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분산 빔포밍을 이용한 OFDM 시스템에서의 동기에러 영향 분석

(Effect of Synchronization Errors with Distributed Beamforming in OFDM Systems)

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요 약

분산 빔포밍의 이득을 얻기 위해서는 심볼 타이밍, 위상, 그리고 주파수 동기가 적절하게 제어가 되어야 한다. 본 논문에서는 OFDM 시스템에서 분산 빔포밍을 이용할 경우 세 가지 동기의 에러에 의한 영향을 분석하였다. 협력신호 사이의 심볼 동기 에러는 저주파수의 부반송파(subcarrier)보다 고주파수의 부반송파에서 큰 영향을 미치고 있다. 위상 및 주파수 에러에 의한 전송신호의 손실은 부반송파 번호와 관련이 없으나, 주파수 오차는 전송신호의 손실 뿐만 아니라 부반송파 사이의 간섭을 야기하기 때문에 OFDM 시스템에서 중요하게 관리되어야 한다. 본 논문에서는 다양한 협력신호 개수 및 세 가지 동기 에러 값에 의한 성능 저하를 보이고 있으며, 성능 분석 결과는 시뮬레이션의 결과와 적절히 일치하는 것을 보이고 있다.

Abstract

Three synchronization issues, i.e., symbol time, phase, and frequency, have to be properly controlled to achieve distributed beamforming gain. In this paper, the impacts of synchronization errors in distributed beamforming are analyzed for OFDM systems. For symbol timing error of cooperating signals, high frequency subcarriers are more susceptible as compared to low frequency ones. The desired signal loss due to phase and frequency offset is independent of subcarrier number. However, frequency offset is critical in OFDM systems since it leads to interference from the other subcarriers as well as power loss in the desired signal. Performance degradation due to three synchronization errors is shown with various numbers of cooperating signals and offset values. It shows that the performance analysis is well matched with simulation results.

Keywords : Cooperative communications, OFDM, distributed beamforming, synchronization

Ⅰ. Introduction

Cooperative communications has received increased interest recently as a means to overcome fading channels with a limited number of antennas and limited power at a portable device. The basic idea behind cooperative communications is that multiple single-antenna devices share their antennas to create a virtual multiple antenna system[1~3]. One approach to improve system performance using cooperative nodes is distributed beamforming[3], where multiple

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cooperative nodes transmit the same signal at the same time after proper preprocessing to obtain beamforming gain at the destination.

In conventional transmit beamforming systems, only phase synchronization of the transmitted signals is considered due to the facts that a single local oscillator is used and the distance difference of the multiple signal paths is small enough to be ignored. In distributed beamforming, however, multiple copies of the transmit signal are generated from different locations with different local oscillators. Therefore, all three synchronization issues, *i.e.*, phase, frequency, and symbol time, have to be properly controlled to achieve beamforming gain. The methods for carrier and phase synchronization were investigated in [4~6], where reference or feedback signal was used for mitigating the effect of synchronization errors.

An efficient cooperative retransmission scheme was proposed which combines packet retransmission and user cooperation[7], where erroneous data packets are retransmitted to the destination using distributed beamforming. The outage probability and packet error rate (PER) performance were shown for the case of perfect synchronization and phase/frequency offsets are estimated. The throughput efficiency and average packet delay were analyzed for the cooperative retransmission scheme where distributed beamforming is used for packet retransmission[8]. However, the effect of synchronization errors in distributed beamforming was not analyzed. The effect of frequency offset was analyzed in orthogonal frequency division multiplexing (OFDM) systems[9]. It was shown that frequency offset is critical in OFDM systems in terms of power loss of desired signal as well as inter-carrier interference (ICI). However, the effect of other synchronization errors was not investigated.

In this paper, the effect of synchronization errors in distributed beamforming is investigated for OFDM systems. The rest of this paper is organized as follows. In Section II, system model using distributed beamforming is described. In Section III, the effect of each synchronization error is analyzed for OFDM systems. The performance simulation for each synchronization error is shown in Section IV to verify the analytical results. Section V concludes this paper.

Ⅱ. System Model

There are M cooperating nodes which deliver the same signal to the destination with distributed beamforming as shown in Fig. 1. It is assumed that cooperating nodes extract information for synchronization from the reference signal which is provided from a master node or the destination[3][7].

