can extend this capacity. Customers arrive at Company 1 according to a Poisson process with rate λ^o. Each customer's rental period is modeled as an exponentially distributed random variable with mean $1/\mu^o$. The exponential rental duration is appropriate when the duration is random with significant variation. In particular, if the rental duration is flexible and extendable, it is known that the exponential approximation is reasonable (Yuan, 1998). It is assumed that rental items are not depreciated in time, all items are identical in terms of rental service, and all items are returned in serviceable condition. Each arriving customer rents exactly one unit of item, provided stock is available. At all times, unfulfilled customer demands are lost and charged a lost sales cost of c_L. Each satisfied customer pays p^o per unit of time during which she is renting an item. The firm incurs a holding cost of c_h^o per unit of time during which each item is held in stock. [Figure 1] shows a typical inventory sample path under a rental operation with capacity return and expansion.



[Figure 1] An Inventory Sample Path Under a Rental Operation with Inventory Flexibility

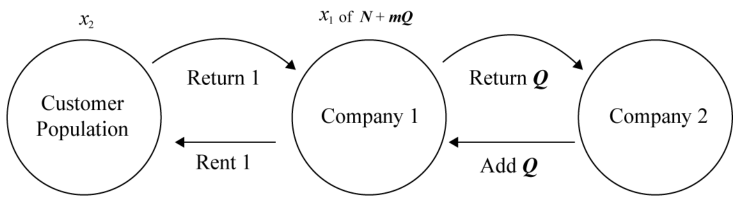
Regarding rental capacity addition/return we make the following assumptions :

(1) Company 1 acquires (rents) batches of $Q(\geq 1)$ items from an upper echelon rental firm, denoted by Company 2, at a fixed lump-sum setup cost c_E and a variable cost c_k^Q per item per unit time that a batch of Q items is rented by Company 1. Our model can be viewed as a (r, nQ) model where if the inventory position drops below r, the system orders a quantity of the smallest integer multiple of batch size Q so as to bring the inventory position above r (see Chen and Zheng [5] for the applications of a (r, nQ) model).

(2) Company 1 reduces its expanded capacity by Q units at a time. When Company 1 returns Q items to Company 2, a fixed lump-sum cost c_R is incurred.

(3) Both the capacity addition/return processes are instantaneous. In other words, if Company 1 decides to rent (return) a batch of Q units, it is immediately added to (removed from) its inventory. Therefore, capacity expansion is allowed only when stock of rental items becomes empty. Although it is a restrictive assumption, our model can be applied to the practical problems, provide that Company 2 has a sufficiently large number of rental items in stock and the delivery time is short. However, if Company 2 needs a production of rental items upon Company 1's rental request, the impact of the stochastic capacity addition/return process on the rental capacity management policy can be crucial.

(4) Company 1 limits the maximum extended capacity level to MQ for some M in the naturals. If Company 1 is already operating MQ extended rental items, no capacity expansion is allowed even though stock is empty. Hence, if a demand occurs when stock is empty, it is lost and a cost of c_L is incurred. In light of the analysis, the assumption of limited maximum extended capacity level makes the size of state space finite, which is necessitated for the existence of the optimal policy of the MDP problem [3].



[Figure 2] A New Rental Management Model with Capacity Addition/Return

A state is described by the vector (x_1, x_2, m) where x_1 and x_2 respectively denote the number of rental items in stock and the number of items rented by customers, and m is the number of batches of size Q that Company 1 is renting from Company 2 (if $m=0$, nothing is rented). The state space is denoted by Γ. At each epoch of customer return, Company 1 decides whether or not to reduce its expanded capacity, if any, by returning to Company 2. [Figure 2] graphically illustrates a rental operation with capacity addition/return.

