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STARLIKENESS OF $q-$DIFFERENTIAL OPERATOR INVOLVING QUANTUM CALCULUS

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Abstract. In the present paper, we investigate starlikeness conditions for $q-$differential operator by using the concept of quantum calculus in the unit disk.

1. Introduction

The study of hypergeometric functions plays a vital role in mathematics and nowadays it becomes popular among the researchers be it in real or complex case. The contributions given by the earliest researchers such as Euler, Gauss, Riemann and many others have great impact to the human kind. They obtained many interesting results which are applicable to many areas such as the combinatorics, numerical analysis, dynamical analysis and the mathematical physics. Here we will study on basic hypergeometric functions (or $q-$hypergeometric functions). The $q-$hypergeometric level can generalize many results for the classical hypergeometric functions. The generalization $q-$Taylor's formula in fractional $q-$calculus introduced by Purohit and Raina [15],

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where certain $q-$generating functions for $q-$hypergeometric functions were derived. Recently, Chung et al. [18] evaluated the q-Laplace images of a number of q-polynomials and generalized basic hypergeometric functions of one and more variables.

Let $\mathcal{H}=\mathcal{H}(\mathbb{U})$ denote the class of functions $f(z)$ which are analytic in the open unit disk

$$\mathbb{U}:\{z:z\in\mathbb{C}\ and\ |z|<1\}.$$

For a positive integer j and $a\in\mathbb{C}$, in [11] the authors defined the following classes of analytic functions:

$$\mathcal{H}[a,j]=\left\{f\in\mathcal{H}:f(z)=a+a_jz^j+a_{j+1}z^{j+1}+...,z\in\mathbb{U}\right\}, \tag{1}$$

and

$$\mathcal{A}_j=\left\{f\in\mathcal{H}:f(z)=z+a_{j+1}z^{j+1}+a_{j+2}z^{j+2}+...,z\in\mathbb{U}\right\}. \tag{2}$$

It is clear that $\mathcal{A}_1=\mathcal{A}$ the normalized class with

$$f(z)=z+\sum_{k=2}^{\infty}a_kz^k. \tag{3}$$

A $q-$hypergeometric function is a power series in one complex variable z with power series coefficients which depend, apart from q on r complex upper parameters $a_1,a_2,...,a_r$ and s complex lower $b_1,b_2,...b_s$ as follows (See Gasper and Rahman [8])

$$\begin{aligned}&{}_r\Omega_s\left(a_1,...a_r;b_1,...b_s,q,z\right)\\&=\sum_{k=0}^{\infty}\frac{(a_1,q)_k...(a_r,q)_k}{(q,q)_k(b_1,q)_k...(b_s,q)_k}\left[(-1)^kq^{\binom{k}{2}}\right]^{1+s-r}z^k,\end{aligned} \tag{4}$$

with $\binom{k}{2}=\frac{k(k-1)}{2}$, where $q\neq0$ when $r>s+1$, $\big(r,s\in\mathbb{N}_0=\mathbb{N}\cup\{0\};z\in\mathbb{U}\big),\mathbb{N}$ denote the set of positive integers and $(a,q)_q$ is the $q-$shifted factorial defined by

$$(a,q)_k=\begin{cases}1, & k=0;\\(1-a)(1-aq)(1-aq^2)...(1-aq^{k-1}), & k\in\mathbb{N}.\end{cases}$$

By using the ratio test, one recognize that, if $|q| < 1$, the series (4) converges absolutely (see Gasper and Rahman [8]) and Ghany [9]) for all z if $r \leq s$ and for $|z| < 1$ if $r = s+1$. For brief survey on $q-$hypergeometric functions, one may refer to [2], [3], [7], [19], [20] and [21].

The $q-$ derivative of a function $h(x)$ is defined by

$$\mathcal{D}_q(h(x)) = \frac{h(qx) - h(x)}{(q-1)x},\ q \neq 1, x \neq 0.$$

For a function $h(z) = z^k$ observe that

$$\mathcal{D}_q(h(z)) = \mathcal{D}_q(z^k) = \frac{1-q^k}{1-q} z^{k-1} = [k]_q z^{k-1}.$$

Note that $lim_{q\to 1}\mathcal{D}_q(h(z)) = lim_{q\to 1}[k]_q z^{k-1} = kz^{k-1} = h'(z)$, where $h'(z)$ is the ordinary derivative and $[k]_q$ is the $q-$number. For more properties of $\mathcal{D}_q$ see [4] and [10].