After adjusting offsets at each cooperating signal with the obtained information, the received signal at the destination can be expressed by

$$r(t)=\frac{1}{\sqrt{M}}\sum_{m=1}^{M}\sqrt{\alpha_m}\,|h_m|x(t-\tau_m)e^{j2\pi f_m t}e^{j\theta_m}+n(t) \tag{1}$$

where α and h_m represent the long term signal loss and channel coefficient of the cooperating signal m, respectively. $x(t)$ is the transmit signal. τ_m, f_m, and θ_m are residual symbol timing offset, frequency offset, and phase offset of cooperating signal m, respectively. $n(t)$ is noise at the destination. It is assumed that τ_m is uniformly distributed in $-\tau_{\max} \le \tau_m \le \tau_{\max}$. It is also assumed that f_m and θ_m are uniformly distributed in

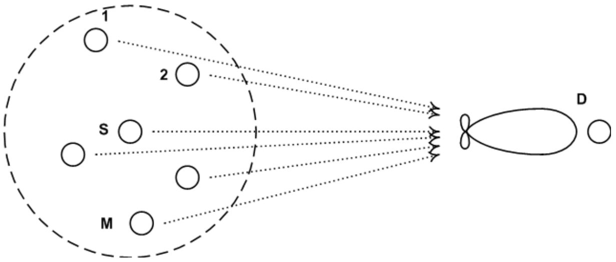

그림 1. 분산 빔포밍을 이용한 협력통신
Fig. 1. Cooperative communications with distributed beamforming.

$-f_{\max} \le f_m \le f_{\max}$ and $-\theta_{\max} \le \theta_m \le \theta_{\max}$, respectively. Note that the total transmit power is normalized by the number of cooperating nodes. The amount of signal to noise ratio (SNR) degradation due to synchronization errors will be investigated for OFDM systems. We will analyze the effect of the three offsets separately since they are generally independent of each other.

In OFDM systems, the information symbols are mapped onto the subcarrier of the inverse discrete Fourier transform (IDFT) and creating an OFDM symbol. The output of the IDFT is converted to a serial sequence and a guard interval is added using cyclic prefix (CP). After the CP has been removed, the receiver performs the inverse operation of the transmitter, *i.e.*, discrete Fourier transform (DFT), to extract the transmitted symbols. When there are N subcarriers, the OFDM symbols can be expressed by

$$x(t) = \frac{1}{N}\sum_{n=0}^{N-1} s(n) e^{j2\pi \frac{n}{T} t} \tag{2}$$

where $s(n)$ is the information symbol on subcarrier n and T is the OFDM symbol duration.

When the cyclic prefix is assumed to be long enough to avoid inter-symbol interference (ISI) between OFDM symbols, the received signal of M cooperating signals after removing the cyclic prefix is given by

$$r(t) = \frac{1}{\sqrt{M}}\sum_{m=1}^{M}\sqrt{\alpha_m}|h_m| x(t-\tau_m) e^{j2\pi f_m t} e^{j\theta_m} + n(t). \tag{3}$$

Note that again the total transmit power is normalized with the number of cooperating signals.

Ⅲ. Performance Analysis

1. Symbol Time Offset

When there is only symbol time offset in M cooperating signals, the received signal in (3) is given by

$$r(t) = \frac{1}{\sqrt{M}}\sum_{m=1}^{M}\sqrt{\alpha_m}|h_m| x(t-\tau_m) + n(t). \tag{4}$$

The received data on subcarrier k can be obtained by

$$r(k) = \frac{1}{T}\int_0^T r(t) e^{-j2\pi\frac{k}{T}t} dt + n(k) \quad \text{for } k = 1, 2, \ldots, N \tag{5}$$

where $n(k)$ is noise on subcarrier k given by

$$n(k) = \frac{1}{T}\int_0^T n(t) e^{-j2\pi\frac{k}{T}t} dt. \tag{6}$$

By substituting (2) and (3) into (5), the received data can be rewritten as

$$\begin{aligned} r(k) &= \frac{1}{\sqrt{M}T}\int_0^T \sum_{m=1}^{M}\frac{1}{N}\sum_{n=0}^{N-1}\sqrt{\alpha_m}|h_m| s(n) \\ &\quad \bullet\, e^{-j2\pi\frac{n}{T}\tau_m t} e^{j2\pi\frac{n-k}{T}t} dt + n(k) \\ &= \frac{1}{\sqrt{M}T}\int_0^T \left\{\sum_{m=1}^{M}\sqrt{\alpha_m}|h_m| s(k) e^{-j2\pi\frac{k}{T}\tau_m}\right\} dt \\ &\quad + n(k) \\ &= s(k)\frac{1}{\sqrt{M}}\sum_{m=1}^{M}\sqrt{\alpha_m}|h_m| e^{-j2\pi\frac{k}{T}\tau_m} + n(k) \end{aligned} \tag{7}$$

