

## Applicability of Coda Wave Interferometry Technique for Measurement of Acoustoelastic Effect of Concrete

Sung Woo Shin

**Abstract** In this study, we examined the applicability of coda wave interferometry (CWI) technique, which was developed to characterize seismic waves, to detect and evaluate change in the velocity of ultrasonic waves in concrete due to acoustoelastic effect. Ultrasonic wave measurements and compressive loading tests were conducted on a concrete specimen. The measured wave signals were processed with CWI to detect and evaluate the relative velocity change with respect to the stress state of the specimen. A phase change due to the acoustoelastic effect of concrete was clearly detected in the late-arriving coda wave. This shows that the relative velocity change of ultrasonic waves in concrete due to the acoustoelastic effect can be evaluated successfully and precisely using CWI.

**Keywords:** Concrete NDE, Coda Wave, Coda Wave Interferometry, Acoustoelastic Effect, Ultrasonic Velocity

### 1. Introduction

Stress states in concrete structures can be changed due to cracking, overloading, support settlements, corrosion of reinforcements, to name a few. The change of the stress state may reduce the load carrying capacity of the structure and can lead to structural failure. Therefore, monitoring the changes of the stress state in a certain critical member of a structure is important task to assess the safety of the structure.

It is well known that the ultrasonic wave velocity of a solid medium changes with the stress state of the medium [1]. This stress dependence of the wave velocity is so called as the acoustoelastic effect [2]. Many research have been devoted to develop the acoustoelastic based stress monitoring techniques for steel structural members [3]. However, those for concrete structural members have not been much interested in research community as the relative velocity change with respect to the stress change in a concrete medium is very small [4]. For example, the relative velocity change due to acoustoelastic

effect of concrete is lower than 0.01%/MPa [5]. On the other hand, nevertheless of constant stress state, an usual velocity measurement error of a conventional time-of-flight (TOF) method, which is commonly used for wave velocity measurement of concrete, is often higher than 1% [6]. Obviously it is unacceptable to monitor the change of stress state in concrete using TOF based ultrasonic wave velocity measurement.

Coda wave interferometry (CWI) is a promising technique to monitor small wave velocity change in complex medium such as soil and concrete. CWI technique was originally developed for detection and quantification of small temporal change between two seismic waves originating from the same source [7]. As originated from seismology, coda wave means a tail of seismogram after an earthquake. When seismic (or ultrasonic) wave propagates through a complex heterogeneous medium, multiple scattering occurs due to heterogeneities of the medium. These multiply scattered wave components (i.e. coda wave) travelled longer distance than the directly propagated wave components (i.e. ballistic wave).

Therefore, coda wave forms later part of measured wave signal while ballistic wave forms early part. Coda wave has more information on the medium than ballistic wave as it travelled longer distance and interacted more with the medium. Therefore, a small change in the medium, that is hardly detected in ballistic wave, becomes detectable in coda wave [7]. CWI technique utilizes a complex medium (e.g. concrete) as an interferometer of coda wave in order to detect and quantify small temporal change of a complex medium [8].

The CWI technique has recently been applied for various problems in damage detection of concrete [9]. In this study, the applicability of the CWI technique to detect ultrasonic wave velocity change due to the stress state change of concrete is investigated.

## 2. Theoretical Basis

### 2.1 Acoustoelastic Based Stress Monitoring

The classical theory of ultrasonic wave propagation assumes a linear elastic behavior of solids. Under this assumption, the wave velocity of a solid medium is independent of the stress state of the medium. However, when a solid material exhibits nonlinear elastic behavior, higher-order terms to describe nonlinear behavior should be included in the stress-strain relation and in this case the wave velocity becomes stress dependent [1]. Hughes and Kelly derived the stress-velocity relationship of uni-axially loaded isotropic medium based on Murnaghan's theory of nonlinear elasticity [10]. Recently, Hughes and Kelly's stress-velocity model has been further generalized using the first order linearization as

$$V_{ij}^{\sigma} = V_{ij}^0(1 + \theta_{ij}\sigma) \quad (1)$$

where subscripts  $i$  and  $j$  denote wave polarization and propagation directions, respectively.  $\sigma$  is normal stress in direction 1.  $V^{\sigma}$  and  $V^0$  are the stressed and the stress-free (or initial) wave velocities of a medium, respectively.  $\theta$  is the acoustoelastic coefficient which depends on the Lamé's second order and the Murnaghan's third order elastic constants [11]. As seen in Eq. (1), it is obvious that the wave velocity is depending on the stress state (i.e.  $\sigma$ ) in the medium. Eq. (1) can be re-written as

$$\sigma = \frac{1}{\theta_{ij}} \frac{V_{ij}^{\sigma} - V_{ij}^0}{V_{ij}^0} = \frac{1}{\theta_{ij}} \frac{\Delta V_{ij}}{V_{ij}^0} \quad (2)$$

Therefore, if the acoustoelastic coefficient of a medium is known, the current stress state ( $\sigma$ ) of the medium can be estimated by measuring the current and the initial wave velocities of the medium. This is the basic idea of the acoustoelastic based stress monitoring technique.