Denote the state at $t=0$ by (x_1, x_2, n) and the interest rate by α. Then, the expected discounted cost given (x_1, x_2, n) over an infinite horizon under a rental capacity management policy, π, can be written as

$$J^{\pi}(x_1, x_2, m) = \lim_{W\to\infty} E\Big[\int_0^W e^{-\alpha t}\big(p^o x_2(t) - c_h^o x_1(t) - c_k^o mQ - c_L 1\{t\in B_1^{\pi}(W)\} - c_E 1\{t\in B_2^{\pi}(W)\} - c_R 1\{t\in B_3^{\pi}(W)\}\big)dt \,|\, (x_1, x_2, m)\Big] \quad (1)$$

where $1\{a\}=1$ if a is true, otherwise, 0. In (1), $B_1^{\pi}(W)$, $B_2^{\pi}(W)$, and $B_3^{\pi}(W)$ respectively denote the set of random instances on $[0, W]$ of the demand rejection, capacity extension, and capacity return under policy π. Then, the goal of this paper is to find an optimal rental capacity management policy π^* that maximizes the following expected discounted costs over an infinite horizon:

$$J(x_1, x_2, m) \equiv J^{\pi^*}(x_1, x_2, m) \equiv \min_{\pi} J^{\pi}(x_1, x_2, m). \quad (2)$$

Our rental capacity management problem can be formulated as a discrete-time Markov decision problem by using uniformization (see [10]). The essence of uniformization makes the transition rates of all states equal by allowing the fictitious self-loop transition. The uniformized version of our model has a transition rate $\gamma=\lambda^o+(N+MQ)\mu^o$ for all states. The expected length of time per state transition becomes γ^{-1} and the discount factor during γ^{-1} is given by $\beta=\gamma/(\gamma+\alpha)$. The goal of this paper is to find a capacity return policy that maximizes Company 1's profit subject to the system costs. Let operators D and I correspond to a customer arrival and a customer return, respectively. Then,

$$D(x_1,x_2)=(x_1-1, x_2+1) \text{ if } x_1>0;\ (x_1,x_2) \text{ otherwise},$$
$$I(x_1,x_2)=(x_1+1, x_2-1) \text{ if } x_2>0;\ (x_1,x_2) \text{ otherwise}.$$

Let $p=\beta p^o/\gamma$, $c_k=\beta c_k^o/\gamma$, $c_h=\beta c_h^o/\gamma$, $\lambda=\beta\lambda^o/\gamma$, and $\mu=\beta\mu^o/\gamma$ for notational convenance and $1(a)=1$ if a is true, 0 otherwise. We define the one stage expected profit in state (x_1, x_2, m) as $g(x_1, x_2, m) = px_2 - c_h x_1 - c_k mQ - \lambda c_L 1(x_1=0\wedge m=M) - \lambda c_E 1(x_1=1\wedge 0\le m<M)$ where px_2 and $c_h x_1$ respectively represent discounted rental revenue and holding cost, and $c_k mQ$ is a discounted cost of renting mQ items until the next state transition. If a customer arrives at the system when $x_1=0$ and $m=M$, it is lost with a cost of c_L. If a customer arrives at the system when $x_1=1$ and $0\le m<M$, the inventory becomes empty and thus Company 1 should expand its capacity with a cost of c_E.

Since the expected profit during the expected transition time is bounded, the optimal total discounted profit function J can be shown to satisfy the following optimality equation which defines the value iteration operator T :

$$\begin{aligned} &TJ(x_1, x_2, m) \quad (3)\\ &= g(x_1, x_2, m)+\lambda[J(Q, N+mQ, m+1)\\ &\quad 1(x_1=1\wedge 0\le m<M)+J(D(x_1, x_2), m)\\ &\quad 1(x_1\ne 1\vee m=M)]+\max[x_2\mu J(I(x_1, x_2), m)\\ &\quad +(N+MQ-x_2)\mu J(x_1, x_2, m), x_2\mu J(I(x_1-Q, x_2), m-1\\ &\quad +(N+MQ-x_2)\mu J(x_1-Q, x_2, m)-(N+MQ)\mu c_R]\\ &\quad 1(x_1\ge Q\wedge x_2>0\wedge 1\le m\le M)+[x_2\mu J(I(x_1, x_2), m)\\ &\quad +(N+MQ-x_2)\mu J(x_1, x_2, m)]1(x_1<Q\vee x_2=0\vee m=0) \end{aligned}$$

