This paper presents dynamic modelling of a virtual object in augmented reality environments when external forces are applied to the object in real-time fashion. In order to simulate a natural behavior of the object we employ the theory of Newtonian physics to construct motion equation of the object according to the varying external forces applied to the AR object. In dynamic modelling process, the physical interaction is taken placed between the augmented object and the physical object such as a haptic input device and the external forces are transferred to the object. The intrinsic properties of the augmented object are either rigid or elastically deformable (non-rigid) model. In case of the rigid object, the dynamic motion of the object is simulated when the augmented object is collided with by the haptic stick by considering linear momentum or angular momentum. In the case of the non-rigid object, the physics-based simulation approach is adopted since the elastically deformable models respond in a natural way to the external or internal forces and constraints. Depending on the characteristics of force caused by a user through a haptic interface and model’s intrinsic properties, the virtual elastic object in AR is deformed naturally. In the simulation, we exploit standard mass-spring damper differential equation so called Newton’s second law of motion to model deformable objects. From the experiments, we can successfully visualize the behavior of a virtual objects in AR based on the theorem of physics when the haptic device interact with the rigid or non-rigid virtual object.

1. Introduction

One of the issues in Augmented Reality (AR) applications is how to interact the overlaid virtual objects feasibly by users. In that sense, recently developed tracking and interaction methods in AR allow users to work with and...
Dynamic Behavior Modelling of Augmented Objects with Haptic Interaction

examine the real physical world, while controlling augmented objects in the system more feasible fashion.

We can generally classify AR technology into two categories, such as marker-based AR and marker-less AR. In marker-based AR a specific marker is used for overlaying an object in the scene. Meanwhile, marker-less AR does not require the forethought of adding markers to a scene in order to render a virtual object. It uses a detected feature from the scene as a marker instead of using a predetermined specific marker.

Regardless of their types of AR systems, most of AR systems need various interactions between users and augmented object in many AR applications. Therefore, the major issue in AR is how to interact with a virtual model in dynamic or convenient way[1,2]. Conventional vision-based interaction techniques in AR are based on 2D image analysis and recognition methods and have a limitation to obtain three dimensional information of the virtual object and user who participates in the interaction.

Both marker-based and marker-less AR systems require some indication of where exactly the virtual objects should be augmented. This has been conventionally accomplished by AR markers such as ARTag or AR Toolkit in marker-based AR. In developing mobile AR system, AR Toolkit Plus is well-adopted.

Once a virtual object is registered on the marker, the users usually want to manipulate or interact with the augmented object. The most of introduced interaction techniques for AR applications allow end users to contact with virtual objects in an intuitive way. Tangible interface and tangible interaction metaphor have become one of the most frequently used AR interaction methods[3]. Tangible AR interaction leads to combining real object input with human gesture interaction and hand gesture interaction methods have been widely investigated from the perspective of computer vision and Augmented Reality[4].

Meanwhile, the advance in Augmented Reality enables haptic exploration of augmented virtual objects through a haptic interface[5]. In haptic augmented reality, haptic rendering enables users to touch and manipulate a virtual object through a haptic interface. It also supports users to feel force feedback during the interaction between the user and the virtual object[6,7]. PHANToM Omni (Sensible Inc.) illustrated in Fig. 1 is a three-degree of freedom haptic device which is widely used for various applications of haptic rendering. In this work we use PHANToM Omni and AR Toolkit in simulating the behavior of both rigid object and elastically deformable object in AR.

(Figure 1) A Haptic Device for Haptic Interaction

Especially, the studies of haptic interaction along with elastically deformable object has been done for various simulations such as simulated medical operation in virtual or augmented reality environments[7,8]. It is known that realistic simulation of physically-based deformation of elastic object is difficult because of complexity of its method and computational expenses of simulation in real-time applications. Moreover, the elastic objects have infinite degrees of freedom and the deformation motion equations impossible to solve analytically.