When the number of cooperating nodes is large enough, (7) can be approximated as

$$\begin{aligned} r(k) &\approx s(k)\sqrt{M} E[\sqrt{\alpha_m}] E[|h_m|] E[e^{-j2\pi\frac{k}{T}\tau_m}] \\ &\quad + n(k). \end{aligned} \tag{8}$$

The average SNR of subcarrier k with symbol timing offset is given by

$$\begin{aligned} SNR &= \frac{ME[\sqrt{\alpha_m}]^2 E[|h_m|]^2}{\sigma_n^2} E[e^{-j2\pi\frac{k}{T}\tau_m}]^2 \\ &= SNR_{perf} \bullet L_{so,k} \end{aligned} \tag{9}$$

where SNR_{perf} is the achievable distributed beamforming gain in OFDM systems. $L_{so,k}$ is average SNR loss of subcarrier k due to the symbol timing offset between the cooperating signals, which is given by

$$L_{so,k} = E[e^{-j2\pi\frac{k}{T}\tau_m}]^2 = \left[\frac{\sin(\pi k\tau_{\max}/T)}{\pi k\tau_{\max}/T}\right]^2 \\ = 20\log\left[\mathrm{sinc}\left(\frac{k\tau_{\max}}{T}\right)\right](dB). \tag{10}$$

In OFDM systems, high frequency subcarriers are more susceptible to symbol timing offset as compared to low frequency ones as shown in (10).

2. Phase Offset

With only phase offset between cooperating signals, the received OFDM signal is given by

$$r(t) = \frac{1}{\sqrt{M}}\sum_{m=1}^{M}\sqrt{\alpha_m}|h_m|x(t)e^{j\theta_m} + n(t). \tag{11}$$

The received signal on subcarrier k, which is transmitted from M cooperating nodes, can be obtained by using the same procedure and given by

$$\begin{aligned} r(k) &= \frac{1}{\sqrt{M}T}\int_0^T\sum_{m=1}^{M}\frac{1}{N}\sum_{n=0}^{N-1}\sqrt{\alpha_m}|h_m|s(n) \\ &\qquad \bullet\ e^{j2\pi\frac{n-k}{T}t}e^{-j2\theta_m}dt + n(k) \\ &= \frac{1}{\sqrt{M}}\sum_{m=1}^{M}\sqrt{\alpha_m}|h_m|s(k)e^{j\theta_m} + n(k) \\ &\approx s(k)\sqrt{M}E[\sqrt{\alpha_m}]E[|h_m|]E[e^{j\theta_m}] + n(k) \end{aligned} \tag{12}$$

The average SNR of subcarrier k with phase offset is given by

$$SNR = \frac{ME[\sqrt{\alpha_m}]^2E[|h_m|]^2}{\sigma_{\mathrm{n}}^2}E[e^{j\theta_m}]^2 \\ = SNR_{perf}\ \bullet\ L_{po,k} \tag{13}$$

$L_{po,k}$ is average SNR loss of subcarrier k due to phase offset between the cooperating signals. Note that it does not depend on the subcarrier number and is given by

$$L_{po,k} = E[e^{j\theta_m}]^2 = 20\log\left[\frac{\sin(\theta_{\max})}{\theta_{\max}}\right](dB) \tag{14}$$

3. Frequency Offset

When there is frequency offset between the cooperating signals, the received signal is given by

$$r(t) = \frac{1}{\sqrt{M}}\sum_{m=1}^{M}\sqrt{\alpha_m}|h_m|x(t)e^{j2\pi f_m t} + n(t). \tag{15}$$

The signal on subcarrier k with frequency offset is obtained by

$$\begin{aligned} r(k) &= \frac{1}{\sqrt{M}T}\int_0^T\sum_{m=1}^{M}\frac{1}{N}\sum_{n=0}^{N-1}\sqrt{\alpha_m}|h_m| \\ &\qquad \bullet\ s(n)e^{j2\pi\frac{n-k}{T}t}e^{j2\pi f_m t}dt + n(k) \\ &= s(k)\frac{1}{\sqrt{M}}\sum_{m=1}^{M}\sqrt{\alpha_m}|h_m|\left\{\frac{1}{T}\int_0^T e^{j2\pi f_m t}dt\right. \\ &\quad \left. + \frac{1}{N}\sum_{\substack{n=0\\n\neq k}}^{N-1}\frac{1}{T}\int_0^T e^{j2\pi\frac{n-k}{T}t}e^{j2\pi f_m t}dt\right\} + n(k) \end{aligned} \tag{16}$$

