

# QoS-Guaranteed Capacity of Centralized Cognitive Radio Networks with Interference Averaging Techniques

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## **Abstract**

It is widely believed that cognitive radio (CR) networks have an opportunistic nature and therefore can only support best-effort traffics without quality-of-service (QoS) guarantees. In this paper, we propose a centralized CR network that adopts interference averaging techniques to support QoS guaranteed traffics under interference outage constraints. In such a CR network, a CR user adaptively adjusts its transmit power to compensate for the channel loss, thereby keeping the receive signal power at the CR base station (BS) at a constant level. The closed-form system capacity of such a CR network is analyzed and derived for a single cell with one CR BS and multiple CR users, taking into account various key factors such as interference outage constraints, channel fading, cell radius, and locations of primary users. The accuracy of the theoretical results is validated by Monte Carlo simulations. Numerical and simulation results show promising capacity potential for deploying QoS-guaranteed CR networks in frequency bands with fixed primary receivers. Our work can provide theoretical guidelines for the strategic planning of centralized CR networks.

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**Keywords:** Cognitive radio network, capacity, interference outage constraint, QoS

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## 1. Introduction

**R**adio spectrum is a precious natural resource, the use of which is strictly licensed by governments. Because most of the usable spectrum is already licensed to the incumbents, it is becoming increasingly difficult to find more spectrum to accommodate the exploding wireless data traffics. However, numerous spectrum monitoring measurements have shown that, in most of the time, many licensed frequency bands remain underutilized [1]-[3]. As a result, cognitive radio (CR) [4]-[6] has been proposed to improve spectrum utilization by “recycling” underutilized licensed spectrum. A CR network operating as a secondary network can exploit the radio spectrum by collectively sensing the radio environment and adaptively adjusting its radio parameters without harming the service of primary networks.

CR networks can be roughly classified into three types: interweave CR, underlay CR and overlay CR networks [7]. In the interweave CR networks, the CR (secondary) user transmits in spectrum holes [5] that are not in use by the licensed (primary) users. In this case, the CR networks coexist with the primary system by operating with orthogonal radio resources. In the underlay CR networks, a very tight transmit power constraint is imposed on CR transmitters to ensure that the interference caused by CR systems is below the noise floor and hence negligible. An underlay network (e.g., ultra-wideband system) typically requires a large bandwidth to compensate for the very low transmit power. In the overlay CR networks, the CR can cause significant interference at the primary receiver, under the condition that the performance degradation at the primary receiver is carefully managed by imposing certain interference constraints, such as peak interference power constraint (e.g., [8][9]), average interference power constraint (e.g., [8]-[10]), and interference outage constraint (e.g., [11]). Compared with interweave CR networks, overlay CR networks have the potential to exploit more spectrum resources (i.e., secondary networks can overlap with primary networks in the same time, frequency, and space). Compared with underlay CR networks, overlay CR networks have the potential to transmit with much greater power. Therefore, the overlay networks represent the “true coexistence” and can achieve better spectrum utilization.

In this paper, we restrict our study to overlay CR networks. More specifically, our studies focus on overlay CR networks with interference outage constraints, which limits the probability that the interference signal power exceeds a certain threshold. Unlike peak and average interference power constraints, the outage constraint takes into account the stochastic nature of radio interference. Therefore it is more appropriate when the quality of service (QoS) of the primary system depends on the instantaneous signal to interference and noise ratio (SINR) (e.g., delay sensitive communication services) [11].

In wireless communication networks, it is an important task to guarantee certain QoS for the users, especially for users with real time services. This task, however, is very challenging because of the random variations in channel conditions and the resulting random fluctuations in received SINRs [12]. The situation is much more exasperated in wireless CR networks due to other factors such as the unpredictable activity of primary users and the availability of CR channels [13][14]. To provide guaranteed QoS in CR networks, much work has been explored through various methods such as routing design (e.g., [15]), MAC design (e.g., [16]), spectrum management (e.g., [17][18]) and power control [13]. These works, however, mostly focus on interweave CR networks. Moreover, these works are restricted to a particular aspect of system design and do not address the fundamental capacity limits of CR networks.

Capacity studies are important to understand the fundamental limits and long-term potential of overlay CR networks. The capacity upper bounds can be used as the first guideline for planning CR networks, e.g., for estimating its range, finding potential applications, and choosing operating frequencies (coexisting primary systems), before substantial efforts are invested to develop a full system. The capacity study of overlay CR networks has a combinatorial nature. A focused study can be performed for different types of networks (centralized or ad-hoc), with different types of interference constraints (peak, average, or outage constraints), at different levels (link, cell, or system levels), and using different capacity metrics (average capacity, outage capacity, bandwidth-guaranteed capacity, effective capacity, or transport capacity). Although there exists a substantial literature devoted to the capacity study of CR networks, all possible combinations of the above aspects have not been thoroughly investigated. The novelty of our work lies in filling in the research gap of the following combination: centralized, outage interference constraint, cell level, and bandwidth-guaranteed capacity. To our best knowledge, this is the first and only work that considers bandwidth-guaranteed capacity of centralized overlay CR system.