As shown in equation (2), the acoustoelastic coefficient of an interesting medium must be known in advance to monitor the stress state of the medium based on wave velocity measurements. Since the acoustoelastic coefficient of a material cannot be determined analytically, an experiment to establish the velocity-stress relation should be performed. For this purpose, an index for relative velocity change ( $\alpha_{ij}$ ) is introduced as

$$\alpha_{ij} = \frac{\Delta V_{ij}}{V_{ij}^0} = \theta_{ij}\sigma \quad (3)$$

This suggests that the acoustoelastic coefficient ( $\theta$ ) can be determined by estimating the slope of  $\alpha - \sigma$  relation [11].

In this study, the applicability of CWI technique for an accurate measurement of  $\alpha$  in concrete is investigated since the conventional TOF method is unacceptable to use for the accurate measurement of  $\alpha$  in concrete.

## 2.2 Coda Wave Interferometry Technique

The theory of CWI technique is briefly introduced here. For more details about the technique, see the state-of-the-art paper by Snieder [8].

Suppose that in a medium the wave velocity is perturbed with the perturbation  $dV$ , and the relative velocity change is the same at every location in the medium. The unperturbed travel time of the wave is given by

$$t = \int_L \frac{1}{V} ds \quad (4)$$

where  $L$  is wave path. The perturbed travel time to the first order in the velocity perturbation is given by

$$t + dt = \int_L \frac{1}{V + dV} ds = \int_L \frac{1}{V} ds - \int_L \frac{dV}{V^2} ds \quad (5)$$

Since  $dV/V$  is assumed to be constant, Eq. (5) can be re-arranged using Eq. (4) as

$$dt = -\left(\frac{dV}{V}\right)t \quad (6)$$

This indicates that the travel time perturbation increases linearly with the arrival time. Therefore, the travel time perturbation such as due to the acoustoelastic effect is more clearly identified in the coda wave than the ballistic wave as the coda wave arrives later than the ballistic wave [8].

Due to the acoustoelastic effect of a medium, the travel time of ultrasonic wave propagated through the medium is perturbed with  $dt$  from the unperturbed wave. Thus, the relationship between the unperturbed and the perturbed signals can be written as

$$u_p(t) = u_0(t + dt) \quad (7)$$

Eq. (7) can be re-written using Eq. (6) as

$$u_p(t) = u_0\left(t - \frac{dV}{V}t\right) \quad (8)$$

Since  $dV/V = \alpha$ , finally we obtain

$$u_p(t) = u_0[t(1 - \alpha)] \quad (9)$$

Eq. (9) suggests that  $\alpha$  can be estimated by stretching (or compressing) the unperturbed wave signal to match a perturbed wave signal [8]. The quality of the match between the stretched unperturbed wave signal and the perturbed wave signal within time-window  $[t_1, t_2]$ , in which possesses coda wave, can be quantified with the following cross-correlation function as

$$CC(\alpha_i) = \frac{\int_{t_1}^{t_2} u_0[t(1 - \alpha_i)]u_p[t]dt}{\sqrt{\int_{t_1}^{t_2} u_0^2[t(1 - \alpha_i)]dt \int_{t_1}^{t_2} u_p^2[t]dt}} \quad (10)$$

where  $\alpha_i$  is a stretching parameter [8]. Finally,  $\alpha$  can be found among all values of  $\alpha_i$  that maximize the correlation coefficient as [8]

$$\alpha = \max CC(\alpha_i) \quad (11)$$

Eqs. (10) and (11) are two keys of CWI technique to detect and quantify small temporal change of ultrasonic waves propagated through a complex medium. In this study, Eqs. (10) and (11) are applied to detect and quantify the relative velocity change of ultrasonic wave propagated through concrete.