where the first term of the final expression represents the loss of the desired signal on subcarrier k and the second term represents ICI due to the residual frequency offset between the cooperating signals. When there is no frequency offset in the received OFDM signal, the full signal power on each subcarrier can be obtained without interference from the other subcarriers since the subcarrier frequencies are chosen to be orthogonal to each other. If there is frequency offset in the received signal, the orthogonality cannot be satisfied and there will be signal loss of the desired subcarrier as well as ICI from the other subcarriers.

The signal loss of subcarrier k due to frequency offset, $D_{fo,k}$, is given by

$$D_{fo,k} = \frac{Si(2\pi f_{\max}T)}{2\pi f_{\max}T} \tag{17}$$

where $Si(x)$ is sine integral defined as

$$Si(x) = \int_0^x\frac{\sin u}{u}du. \tag{18}$$

$L_{fo,k}$ is average SNR loss of subcarrier k due to frequency offset between the cooperating signals and given by

$$L_{fo,k} = 20\log D_{fo,k} \quad (dB) \tag{19}$$

$ICI(k)$ is interference due to frequency offset on subcarrier k which is given by

$$\begin{aligned} ICI(k) &= \frac{1}{N}\sum_{\substack{n=0,\\ n\neq k}}^{N-1} \frac{1}{j2\pi f_{\max}T}\{Ei(j2\pi\alpha) \\ &\qquad - Ei(j2\pi\beta) + \log(\beta/\alpha)\} \\ &\approx \frac{1}{2}(1-D_{fo}) \end{aligned} \tag{20}$$

where $Ei(z)$ is exponential integral defined as

$$Ei(z) = \int_{-z}^{\infty} \frac{e^{-t}}{t}dt \tag{21}$$

and $\alpha = n-k+f_{\max}T$, $\beta = n-k-f_{\max}T$. D_{fo} is the amount of signal loss which is independent of subcarrier number as shown in (17). The detailed derivation of the signal loss and ICI due to frequency offset is given in [9].

Using the approximation of ICI, the signal to interference-plus-noise ration (SINR) of subcarrier k with frequency offset is given by

$$\begin{aligned} &SINR(k) \\ &= \frac{ME[\sqrt{\alpha_m}]^2 E[|h_m|]^2 L_{fo,k}}{ME[\sqrt{\alpha_m}]^2 E[|h_m|]^2 (1-D_{fo})^2/4 + \sigma_n^2} \\ &= \frac{SNR_{perf}L_{fo,k}}{SNR_{perf}(1-D_{fo})^2/4+1}. \end{aligned} \tag{22}$$

Finally, distributed beamforming gain in OFDM systems with M cooperating nodes is given by

$$\begin{aligned} &SINR(k) \\ &= \frac{ME[\sqrt{\alpha_m}]^2 E[|h_m|]^2 L_{so,k}L_{po,k}L_{fo,k}}{ME[\sqrt{\alpha_m}]^2 E[|h_m|]^2 L_{so,k}L_{po,k}(1-D_{fo})^2/4 + \sigma_n^2} \\ &= \frac{SNR_{perf}L_{so,k}L_{po,k}L_{fo,k}}{SNR_{perf}L_{so,k}L_{po,k}(1-D_{fo})^2/4+1}. \end{aligned} \tag{23}$$

When there is no offsets among the cooperating signals (*i.e.*, τ_m=0, θ_m=0, and f_m=0), $L_{so,k}$, $L_{po,k}$, $L_{fo,k}$, and D_{fo} are one from (10), (14), (17), and (19), and the achievable SINR of subcarrier k in (23) will be SNR_{perf}.

Ⅳ. Performance Simulation

To verify the analytical result, simulation was performed with multiple OFDM cooperating signals. Simulation parameters are given in Table 1. The channel coefficient h_m was assumed to be independent and identically distributed (i.i.d.) complex Gaussian random variable with zero mean and 0.5 variance per dimension.