Now for $z \in \mathbb{U}, |q| < 1$, and $r = s+1$, the $q-$ hypergeometric function defined in (4) takes the form

$${}_r\Psi_s\left(a_1, ..., a_r; b_1, ..., b_s, q, z\right) = \sum_{k=0}^{\infty} \frac{(a_1, q)_k ... (a_r, q)_k}{(q, q)_k (b_1, q)_k ... (b_s, q)_k} z^k,$$

which converges absolutely in the open unit disk $\mathbb{U}$.

Corresponding to a function ${}_r\Lambda_s(a_i; b_j; q, z)$ defined by

$${}_r\Lambda_s(a_i; b_j; q, z) = z\, {}_r\Psi_s(a_i; b_j; q, z) = z + \sum_{k=2}^{\infty} \frac{(a_1, q)_{k-1} ... (a_r, q)_{k-1}}{(q, q)_{k-1}(b_1, q)_{k-1} ... (b_s, q)_{k-1}} z^k,$$

where $i = 1, ..., r,\ j = 1, ..., s,\ a_i, b_j \in \mathbb{C}, b_j \in \mathbb{C}\backslash\{0, -1, -2, ...\}$.

Mohammed and Darus [13] defined the operator $\mathcal{M}_s^r f(z)$ by using $q-$hypergeometric function. In this paper, we modify the operator $\mathcal{M}_s^r f(z)$ as follows:

for $z \in \mathbb{U}$, $|q| < 1$ and $r = s+1$, then

$$\begin{aligned}
\mathcal{H}^0_{r,s,\lambda}(a_i, b_j; q)f(z) =& f(z) = z + \sum_{k=2}^{\infty} a_k z^k \\
\mathcal{H}^1_{r,s,\lambda}(a_i, b_j; q)f(z) =& (\beta - \lambda)\mathcal{M}^r_s f(z) + \lambda z D_q(\mathcal{M}^r_s f(z)) + (1-\beta)z \\
=& z + \sum_{k=2}^{\infty} ((\beta + \lambda([k]_q - 1))\Upsilon_k)\, a_k z^k \\
&\vdots \\
\mathcal{H}^n_{r,s,\lambda}(a_i, b_j; q)f(z) =& \mathcal{H}^1_{r,s,\lambda}\left(\mathcal{H}^{n-1}_{r,s,\lambda}(f(z))\right),
\end{aligned}$$

$$= z + \sum_{k=2}^{\infty} ((\beta + \lambda([k]_q - 1))\Upsilon_k)^n\, a_k z^k, \tag{5}$$

for $\Upsilon_k = \frac{(a_1,q)_{k-1} \cdots (a_r,q)_{k-1}}{(q,q)_{k-1}(b_1,q)_{k-1} \cdots (b_s,q)_{k-1}}$, $\lambda \geq 0$, and $\beta \geq 1$.

Remark

- For $n = 1, \lambda = 0$ and $\beta = 1$ we get the linear operator introduced and studied recently by Mohammed and Darus [13].
- For $r = s+1$, $n = 1, a_i = q^{\alpha_i}, b_j = q^{\beta_j}, \alpha_i, \beta_j \in \mathbb{C}$, $\beta_j \neq 0, -1, -2, ..., (i = 1, ..., r, j = 1, ..., s), \lambda = 0, \beta = 1$ and $q \to 1$, we receive the well-known Daziok-Srivastava linear operator [6].
- For $r = 1, s = 0, a_1 = q, \lambda = 1, \beta = 1$ and $q \longrightarrow 1$, we obtain *Sălăgean* differential operator (see [17]).
- For $r = 1, s = 0, a_1 = q, \beta = 1$ and $q \longrightarrow 1$, we obtain Al-Obudi differential operator (see [1]). Many differential operators studied by various authors can be seen in the literature (see for examples [5], [16]).

DEFINTION 1.1. A function $f(z) \in \mathcal{A}$ is said to be starlike of order α if it satisfies

$$Re\left\{\frac{z\mathcal{D}_q f(z)}{f(z)}\right\} > \alpha \quad for \quad 0 \leq \alpha < 1,$$

where $\mathcal{D}_q$ is the $q-$derivative and we denote the set of all starlike functions of order α by $\mathcal{S}^*_q(\alpha)$.