In (3), the terms multiplied by λ represent transitions generated by the arrival of a demand. A state transition associated with a demand arrival in state (x_1, x_2, m) can be one of the following three cases : (i) If a demand occurs when $x_1 = 1$ and $0 \le m < M$, the stock level becomes zero and Company 1 immediately rents a batch of Q units from Company 2. (ii) If a demand occurs when $x_1 = 0$ and $m = M$, it is lost at a cost of c_L. (iii) Otherwise, a demand arrival decreases x_1 by one and increases x_2 by one. A state transition associated with a customer return in state (x_1, x_2, m) can be one of the following two cases : (i) If a customer return occurs when $x_1 \ge Q$, $x_2 > 0$, and $1 \le m \le M$, x_1 is increased by one and x_2 is decreased by one. And then, Company 1 should decide whether or not to reduce its expanded capacity. If Company 1 chooses to return, Q items are immediately returned to Company 2, which incurs c_R. (ii) When $x_1 < Q$ or $m = 0$, a customer return increases x_1 by one and decreases x_2 by one without capacity return considered. In both cases, the terms multiplied by $(N + MQ - x_2)\mu$ represents a self-transition to make the state transition rate equal.

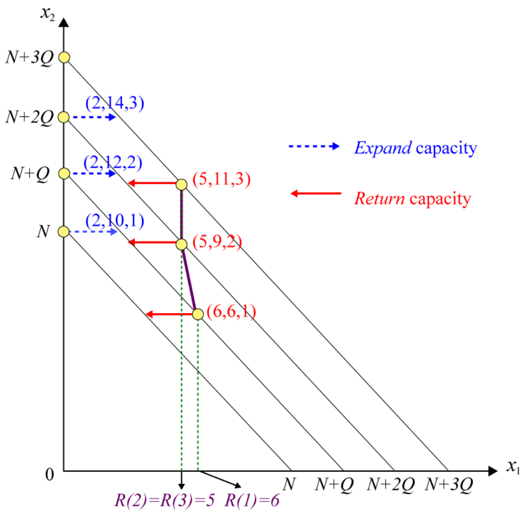
In addition to characterizing the structure of the optimal policy, another important problem that Company 1 faces is how to select the optimal values N^*, M^*, and Q^* that maximize Company 1's profit. To find these optimal parameters, we need to solve (1) for all possible values in N, M, and Q because concavity of J with respect to these parameters cannot be guaranteed. This issue will be discussed in Section 4.

3. Optimal Rental Capacity Management Policy

Let

$$R(m) := \min\left\{ \begin{matrix} x_1 > Q: J(x_1, x_2 + Q, m) \\ - J(x_1, x_2, m-1) < -c_R \end{matrix} \right\}, \ 1 \le m \le M.$$

$R(m)$ is the smallest value of the inventory level which makes the action of returning Q rented units to Company 2 more profitable than the action of not returning them. [Figure 3] graphically illustrates the optimal return policy for an example with $N = 10$, $M = 3$, $Q = 2$, $p^o = 100$, $c_h^o = 20$, $c_k^o = 60$, $c_E = 400$, $c_R = 400$, $c_L = 80$, $\lambda^o = 0.7$, $\mu^o = 0.05$, and $\beta = 0.95$. When state $(5, 11, 3)$ is reached while Company 1 is renting $3Q\ (= 6)$ units from Company 2, the optimal policy is to reduce Company 1's expanded capacity by Q units and the state is transited to state $(3, 11, 2)$. In state $(0, 14, 2)$, Company 1 expands its capacity by Q units and the state immediately is transited to $(2, 14, 3)$. In state $(6, 6, 1)$, it is optimal to reduce Company 1's expanded capacity by Q units and the state is transited to $(4, 6, 0)$. In this example, $R(1) = 6$ and $R(2) = R(3) = 5$. The optimal return policy of [Figure 3] was found using value iteration, which will be presented in Section 4



[Figure 3] Optimal Capacity Expansion and Return Policy

If the system starts in a state on the left side of the threshold $R(m)$, $1 \le m \le M$, we note that the stock level will never move right across the boundary $R(m)$. To see this, suppose that Company 1 is renting $3Q$ units from Company 2 and the system starts in state (4, 12, 3). There are following two possible transitions. First, corresponding to a demand occurrence the stock level decreases by one and (4, 12, 3) is transited to (3, 13, 3). Second, corresponding to a customer return, the stock level increases by one and the number of items in rent by customer decreases by one (i.e. (5, 11, 3)). Hence, (4, 12, 3) is transited to (3, 11, 2). All the transition makes the system move within the left side of $R(m)$.