Many researchers have been proposed different techniques to speed up deformation computation using finite element methods (FEM)[9]. However, the elastic theory to model the differential equations that govern the non-rigid elastic model can be adequate for achieving a relatively realistic model deformation in real time fashion. In that sense, a simple mass-spring system (MSS) is appropriate for simulating deformable behavior of the elastic object because of the simplicity of the motion equation and of the implementation.

In this work, we present natural behavior of an augmented object in AR environments when external forces are applied to the object In order to simulate a natural behavior of the object we employ the theory of Newtonian physics to construct motion equation of the object. As virtual 3D objects both rigid and non-rigid(elastic) objects are used for the dynamic modelling.
2. Proposed Approach

The overall procedures of haptic interaction with a virtual object in a marker-based AR. ARToolkit can detect and identify a specific marker from an input video image and register a virtual 3D object on the marker as illustrated in Fig 2. When moving the haptic stick around in 3D physical space, the haptic interface point so called HIP moves accordingly in AR space. The haptic device can send forces in three dimensions, thus allowing a user to interact with objects in the computer. The user can feel the force feedback when the device is approaching the virtual object and collides with the boundary of the object. The magnitude of the force can be evaluated by considering the ideal haptic interface point located on the boundary surfaces of the object.

(Figure 2) The procedure for dynamic modelling using a haptic device in AR

2.1 Modeling Method for Rigid Object

In this work, the dynamic behaviors of the rigid objects are simulated by applying simplified Newtonian physics in the AR space. Both linear momentum and angular momentum for motion simulation of the rigid objects are taken into consideration. The physics of each kind of momentum are described as follow[10].

The first case study of simulation is based on linear momentum for collision of objects. In physics, linear momentum ($P$) is the product of mass ($m$) and velocity ($v$) of an object. Because the momentum has direction and magnitude, it can be used to predict the resulting direction and speed of objects when the objects collide. According to the Newton’s law of motion, the momentum is conserved if a closed system is not affected by any external forces.

In this simulation, we assume an augmented virtual object in AR is hit by a physical haptic stick, both of which are also modeled as particles. Fig. 3 shows the elastic collision between the virtual object ($m_1$) and the haptic stick($m_2$).

(Figure 3) Collision between the virtual object ($m_1$) and haptic stick($m_2$)

The equation for conservation of linear momentum for the collision between($m_1$) and ($m_2$) in augmented sphere system can be expressed as

$$m_1v_1 = m_2v_1' + m_2v_2'$$

(1)

In the equation (1), the velocity and direction of the each object after collision can be defined as follow.

$$v_1' = \left(\frac{b}{2r}\right)v_1$$

$$v_2' = \sqrt{1 - \left(\frac{b}{2r}\right)^2}v_1$$

$$\theta_1 = \sin^{-1}\left(\sqrt{1 - \left(\frac{b}{2r}\right)^2}\right)$$

$$\theta_2 = \sin^{-1}\left(\frac{b}{2r}\right)$$

(2)

In the ideal condition without any kinds of friction, energy should also be conserved based on the following equation:
which incorporates both the $x$ and $y$ directional movements of the particles. The kinetic energy of the particle with mass $m_1$ before and after the collision are each given as $E_{KE}^{1}$ and $E_{KE}'$ and the kinetic energy of the particle with mass $m_2$ after the collision as $E_{KE}^{2}$. The second case study of simulation is based on the angular momentum ($L$) for collision of objects. Angular momentum is a vector quantity that represents the product of object’s rotational inertia ($I$) and rotational velocity ($\omega$) about particular axis. For the cases where the object is small compared to the radial distance to its axis of rotation, the angular momentum can be expressed by

$$L = r \times p = r \times mv$$

(4)

Based on the conservation of angular momentum, the angular momentum of the system when the haptic stick collides with the virtual object is given as

$$r m_1 v_1 = r m_1 v_1' + L_2 \omega_2$$

(5)

where $r m_1 v_1$ and $r m_1 v_1'$ are the angular momentum of the haptic stick before and after the collision, respectively, with its distance from the $x$-axis ($r$) given as a constant, and $L_2 \omega_2$ is the angular momentum of the augmented sphere that has the moment of inertia ($I = m_2 r^2$) and spins with the angular velocity ($\omega$).