## 3. Experimental Program

### 3.1 Experimental Setup

A test concrete specimen was prepared. Type I Portland cement, fine sand, and crushed gravel with nominal maximum size of 20 mm were used to make a concrete specimen. The length of the specimen is 300 mm and the square sectional area is 150 mm by 150 mm. Water/cement ratio of the specimen is 0.4 by weight. The specimen was casted and cured in a



Fig. 1 Test specimen with ultrasonic transducers

controlled curing room for 28 days before testing. A companion cylinder specimen was also produced for the compressive strength test. Measured compressive strength of companion cylinder specimen is 29.2 MPa.

A pair of ultrasonic transducers with a central frequency 100 kHz (Model : Olympus V1011) was used for transmitting and receiving of ultrasonic waves. Both transducers were attached on the middle of two opposite sides of the specimen (see Fig. 1). Therefore, the direction of wave propagation (2 direction) is set to be perpendicular to the direction of loading (1 direction). Note that since the transducer used in this study generates compressional ultrasonic wave, the mode of the wave velocity becomes  $V_{22}$ .

8 cycles of Hanning-windowed sine function with an oscillation frequency of 100 kHz was used as an input wave signal. A waveform generator and a power amplifier are used to generate the input. The received wave signal is amplified with a preamplifier and digitized with a sampling rate of 1  $\mu s$  and a duration of 2  $ms$ . The digitized signals are saved in a personal computer for further signal processing.

### 3.2 Test Procedures

In order to obtain the relative velocity change ( $\alpha$ ) with respect to the stress state of a concrete specimen, ultrasonic wave measurements are executed during the uni-axial compression tests on the specimen. Load-controlled compression test was performed on the test specimen. Step loading with 0.1 MPa increment ranging from 5 to 6 MPa was applied. Since formation of crack can affect wave behaviors in concrete, stress ranges from 5 to 6 MPa, which is well below the matrix cracking strength (about 14 to 18 MPa) of the test specimen, was chosen in order to ensure elastic behavior of the concrete specimen. Ultrasonic tests were performed at each loading step. The applied load is maintained constant during the ultrasonic measurement. After the ultrasonic test, the compressive load is slowly increased to the next step load and then the ultrasonic measurement is repeated.

## 4. Results and Discussions

### 4.1 Detection of Small Temporal Change of Ultrasonic Wave in Concrete

Typical ultrasonic signals measured at different stress states (5.0 MPa, 5.5 MPa and 6.0 MPa) are shown in Figs. 2-4. As seen in Fig. 2, those three signals seem to be very similar over the entire duration of the signals. Especially in the early part (ballistic wave part) of measured signals as shown in Fig. 3, it is observed that the arrival of ultrasonic wave is not visibly discernible (almost identical) nevertheless of the different stress states. However, as shown in Fig. 4, notable phase changes of ultrasonic waves with respect to stress state are clearly seen in the later part (coda wave part) of the measured wave signals. The results suggest that the conventional TOF

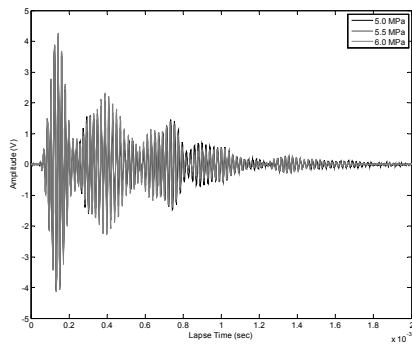


Fig. 2 Measured full ultrasonic wave signals for three different stress states

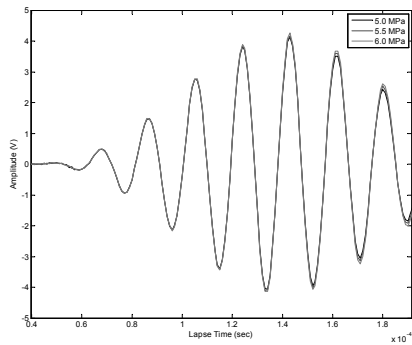


Fig. 3 Early part (from 40 to 190  $\mu s$ ) of full wave signals presented in Fig 2

method, which estimates the time of arrival of the fastest wave (i.e. ballistic wave) from which the wave velocity is calculated, may not be effective to detect small velocity change in concrete. On the contrary, a small temporal change of ultrasonic wave is clearly identify in coda wave part. This result verifies the effectiveness of utilizing coda wave to detect a small velocity change of ultrasonic wave in concrete. Finally, as seen in Fig. 4, the lapse time of coda wave decreases as compressive stress in concrete increases. It was reported that the wave velocity of  $V_{22}$  mode in concrete increases as compressive stress (perpendicular to the direction of  $V_{22}$  mode) increases due to acoustoelastic effect of concrete [12]. In this