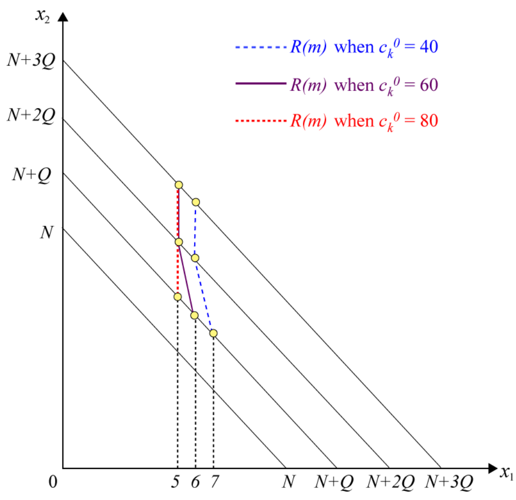
Based on the numerical investigation, we conjecture the following :

(i) Whenever the stock level reaches $R(m)$ upon a customer return, Company 1 should reduce the currently expanded capacity of mQ to $(m-1)Q$ by returning Q units to Company 2, and

(ii) $R(m) \geq R(m+1)$, $1 \leq m \leq M-1$, that is, the optimal return curve becomes non-strictly lower as Company 1 operates more expanded capacity.

The first part of the conjecture implies that the optimal return curve $R(m)$ separates two regions in the state space when $m \geq 1$. Part (ii) shows that as the size of the expanded capacity rent by Company 1 increases, it is optimal to advance the timing to reduce the expanded capacity.

Our numerical investigation also indicates that $R(m)$ will change as a function of the system costs. For example, [Figure 4] illustrates that $R(m)$ might be non-decreasing as c_k^o decreases. This result can be explained intuitively. The decrease in c_k^o makes operating expanded capacity less expensive. Hence, it may become optimal to increase $R(m)$ (i.e. delay capacity return) and keep expanded capacity longer. The example used in [Figure 4] is identical to the one used in [Figure 3] except that c_k^o varies.



[Figure 4] Effect of c_k^o on $R(m)$

In addition to this, we observe that $R(m)$ is non-decreasing as p^o and c_E increase, respectively, and it is non-increasing as c_h^o increases. The explanations of these observation are intuitively clear. If the setup cost c_E gets larger, the policy will work towards making less use of capacity expansion before. Since the size of expanded capacity is reduced, however, this action can break the balance of rental operation established by the optimal combination of native and expanded rental capacity. To minimize it, the policy will prefer to delaying the return timing of expanded capacity by increasing $R(m)$. Gaining insight into the sensitivity of $R(m)$ for changes in c_h^o is similar to the case of c_k^o. Finally, the increase in p^o will push Company 1 to keep expanded capacity to a greater degree to reduce the possibility of stockout. Hence, it is likely that the optimal return curve will be higher than before.

4. Numerical Study

In this section, we discuss the issue of simultaneously finding N^*, M^*, and Q^* that maximize Company 1's rental operation profit. Since it is intractable to analytically find these optimal values, they should be found based on a three-dimensional search. Our numerical study focuses on examining (i) to what extent the proposed new capacity management policy is effective(i.e. the value of capacity/inventory flexibility) and (ii) how much N^*, M^*, and Q^* are affected when the values of system parameters are changed. For the sake of explanation, we evaluate the optimal average profit per unit time, denoted by g, rather than the optimal discounted profit J.