Observe that $lim_{q\to 1}\mathcal{S}_q^*(\alpha)=\mathcal{S}^*(\alpha)$.

Our considerations are based on the following results which can be found in [11].

LEMMA 1.1. *Let $f(z)\in\mathcal{A}_j$ and let $0\le\alpha<1$. If f satisfies*

$$|zf''(z)|<\frac{j(j+1)(1-\alpha)}{j+1-\alpha},$$

then $f(z)$ is starlike of order α in $\mathbb{U}$.

LEMMA 1.2. *Let $f(z)\in\mathcal{A}_j$, $0\le\gamma<j$ and $0\le\alpha<1$. If f satisfies*

$$|zf''(z)-\gamma(f'(z)-1)|<\frac{(j+1)(1-\alpha)(j-\gamma)}{j+1-\alpha},$$

then $f(z)$ is starlike of order α in $\mathbb{U}$.

Miller and Mocanu [12] have investigated some second-order differential inequality that implies starlikeness and deduced the following lemma.

LEMMA 1.3. *Let $f(z)\in\mathcal{A}_j$ and let $0\le\gamma<j$. If f satisfies*

$$|zf''(z)-\gamma(f'(z)-1)|<j-\gamma,$$

then $f(z)$ is starlike in $\mathbb{U}$.

In this paper, we introduce a generalization for the previous lemmas by employing q-derivative. Some analytic and geometric properties are obtained.

2. Main Results

First, we generalize lemmas 1.1, 1.2 and 1.3 by using q-derivative as following.

LEMMA 2.1. *Let $f(z)\in\mathcal{A}_j$ and let $0\le\alpha<1$. If f satisfies*

$$|z\mathcal{D}_q^2 f(z)|<\frac{j(j+1)(1-\alpha)}{j+1-\alpha},$$

then $f(z)$ is starlike of order α in $\mathbb{U}$.

Proof. When $q \to 1$, then we have

$$|zf''(z)| < \frac{j(j+1)(1-\alpha)}{j+1-\alpha},$$

In view of Lemma 1.1, we deduce that $f(z)$ is starlike of order α in $\mathbb{U}$. □

LEMMA 2.2. *Let $f(z) \in \mathcal{A}_j$, $0 \le \gamma < j$ and $0 \le \alpha < 1$. If f satisfies*

$$|z\mathcal{D}_q^2 f(z) - \gamma(\mathcal{D}_q f(z) - 1)| < \frac{(j+1)(1-\alpha)(j-\gamma)}{j+1-\alpha},$$

then $f(z)$ is starlike of order α in $\mathbb{U}$.

Proof. . The proof is similar to the proof of lemma 2.1. □

LEMMA 2.3. *Let $f(z) \in \mathcal{A}_j$ and let $0 \le \gamma < j$. If f satisfies*

$$|z\mathcal{D}_q^2 f(z) - \gamma(\mathcal{D}_q f(z) - 1)| < j - \gamma,$$

then $f(z)$ is starlike in $\mathbb{U}$.

Proof. . The proof is similar to the proof of lemma 2.1. □

Next, we establish the sufficient conditions to obtain a starlikeness for analytic functions involving the differential operator (5).

THEOREM 2.1. *Let $f(z) \in \mathcal{A}$. If for $\lambda \ge 0$*

$$(1+q)(\beta+\lambda q)^n |\Upsilon_2|^n |a_2| < \frac{1}{2}, \quad 0 < q < 1,$$

and

$$\sum_{k=2}^{\infty} [k]_q([k]_q + q^k)\left(\beta + \lambda([k]_q - 1 + q^k)\right)^n |\Upsilon_{k+1}|^n |a_{k+1}| \le (1+q)(\beta+\lambda q)^n |\Upsilon_2|^n |a_2|. \tag{6}$$

Then the operator $\mathcal{H}^n_{r,s,\lambda}(a_i, b_j; q) f(z)$ is starlike of order α, where

$$\alpha < \frac{1 - 2(1+q)(\beta+\lambda q)^n |\Upsilon_2|^n |a_2|}{1 - (1+q)(\beta+\lambda q)^n |\Upsilon_2|^n |a_2|}, \tag{7}$$

and Υ_2 is given by $\frac{(1-a_1)\dots(1-a_r)}{(1-q)(1-b_1)\dots(1-b_s)}$.