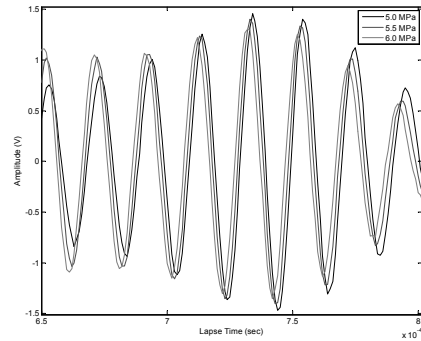


Fig. 4 Later part (from 650 to 800  $\mu s$ ) of full wave signals presented in Fig 2.

study, the same wave mode ( $V_{22}$ ) and loading condition were used. Therefore, it may be concluded that the phase change of coda wave in this study is resulting from the acoustoelastic effect of concrete.

#### 4.2 Evaluation of Acoustoelastic Velocity Change in Concrete

In order to evaluate the acoustoelastic velocity change in concrete,  $\alpha$  values for various stress states were estimated. A stretching CWI technique, represented in Eqs.(10) and (11), was applied for this purpose. It is worth noting that the ultrasonic signal measured at the stress level of 5 MPa was selected as the reference signal,  $u_0(t)$ , in CWI processing. In order to include coda wave part (see Fig. 4) in processing of Eq. (10), the time range ( $[t_1, t_2]$ ) in Eq. (10) was set as  $[600 \mu s, 850 \mu s]$ . Using Eq. (10),  $CC(\alpha_i)$  were searched for all  $\alpha_i$  ranging from 0 to 0.05 with increment of  $10^{-5}$ . Then, as shown in Fig. 5,  $\alpha$  value for a certain stress level was determined using Eq.(11).

Fig. 6 shows the estimated  $\alpha$  values for various levels of compressive stress in the test concrete specimen. It is clearly seen that the  $\alpha$  value increases as compressive stress increases. This result suggests that CWI technique can

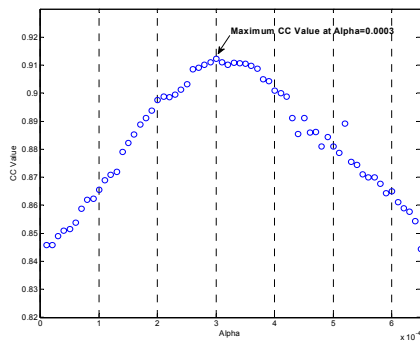


Fig. 5 Example of  $\alpha$  value determination (for a case with the stress level of 5.1 MPa)

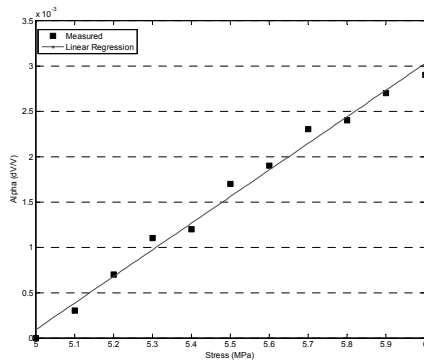


Fig. 6 Estimated relative velocity change ( $\alpha$ ) with respect to the level of compressive stress in the test specimen

quantify the relative velocity change of concrete due to the acoustoelastic effect of concrete precisely (with resolution of 0.1 MPa). Using linear regression analysis, the acoustoelastic coefficient ( $\theta$  in Eq. 3), which is represented as a slope in Fig. 6, was estimated. The estimated acoustoelastic coefficient is 0.002936 MPa<sup>-1</sup> with high value of correlation coefficient (0.9895). This result indicates that the  $\alpha - \sigma$  relation of concrete is linear and also suggests that CWI technique can effectively be used both to establish  $\alpha - \sigma$  relation of various types of concrete and to monitor stress change of concrete structural members.

## 5. Conclusion

In this study, the applicability of CWI technique for measurements of ultrasonic wave velocity change due to the acoustoelastic effect of concrete was investigated. On the basis of the results, following conclusions could be drawn.

- (1) Nevertheless of change in stress state in concrete, the first-arrival ballistic wave part of measured ultrasonic signals does not change much (visibly indiscernible) with the stress state of concrete.
- (2) Significant (visible) phase change with respect to the stress state of concrete is occurred in the late-arriving coda wave part of measured ultrasonic signals.
- (3) The relative velocity change due to the acoustoelastic effect of concrete can be evaluated precisely using CWI technique with a resolution of 0.1 MPa.
- (4) The CWI technique can be used to establish  $\alpha - \sigma$  relation of concrete and to monitor stress change of concrete.

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