We generate four sets of scenarios by varying the values of these parameters. In <Table 1>, we cover situations where the holding cost rate for rental units in stock varies. <Table 2> allows us to consider cases where capacity addition/return setups are inexpensive (50), medium (200), and very expensive (400). In <Table 3>, the variable costs of renting the extended capacity are such that they are 50% of, 70% of, and 90% of the rental revenue rate. We do not test a case where c_k is higher than p because it may not be realistic to consider that situation. Finally, we are able to examine the effect of the rental revenue rate by varying its values in <Table 4>. With these scenario setting we also cover situations with five types of customer demand rates. The expected rental duration is fixed to $1/\mu^o$ = 20 and a lost sales cost is set to 400. Since the expected rental duration is 20 and the rental revenue rate is 100, a lost sales cost of 400 can be a reasonable assumption.

Since it is not guaranteed that he optimal total discounted profit function J can be convex or concave with respect to N, M, and Q, a three dimensional search is required for searching N^*, M^*, and Q^*. For this three dimensional search, we set $1 \le N \le 30$, $0 \le M \le 8$, and $1 \le Q \le 3$. The CPU times of a personal computer to find N^*, M^*, and Q^* are within seconds for the examples in tables 1-4%. in tables 1-4 is defined as the average profit change in percent between rental operations with and without capacity addition/return to the performance under rental operation without capacity addition/return.

〈Table 1〉 Performance Evaluation as a Function of c_h^o

Ex	$p^o=100,\ c_k^o=60,\ c_E=200,$ $c_R=200,\ c_L=400,\ \mu^o=0.05$		With Flexibility				Without Flexibility		
	c_h^o	λ^o	N^*	M^*	Q^*	g	N^*	g	%
1	20	0.4	12	5	2	677	13	666	1.7
2		0.5	15	4	2	865	15	849	1.9
3		0.6	17	4	2	1055	18	1035	1.9
4		0.7	19	5	2	1245	21	1222	1.9
5		0.8	22	5	2	1436	23	1410	1.8
6	40	0.4	11	6	1	597	12	574	3.9
7		0.5	13	6	2	779	14	749	3.8
8		0.6	15	5	2	962	16	924	3.9
9		0.7	18	5	2	1146	19	1101	3.9
10		0.8	20	5	2	1332	21	1280	3.9
11	50	0.4	10	6	1	564	11	539	4.3
12		0.5	13	8	2	741	13	707	4.7
13		0.6	15	5	2	923	16	877	5.0
14		0.7	17	5	2	1106	18	1051	5.0
15		0.8	19	6	2	1289	20	1225	5.0

〈Table 2〉 Performance Evaluation as a Function of c_E^o and c_R^o

Ex	$p^o=100,\ c_h^o=40,\ c_k^o=60,\ c_L=400,\ \mu^o=0.05$			With Flexibility				Without Flexibility		
	c_E^o	c_R^o	λ^o	N^*	M^*	Q^*	g	N^*	g	%
1	50	50	0.4	10	8	1	625	11	576	7.8
2			0.5	12	8	1	808	14	749	7.3
3			0.6	15	8	1	993	16	924	6.9
4			0.7	17	8	1	1179	19	1101	6.6
5			0.8	19	8	1	1366	21	1280	6.3
6	200	200	0.4	11	6	1	597	11	576	3.6
7			0.5	13	6	2	779	14	749	3.8
8			0.6	15	5	2	962	16	924	3.9
9			0.7	18	5	2	1146	19	1101	3.9
10			0.8	20	5	2	1332	21	1280	3.9
11	400	400	0.4	11	3	2	579	11	576	0.5
12			0.5	14	3	2	758	14	749	1.2
13			0.6	16	3	2	940	16	924	1.7
14			0.7	18	5	2	1123	19	1101	2.0
15			0.8	20	5	2	1307	21	1280	2.0