OFDM systems have an advantage of robustness to symbol timing error using a guard interval with a cyclic prefix. If symbol timing error is less than the guard interval in the typical OFDM system, it is converted into a phase shift and can be compensated by a simple method. In distributed beamforming, however, phase offset due to symbol timing error cannot be adjusted when cooperating signals arrived with different timing offsets. Fig. 2 shows the SNR degradation due to symbol timing error for OFDM systems. High frequency subcarriers in OFDM systems are more susceptible to timing offset since they are affected relatively more by the same amount of phase shift. However, the SNR loss due to symbol offset is pretty small even in high frequency subcarriers.

Fig. 3 shows the SNR loss due to phase offset for OFDM systems when the number of cooperating

표 1. 모의실험 파라미터
Table 1. Simulation parameters.

Parameters	Values
Data rate(R)	1, 2 Mbps
Number of subcarriers	64
OFDM symbol duration	64 μs
Cyclic perfix	8 μs
Number of simulation packets	4000
Number of cooperating signals	2~10
Average SNR of cooperating signals	5.5 dB

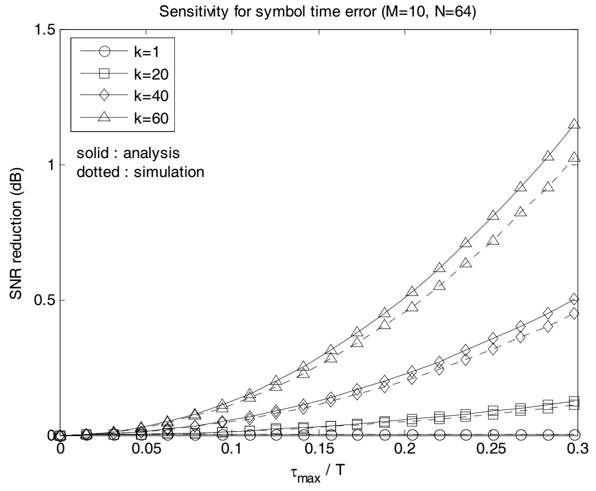

그림 2. 분산 빔포밍에서 심볼 동기 에러에 의한 SNR 감소

Fig. 2. SNR reduction due to symbol time error in distributed beamforming.

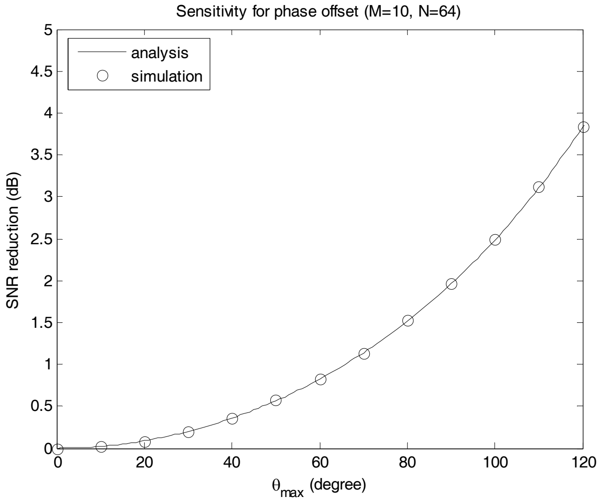

그림 3. 분산 빔포밍에서 위상 오차에 의한 SNR 감소

Fig. 3. SNR reduction due to phase offset in distributed beamforming.

signals is 10. The performance degradation due to phase offset is not large when the phase offset is moderate. However, the SNR loss increases rapidly if it is not controlled properly.

In distributed beamforming, OFDM systems are fairly robust to symbol timing error and phase offsets at the moderate ranges. However, it is very sensitive to carrier frequency offset. The performance degradation due to frequency offset in OFDM systems results from not only power loss of the desired signal but also ICI from the other subcarriers. Fig. 4 and Fig. 5 show signal loss and ICI due to frequency offset in OFDM systems. It is shown that the SNR loss and ICI increase dramatically at the certain range of frequency offset. From the simulation results, we verity that the analytical results for three synchronization errors are well matched with the numerical ones. The small difference between analytical and numerical results in Fig. 4 comes from

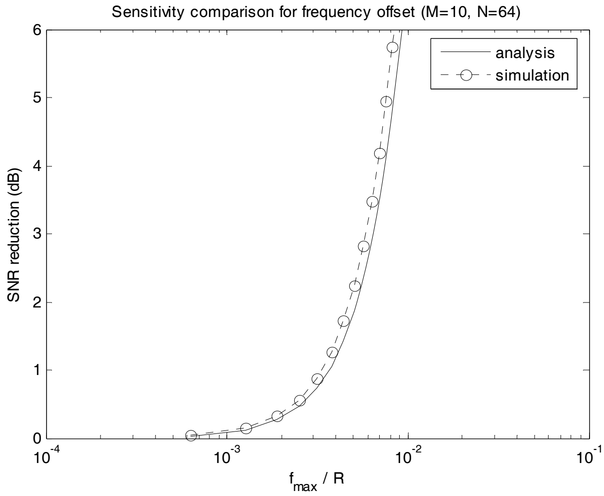

그림 4. 분산 빔포밍에서 주파수 오차에 의한 SNR 감소

Fig. 4. SNR reduction due to frequency offset in distributed beamforming.