Proof. For $f(z) \in \mathcal{A}$, we have

$$
\begin{aligned}
&|z\mathcal{D}_q^2\left(\mathcal{H}_{r,s,\lambda}^n(a_i,b_j;q)f(z)\right)| \\
&= \left|\sum_{k=1}^{\infty}[k]_q[k+1]_q\left((\beta+\lambda([k]_q-1+q^k))\Upsilon_{k+1}\right)^n a_{k+1}z^k\right| \\
&\leq \sum_{k=1}^{\infty}[k]_q[k+1]_q\left(\beta+\lambda([k]_q-1+q^k)\right)^n|\Upsilon_{k+1}|^n|a_{k+1}| \\
&= (1+q)(\beta+\lambda q)^n|\Upsilon_2|^n|a_2| + \sum_{k=2}^{\infty}[k]_q[k+1]_q\left(\beta+\lambda([k]_q-1+q^k)\right)^n \\
&\qquad |\Upsilon_{k+1}|^n|a_{k+1}|.
\end{aligned}
$$

By the conditions (6) and (7) and since $[k+1]_q = [k]_q + q^k$, then in view of Lemma 2.1, we impose

$$
\begin{aligned}
|z\mathcal{D}_q^2\left(\mathcal{H}_{r,s,\lambda}^n(a_i,b_j;q)f(z)\right)| \leq& 2(1+q)(\beta+\lambda q)^n|\Upsilon_2|^n|a_2| \\
<& \frac{2(1-\alpha)}{2-\alpha},
\end{aligned}
$$

which proves that $\mathcal{H}_{r,s,\lambda}^n(a_i,b_j;q)f(z)$ is starlike of order α in $\mathbb{U}$. □

For the case $\lambda = 0, \beta = 1, r = 1, s = 0, a_1 = q$ and $q \to 1$ in the above theorem, we find the following result.

Corollary 2.1. *For $f(z) \in \mathcal{A}$, and let $0 \leq \alpha < 1$, we have that*

$$
|zf''(z)| < \frac{2(1-\alpha)}{2-\alpha} \Rightarrow f(z) \in S^*(\alpha),
$$

where was defined by [11].

The case $\alpha = 0$ was first discussed by Obradović [14].

Theorem 2.2. *Let $f(z) \in \mathcal{A}$. If for $\lambda \geq 0$*

$$
(1+q)(\beta+\lambda q)^n|\Upsilon_2|^n|a_2| < \frac{1}{2}, \quad 0 < q < 1,
$$

and

(8)

$$
\begin{aligned}
&\sum_{k=2}^{\infty}\left([k]_q\left([k]_q-\gamma+q^k\right)-\gamma q^k\right)\left(\beta+\lambda([k]_q-1+q^k)\right)^n|\Upsilon_{k+1}|^n|a_{k+1}| \\
&\quad \leq (1-\gamma)(1+q)(\beta+\lambda q)^n|\Upsilon_2|^n|a_2|.
\end{aligned}
$$

Then the operator $\mathcal{H}^n_{r,s,\lambda}(a_i, b_j; q)f(z)$ *is starlike of order* α*, where*

$$\alpha < \frac{1-2(1+q)(\beta+\lambda q)^n|\Upsilon_2|^n|a_2|}{1-(1+q)(\beta+\lambda q)^n|\Upsilon_2|^n|a_2|},\ 0 \le \gamma < 1, \tag{9}$$

and Υ_2 *is given by* $\frac{(1-a_1)...(1-a_r)}{(1-q)(1-b_1)...(1-b_s)}$.

Proof. We note that

$$\begin{aligned}
&|z\mathcal{D}_q^2\left(\mathcal{H}^n_{r,s,\lambda}(a_i, b_j; q)f(z)\right) - \gamma\left(\mathcal{D}_q\left(\mathcal{H}^n_{r,s,\lambda}(a_i, b_j; q)f(z)\right)-1\right)| \\
&= \left|\sum_{k=1}^{\infty}[k+1]_q([k]_q-\gamma)\left(\beta+\lambda([k]_q-1+q^k)\Upsilon_{k+1}\right)^n a_{k+1}z^k\right| \\
&\le \sum_{k=1}^{\infty}[k+1]_q([k]_q-\gamma)\left(\beta+\lambda([k]_q-1+q^k)\right)^n|\Upsilon_{k+1}|^n|a_{k+1}| \\
&= (1-\gamma)(1+q)(\beta+\lambda q)^n|\Upsilon_2|^n|a_2| \\
&\quad + \sum_{k=2}^{\infty}[k+1]_q([k]_q-\gamma)\left(\beta+\lambda([k]_q-1+q^k)\right)^n|\Upsilon_{k+1}|^n|a_{k+1}|
\end{aligned}$$