Given that the collision within the system is elastic, Energy should also be conserved:

$$E_{KE} = E_{KE}' + E_{KE}''$$

(6)

$$\frac{1}{2} m_1 v_1 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_2 \omega^2$$

In formula (6), $E_{KE}$ is kinetic energy of the haptic stick before collision, $E_{KE}'$ is kinetic energy of the haptic stick, and $E_{KE}''$ is rotational kinetic energy of the ball after collision. The angular velocity ($\omega = v/r$) of the object can be derived from solving the formula (5) and (6). The direction of torque $\tau$ which is the axis of rotation of the virtual object can be determined as follows.

$$\tau = \tau \times F$$

(7)

(Figure 4) Direction of rotational axis based on angular torque

However, in both simulation it is assumed that a certain type of friction (cf. air friction) exists and dissipates the energy of the system that energy is not conserved and the linear and the rotational movements of the object eventually stop.

2.2 Modeling Method for Elastic Object

The elastic object manipulated in AR consists of the mesh of mass points connected by elastic links so called a mass spring connected system. The motion of the system can be evaluated by integrating Newton’s second law of motion:

$$M \frac{d^2 x}{dt^2} + D \frac{dx}{dt} + Kx = F(t)$$

(8)

where $M$ is the mass matrix, $D$ is damping matrix due to the viscous damper, $K$ is spring stiffness matrix, and $F(t)$ is external forces, respectively. The internal force $Kx$, which is restoring force due to the spring, is computed based on Hook’s law. The external forces are applied to the object through a haptic device. When a system which has $n$ numbers of mass is considered, the equation of motion of
Dynamic equilibrium can be obtained by equating the sums of the forces and moments of each mass of the system to zero and given by

\[ F_i - \frac{\partial}{\partial t} \left( M_i \frac{\partial x}{\partial t} \right) = 0, \quad \text{for} \ i = 1, 2, \ldots, n \quad (9) \]

As the most commonly used generalized coordinates for an elastic object, the vibration modes of the object can be considered. When the masses of a system are oscillating in such a manner that they reach maximum displacements simultaneously and pass their equilibrium points at the same time, or all moving particles of the system are oscillating in phase with one frequency, such a state of motion is called normal mode.

Since the elastic object in AR is considered as a spring connected mesh model, the equations of motion of the system are not entirely independent because of coupling actions. Thus, it is necessary to solve these equations at the same time. If \( n \) independent coordinates are required to specify the positions of mass of the system, the motion of the system is represented by \( n \) differential equations of motion based on the formula of Newton’s second law (8).

For the solution, this second-order differential equation which governs the motion of the system can be changed into

\[ \frac{\partial^2 x}{\partial t^2} + \frac{D}{M} \frac{\partial x}{\partial t} + \frac{K}{M} x = \frac{F(t)}{M} \quad (10) \]

Subsequently, the equation (10) can be converted into following matrix form of the differential equation

\[
\begin{pmatrix}
\frac{\partial x}{\partial t} \\
\frac{\partial^2 x}{\partial t^2}
\end{pmatrix} =
\begin{pmatrix}
0 & 1 \\
-k/M & -D/M
\end{pmatrix}
\begin{pmatrix}
x \\
\frac{\partial x}{\partial t}
\end{pmatrix} + \begin{pmatrix}
0 \\
\frac{F(t)}{M}
\end{pmatrix} \quad (11)
\]

As a result, dynamic motions of any multi-degree freedom systems can be explained by the linear differential solution of

\[ \dot{x}(t) = Ax(t) + F(t); \quad x(t_0) = x_0 \quad (12) \]

Once an elastically deformable virtual object is overlaid on a specific marker then the haptic interaction between the object and the haptic device is taken placed. Using the haptic device (often called the haptic interface pointer “HIP”) of PHANToM we can contact with the object and feel the force feedback when we touches the object.

(Figure 5) Haptic interaction in spring-mass connected system.