〈Table 3〉 Performance Evaluation as a Function of c_k^o

Ex	$p^o=100,\ c_h^o=40,\ c_E=200,\ c_R=200,\ c_L=400,\ \mu^o=0.05$		With Flexibility				Without Flexibility		
	c_k^o	λ^o	N^*	M^*	Q^*	g	N^*	g	%
1	50	0.4	11	8	2	604	11	576	4.6
2		0.5	13	5	2	786	14	749	4.7
3		0.6	15	5	2	971	16	924	4.8
4		0.7	17	5	2	1155	19	1101	4.7
5		0.8	19	6	2	1341	21	1280	4.5
6	70	0.4	11	5	1	593	11	576	2.8
7		0.5	13	5	2	771	14	749	2.9
8		0.6	16	5	2	954	16	924	3.2
9		0.7	18	5	2	1139	19	1101	3.4
10		0.8	20	5	2	1325	21	1280	3.3
11	90	0.4	11	3	1	584	11	576	1.4
12		0.5	14	4	1	762	14	749	1.7
13		0.6	16	4	2	943	16	924	2.0
14		0.7	18	4	2	1126	19	1101	2.2
15		0.8	20	4	2	1309	21	1280	2.2

〈Table 4〉 Performance Evaluation as a Function of p^o

Ex	$c_h^o=40$, $c_k^o=60$, $c_E=150$, $c_R=150$, $c_L=400$, $\mu^o=0.05$		With Flexibility				Without Flexibility		
	p^o	λ^o	N^*	M^*	Q^*	g	N^*	g	%
1	100	0.4	11	8	1	605	11	576	4.8
2		0.5	13	8	1	786	14	749	4.7
3		0.6	15	8	2	969	16	924	4.7
4		0.7	17	7	2	1154	19	1101	4.6
5		0.8	19	6	2	1340	21	1280	4.5
6	120	0.4	11	8	1	765	12	726	5.1
7		0.5	13	8	1	986	14	938	4.9
8		0.6	15	8	2	1209	17	1152	4.8
9		0.7	17	7	2	1434	19	1369	4.6
10		0.8	19	7	2	1660	21	1585	4.5
11	140	0.4	11	8	1	925	12	878	5.1
12		0.5	13	8	1	1186	15	1127	5.0
13		0.6	15	8	2	1449	17	1382	4.7
14		0.7	17	7	2	1714	19	1636	4.6
15		0.8	19	7	2	1980	22	1895	4.3

4.1 Effectiveness of Capacity Expansion/Return Strategy

Test results in tables 1-4 suggest that rental operation with capacity expansion/return can be economically favorable over rental operation without capacity expansion/return. They show that rental operation with capacity expansion/return can be more effective particularly when (i) inventory holding cost is higher (see <Table 1>), (ii) lump-sum setup costs of capacity addition/return are smaller (see <Table 2>), (iii) variable cost of renting expanded units is lower (see <Table 3>), and (iv) rental revenue rate is larger(see <Table 4>). The average profit difference of 60 examples for the rental operation with and without flexibility is 3.9%. In particular, when the lump-sum setup costs for capacity addition/return are inexpensive, this figure reaches at about 7%. This computational experiment demonstrates that rental operation with flexibility has the economical advantages over rental operation without flexibility.

In addition to the increase in rental operation profit, numerical results show that under rental operation with capacity expansion/return it may be possible to reduce the size of native rental capacity. In most of test examples, rental operation with capacity expansion/return has a lower N^* than rental operation without capacity expansion/return. This tendency becomes more apparent when either c_E and c_R are small or c_k is low. We have no other test examples where rental operation with capacity expansion/return has a higher N^* than rental operation without capacity expansion/return. When rental companies increase the level of native rental capacity from the acquisition, it incurs large investment cost and extra costs of inventory and maintenance. In contrast, by allowing capacity flexibility, companies can reduce their fixed costs and face less risk associated with lower native capacity level. The trade-off is in the variable and setup costs related to rental capacity flexibility.

4.2 Observations on the Sensitivity of N^*, M^*, and Q^* to Model Parameters

Optimal selection of N, M, and Q should be determined based on how best to use the option of rental capacity addition/return, which can be also varied according to what the values of system parameters are. Our numerical experience suggests several meaningful properties that are held between system parameters and N^*, M^*, and Q^*, which we briefly mention. These tendencies add to our intuition and may be very useful in developing heuristic formulas in the future research.