From the assumptions (8) and (9) and taking in mind $[k+1]_q = [k]_q + q^k$, then by Lemma 2.2, we obtain

$$\begin{aligned}
&|z\mathcal{D}_q^2\left(\mathcal{H}^n_{r,s,\lambda}(a_i, b_j; q)f(z)\right) - \gamma\left(\mathcal{D}_q\left(\mathcal{H}^n_{r,s,\lambda}(a_i, b_j; q)f(z)\right)-1\right)| \\
&\le 2(1-\gamma)(1+q)(\beta+\lambda q)^n|\Upsilon_2|^n|a_2| \\
&< \frac{2(1-\gamma)(1-\alpha)}{2-\alpha}.
\end{aligned}$$

Then $\mathcal{H}^n_{r,s,\lambda}(a_i, b_j; q)f(z)$ is starlike of order α. □

If we take $\lambda = 0, \beta = 1, r = 1, s = 0, a_1 = q$ and $q \to 1$ in the Theorem 2.2, we find the following result.

Corollary 2.2. *Let* $f(z) \in \mathcal{A}$*, and let* $0 \le \alpha < 1$ *and* $0 \le \gamma < 1$*, if* $f(z)$ *satisfies*

$$|zf''(z) - \gamma(f'(z)-1)| < \frac{2(1-\gamma)(1-\alpha)}{2-\alpha},$$

then $f(z)$ *is starlike of order* α *in* $\mathbb{U}$.

Theorem 2.3. *Let* $f(z) \in \mathcal{A}$*. If for* $\lambda \ge 0$

$$(1+q)(\beta+\lambda q)^n|\Upsilon_2|^n|a_2| < \frac{1}{2},\quad 0 < q < 1, \tag{10}$$

and

(11)

$$\sum_{k=2}^{\infty}\left([k]_q\left([k]_q-\gamma+q^k\right)-\gamma q^k\right)\left(\beta+\lambda([k]_q-1+q^k)\right)^n|\Upsilon_{k+1}|^n|a_{k+1}|$$
$$\leq(1-\gamma)(1+q)[\beta+\lambda q]^n|\Upsilon_2|^n|a_2|.$$

Then the operator $\mathcal{H}^n_{r,s,\lambda}(a_i,b_j;q)f(z)$ *is starlike in* $\mathbb{U}$, *where* $0\leq\gamma<1$, *and* Υ_2 *is given by* $\frac{(1-a_1)\dots(1-a_r)}{(1-q)(1-b_1)\dots(1-b_s)}$.

Proof. By setting

$$\begin{aligned}
&|z\mathcal{D}^2_q\left(\mathcal{H}^n_{r,s,\lambda}(a_i,b_j;q)f(z)\right)-\gamma\left(\mathcal{D}_q\left(\mathcal{H}^n_{r,s,\lambda}(a_i,b_j;q)f(z)\right)-1\right)|\\
&\quad=\left|\sum_{k=1}^{\infty}[k+1]_q([k]_q-\gamma)\left(\beta+\lambda([k]_q-1+q^k)\Upsilon_{k+1}\right)^n a_{k+1}z^k\right|\\
&\quad\leq\sum_{k=1}^{\infty}[k+1]_q([k]_q-\gamma)\left(\beta+\lambda([k]_q-1+q^k)\right)^n|\Upsilon_{k+1}|^n|a_{k+1}|\\
&\quad=(1-\gamma)(1+q)(\beta+\lambda q)^n|\Upsilon_2|^n|a_2|\\
&\qquad+\sum_{k=2}^{\infty}[k+1]_q([k]_q-\gamma)\left(\beta+\lambda([k]_q-1+q^k)\right)^n|\Upsilon_{k+1}|^n|a_{k+1}|
\end{aligned}$$

Making use of (10) and (11) and applying the fact that $[k+1]_q=[k]_q+q^k$, then note that by Lemma 2.3, we define

$$\begin{aligned}
&\leq 2(1-\gamma)(1+q)(\beta+\lambda q)^n|\Upsilon_2|^n|a_2|\\
&<1-\gamma.
\end{aligned}$$