In haptic interaction, the collision between the object, which is modeled as a spring-mass connected mesh system, and HIP is detected at just one position (vertex) of the object and the external force is applied to the point only as illustrated in Fig. 1. When a certain mesh point of the object is collided with a haptic interface, a force of spring damper is induced by the movement of the mass and the force acts on the spring connected neighboring vertices around the collided vertex with the haptic interface. Therefore, we need to evaluate the range of the force affected and the magnitude of the acting force to the neighboring vertices to make the deformation of the entire system in real-time.

If the intrinsic properties such as stiffness \( k \) and damping constant \( D \) are homogeneous for overall system then each force of the neighboring vertex can be determined by the velocity and displacement between vertices connected with springs and damper and expressed by

\[ F_i = \sum \left( I_{ij} \cdot \left( k \langle [\dot{u}_{ij}] - [u_{ij}] \rangle + D [\dot{u}_{ij}] \right) \right) \quad (13) \]

where \( l_{ij} \) is length of spring between vertex \( i \) and \( j \) when the spring is elongated, \( l_{ij}^0 \) is initial length of spring
and \( v_i \) is the velocity of vertex \( i \).

3. Results

In this experiment, as for the experimental environments, OpenGL, ARToolkit and Omni-PHANToM are used for modeling 3D object, tracking and registering the model on a marker and interacting with the AR object, respectively.

For modeling rigid object, we simulate two cases of motion when a collision occurs. The first one is the collision between a cubic object and the haptic stick based on linear momentum. The second one is the collision that incorporates the angular momentum of a spherical rigid object. Following figure shows the snapshots of dynamic motion of the both AR object after collision based on each momentum.

(Figure 6) Results of dynamic modeling for rigid objects: cube and sphere

As elastic objects we model a simple mass-spring connected planar surface and semi-sphere and use for the deformable modeling. We also assume the intrinsic properties of the model such as mass, coefficient of spring and damper are homogeneous and constant during the deformation.

(Figure 7) Results of dynamic modeling for elastic objects: semi-sphere (top) and plane sheet (middle and bottom)

4. Conclusion

In this paper, we present a dynamic behavior of both rigid objects and elastically deformable objects in haptic AR environments. In the case of rigid object, we can visualize the natural movement of the augmented object which
collides with a physical haptic stick interface by applying the Newtonian physics to the models. Linear momentum and angular momentum for dynamic motion simulation of rigid objects are utilized. In the case of elastically deformable object, the virtual elastic object is deformed naturally based on the characteristics of force by a user through a haptic interface and model’s intrinsic properties. In this study, we employ Newton’s second law of motion to model deformable objects. From the experiments, we can show the natural behavior of both rigid and elastic object in AR based on the theory of physics.

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저 자 소 개

이 선 호 (Seonho Lee)
2012년 경기대학교 컴퓨터과학과 졸업 (학사)
2012년~현재 경기대학교 대학원 컴퓨터과학과 석사과정
관심분야 : 증강현실, 비전기반상호작용, 물리기반 모델링
E-mail : sunho36@naver.com

전 준 철 (Junchul Chun)
1984년 중앙대학교 전자계산학과 (학사)
1986년 중앙대학교 대학원 전자계산학과 (석사)
1987년 2월~1988년 6월 삼성방도체 통신연구소 S/W 공학 연구실 연구원
1992년 The Univ. of Connecticut, 컴퓨터공학과 (석사)
1995년 The Univ. of Connecticut, 컴퓨터공학과 (박사)
1995년~현재 경기대학교 컴퓨터과학과 교수
2010년 7월~현재 경기대학교 CCSRC (콘텐츠융합 소프트웨어 연구센터) 센터장
2001년~2002년 2월 Michigan State Univ. 컴퓨터공학과 Pattern Recognition and Image Processing Lab. (PRIP) 방문 연구원
2009년~2010년 2월 Univ. of Colorado, Boulder, 컴퓨터과학과 Wellness Innovation and Interaction Lab. 방문 연구원
관심분야 : 비전기반 HCI, 컴퓨터 애니메이션, 증강현실, 의료영상처리 등
E-mail : jcchun@kgu.ac.kr