Test results in tables 1-4 indicate that as λ increases N^* increases under both rental operations with and without capacity expansion/return. They also indicate that M^* tends to be non-decreasing (but not monotonically) for many examples, which means that Company 1 should utilize more extended capacity to accommodate increased customer demands. In the numerical study, we observe that Q^* is hardly affected by the change of values in system parameters. In most of test examples, $Q^*=2$. Effectively corresponding to increased demands can be done by two different types of actions. One option is to increase the batch size Q and the other is to increase the maximum extended capacity level M. The former increases rental variable cost and holding cost, and the latter increases the lump-sum setup costs related to capacity expansion and return. Test results imply that the second option is more cost-effective than the first option. Based on the observations of the numerical study, we further provide the following additional managerial insights for rental operation with capacity expansion/return :

(1) As c_h increases, N^* and Q^* are non-increasing and % value is larger : If c_h increases, the optimal size of native rental capacity will be reduced for both rental operations with and without capacity addition/return in order to avoid excessive holding costs of native rental capacity. In case of rental operation with capacity addition and return, this deficit in rental capacity could be made up by increasing the maximum extended capacity level, M. Even though increasing M^* implies a greater cost related to capacity addition/return, it is reasonable to expect that increased rental revenue will sufficiently offset this incremental cost. Further, the value of inventory flexibility will become greater as the optimal size of native rental capacity becomes reduced. However, our numerical test indicates that M^* is not monotonically increased in c_h.

(2) As c_E and c_R increase, N^* and Q^* are non-decreasing, M^* is non-increasing, and % value is smaller : When c_E and c_R increase, Company 1 has a motivation to cut down total setup costs related to capacity addition/return by lowering the maximum extended capacity level M. To make up this deficit in expanded capacity, the size of native capacity will be increased. In addition, as c_E and c_R increase, rental operation with capacity addition/return becomes expensive and thus its performance becomes worse.

(3) As c_k increases, N^* is non-decreasing, M^* and

Q^* are non-increasing, and % value is smaller : Like increasing c_E and c_R, increasing c_k will result in less use of capacity addition/return. In addition, smaller batch size of capacity addition will be preferred because operating expended capacity becomes expensive. The combined result will deteriorate the performance of rental operation with capacity addition/return because the effect of reducing the size of native rental capacity is diminished

(4) As p increase, M^* is non-decreasing, and % value is larger : If p increases, it will be desirable to utilize more capacity addition/return by increasing the maximum extended capacity level M instead of increasing the size of native rental capacity N.

5. Conclusions

In this paper, we have developed a stochastic capacity management model for a rental firm with random demand and return processes. The proposed model was tested in an extensive numerical study. When the system has a higher inventory holding cost, a smaller setup cost of capacity addition/return, a lower variable cost of renting expanded units, or a larger rental revenue rate, our computational results show that the economical favor of rental operation with capacity addition/return can be much larger over rental operation without capacity expansion/return. The numerical experiment also reveals that under rental operation with capacity expansion/return it may be possible to reduce the size of native rental capacity. Combined with the simple structure, these numerical results support the value of rental capacity flexibility in practice.

The model presented in this paper can be applied to the practices with industrial and commercial products such as trucks, cars, and computers. Typically, a rental company generates revenue when the rental capacity is available upon a customer's demand and experiences a loss when the stock is empty. Therefore, the suggested dynamics of managing rental capacity can contribute to the enhancement of the profits of the rental companies that face uncertain customer rental demands and returns.

The primary contributions of our work to the reverse logistics and capacity management literature are summarized as follows. We analyzed a model which considers the dependency of return process on demand process and the capacity management control simultaneously and deals with both capacity augmentation and reduction. Our model also differs from the capacity management literature by treating a situation where the product is returned after use. Several issues remain to be explored. First, we assume that the batch size of capacity expansion is fixed. However, in many instances, it may be desirable to change this batch size dynamically. Second, this paper focuses on the instantaneous delivery process upon capacity expansion. Another important future research is to examine stochastic delivery processes. In such cases, there is a richer issue of the proper timing of capacity expansion.

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