Then $\mathcal{H}^n_{r,s,\lambda}(a_i,b_j;q)f(z)$ is starlike in $\mathbb{U}$. □

Putting $\lambda=0,\beta=1,r=1,s=0,a_1=q$ and $q\to1$ in the Theorem 2.3, we have

Corollary 2.3. *Let* $f(z)\in\mathcal{A}$, *and let* $0\leq\gamma<1$, *if* $f(z)$ *satisfies*

$$|zf''(z)-\gamma(f'(z)-1)|<1-\gamma,$$

then $f(z)$ *is starlike in* $\mathbb{U}$.

References

[1] F. M. Al-Oboudi, *On univalent functions defined by a generalized Salagean operator*, Int. J. Math and Math Sci **25-28** (2004), 1429–1436.

[2] H. Aldweby and M. Darus, *Some subordination results on $q-$Analogue of Ruscheweyh differential operator*, Abstr. Appl. Anal. (2014) Article ID958563, 6 pages.

[3] I. Aldawish and M. Darus, *New subclass of analytic function associated with the generlalized hypergeometric functions*, Electronic J. Math. Anal. Appl. **2** (2) (2014), 163–171.

[4] A. Aral, V. Gupta Ravi and P. Agarwal, *Applications of q Calculus in operator Theory*, New york, NY: Springer, 2013.

[5] B. C. Carlson and D. B. Shaffer, *Starlike and prestarlike hypergeometric functions*, SIAM J. Math. Anal. **15** (4) (1984), 737–745.

[6] J. Dziok and H. M. Srivastava, *Classes of analytic functions associated with the generalized hypergeometric function*, Appl. Math. Comput. **103** (1999), 1–13.

[7] H. Exton, *q-hypergeometric Functions and Applications*, Ellis Horwood Limited, Chichester, 1983.

[8] G. Gasper and M. Rahman, *Basic Hypergeometric Series*, Encyclopedia of Mathematics and its Application, Vol. 35, Cambridge University Press, Cambridge, 1990.

[9] H. A. Ghany, *q-derivative of basic hypergeometric series with respect to parameters*, Int. J. Math. Anal. (Ruse) **3** (33-36) (2009), 1617–1632.

[10] F. H. Jackson, *On $q-$functions and a certain difference operator*. Tranc. Roy. Soc. Edin **46** (1908), 253–281.

[11] K. Kuroki and S. Owa, *Double integral operators concerning starlike of order β*, Int. J. Differ. Equ. (2009), 1–13.

[12] S. S. Miller and P. T. Mocanu, *Double integral starlike operators*, Integral Transforms Spec. Funct. **19** (7-8) (2008), 591–597 .

[13] A. Mohammed and M. Darus, *A generalized operator involving the $q-$hypergeometric function*, Mat. Vesnik , **65** (4) (2013), 454–465.

[14] M. Obradović, *Simple sufficient conditions for univalence*, Mat. Vesnik, **49** (3-4) (1997), 241–244.

[15] D. Purohit and R. K. Raina, *Generalized $q-$Taylor's series and applications*, Gen. Math. **18** (3) (2010), 19–28.

[16] S. Rusheweyh, *New criteria for univalent functions*, Proc. Amer. Math. Soc. **49** (1975), 109–115.

[17] G. S. Salagean, *Subclasses of univalent functions*, Lecture Notes in Math. (Springer Verlag), **1013** (1983), 362–372.

[18] W.S. Chung, T. Kim, H. I. Kwon, *On the q-analog of the Laplace transform*, Russ. J. Math. Phys. **21** (2) (2014), 156–168.

[19] J. Yang, U$(n+1)$ *extensions of some basic hypergeometric series identities*. Adv. Stud. Contemp. Math. (Kyungshang) **18** (2) (2009), 201–218.

[20] Y. S. Kim, C. H. Lee, *Exton's triple hypergeometric series associated with the Kamp De Friet function*, Proc. Jangjeon Math. Soc. **14** (4) (2011), 447–453.

[21] K. R. Vasuki, A. A. Kahtan, G. Sharath, *On certain continued fractions related to ${}_3\psi_3$ basic bilateral hypergeometric functions*, Adv. Stud. Contemp. Math. (Kyungshang) **20** (3) (2010) 343–357.

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