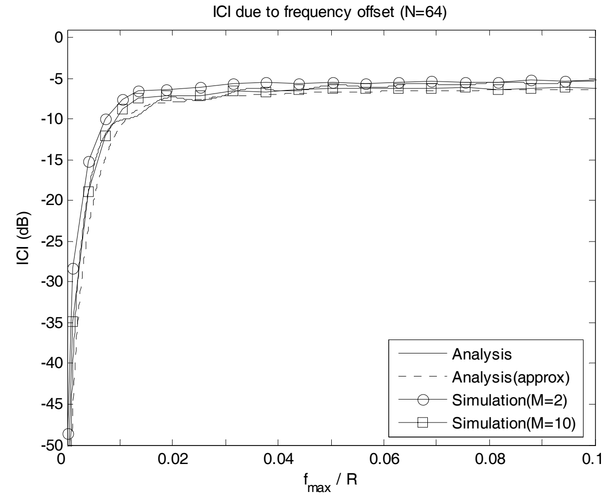

그림 5. 분산 빔포밍을 이용한 OFDM 시스템에서 주파수 오차에 의한 ICI

Fig. 5. ICI in OFDM systems due to frequency offset in distributed beamforming.

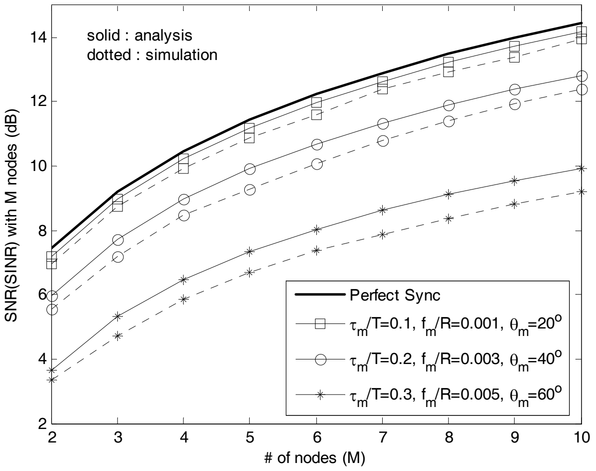

그림 6. 협력노드 수 및 오차 값들에 따른 SINR
Fig. 6. Achievable SINR varying the number of cooperating nodes with various offset values.

the approximation of ICI shown in (20).

Fig. 6 shows the achievable SINR with M cooperating signals for various offset values. In real scenario, the cooperating nodes might be located randomly around source and destination nodes. To include this case in simulation, the long term signal loss is chosen randomly from 1 dB to 10 dB for each packet. When there are two cooperating nodes, for example, 4000 packets are received at the destination with different α values. To compare with numerical results, the average received SNR of cooperating signals is assumed to be 5.5 dB in analysis.

The analytical results are pretty well-matched with the simulated results with small offset values. However, there is about 0.8 dB difference with higher offset values. The difference between the analytical and simulated results comes from the approximation of ICI in the analysis. Distributed beamforming in OFDM systems shows good performance when there are small offsets among the cooperating signals. However, its performance degrades very sharply when synchronization errors are not controlled properly, especially for frequency offsets.

Ⅴ. Conclusion

When distributed beamforming is used in cooperating signals, the effects of synchronization errors between cooperating signals were investigated for OFDM systems. For symbol timing offset in OFDM systems, high frequency subcarriers are more susceptible as compared to low frequency ones. However, the effect of symbol timing offset and phase offset is pretty small at the moderate offset ranges. Frequency offset is critical in OFDM systems since it leads to interference from the other subcarriers as well as power loss in the desired signal. The benefit of distributed beamforming with multiple cooperating nodes will be diminished if synchronization of cooperating signals are not controlled properly.

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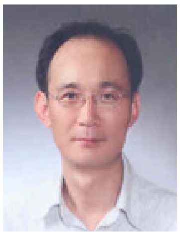

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