The capacity of overlay CR networks was investigated in [19] for a special case (GSM uplink band) under peak interference power constraints and in [10][20] under averaged-interference power constraints. However, the work in [10][19][20] did not address the case of interference outage constraint. In [12][14][21], S. Akin and M. C. Gursoy have extended the former research to the study of effective capacity [22] of QoS-guaranteed CR networks, but their works are also limited to the case of averaged-interference power constraints. Moreover, no closed-form capacity formula was provided. To our best knowledge, the capacity of QoS guaranteed overlay CR networks under interference outage constraints has rarely been investigated.

In this paper, we study a single CR cell with multiple CR users and one CR base station (BS). Our goal is to investigate the uplink system capacity of such a CR network under two constraints: (1) the interference outage constraint at primary users and (2) the QoS constraint for each CR user. Our contributions are summarized as follows. First, we propose a centralized CR network that uses interference averaging techniques to support QoS-guaranteed traffic under interference outage constraints. The closed-form capacity of such a CR network for non-fading channels is derived for the first time. Second, for the above CR network, we provide a theoretical framework to facilitate systematic investigation on the impacts of outage probability limit, channel fading and relative location of primary users in the cell on the system capacity. The accuracy of the theoretical framework is validated by Monte Carlo simulations. We further use an example to demonstrate the useful application of our theoretical frameworks for the strategic planning of centralized CR networks. Important guidelines for deploying the proposed networks in a practical band are provided.

The rest of this paper is organized as follows. The system model is introduced in Section 2. Section 3 studies the system capacities of the CR networks designed to support QoS guaranteed traffics. Numerical results, simulation results, and discussions are presented in Section 4. Finally, conclusions are drawn in Section 5.

## 2. System Model

### 2.1 Coexistence Scenario and Channel Model

The coexistence scenario we considered is shown in Fig. 1, where primary users (illustrated as disk antennas) and CR users (illustrated as mobile phones) coexist on a plane. We consider a

centralized circular CR cell with a BS located at the center and  $N$  CR users uniformly distributed within the cell. The cell radius is denoted as  $R$ . In this paper, we consider the uplink capacity analysis of the CR cell, while the same approach can be extended for downlink analysis.

The interference outage constraint can be specified in two cases. In the first case, the CR network is unaware of the location of primary receivers. Therefore, the interference outage constraint should be applied to any point within the cell. In the second case, if the exact locations of the primary users are known, the interference outage constraint can be applied only to the primary user locations. Without loss of generality, we focus on an arbitrary primary receiver within the cell. This primary receiver is illustrated as a disk antenna in **Fig. 1**. The distance from the primary receiver to the CR BS is denoted as  $r$  ( $0 \leq r \leq R$ ).

## 2.2 Channel Model

The channels from the CR transmitters to the primary receivers are referred to as interference channels. The instantaneous channel power gains from the  $j$ th ( $1 \leq j \leq N$ ) CR user to the  $i$ th ( $1 \leq i \leq \infty$ ) primary receiver is denoted as  $h_{i,j}^I$ . On the other hand, the channels from the CR transmitters to the center BS are referred to as access channels. The instantaneous channel power gain from the  $j$ th CR user to the BS is denoted as  $h_j^A$ . All radio channels under consideration are narrowband channels (or single carriers of a multi-carrier system) and are assumed to be stationary and ergodic.

In previous works such as [10][20][23], only the path loss effect is considered in the channel, ignoring other effects of shadowing and fading. Although such an assumption is not very realistic, it is convenient since they often lead to elegant analytical results which can reveal important insights without over-complicating the problem. Considering only the path loss we have

$$h_{i,j}^I = K_I / (d_{i,j}^I)^\alpha \quad (1)$$

$$h_j^A = K_A / (d_j^A)^\alpha \quad (2)$$

where  $K_I$  and  $K_A$  are constants related to path loss and antenna gains in the interference and access channels, respectively,  $d_{i,j}^I$  is the distance between the  $i$ th primary receiver and the  $j$ th CR transmitter,  $d_j^A$  is the distance between the  $j$ th CR transmitter and the BS, and  $\alpha$  is the path loss exponent typically ranging from 2 to 5 [24].

If shadowing and fading are further taken into account, (1) and (2) should be modified as

$$h_{i,j}^I = K_I \xi_{i,j}^I \eta_{i,j}^I / (d_{i,j}^I)^\alpha \quad (3)$$

$$h_j^A = K_A \xi_j^A \eta_j^A / (d_j^A)^\alpha \quad (4)$$

where  $\xi_{i,j}^I$  and  $\eta_{i,j}^I$  are random variables which model the effects of the shadowing and multipath fading in the interference channels, respectively. Similarly,  $\xi_j^A$  and  $\eta_j^A$  represent random shadowing and fading factors in the access channels, respectively. We assume that the shadowing factors  $\xi_{i,j}^I$  and  $\xi_j^A$  are mutually independent, each following a log-normal

distribution with zero mean and a standard deviation  $\sigma_\xi$  ranging from 5 to 12 dB [24, pp. 99] with 8 dB being a typical value for macrocellular applications. We further assume that the fading factors  $\eta_{i,j}^I$  and  $\eta_j^A$  are also mutually independent and follow identical distributions  $f_\eta(x)$ . When Nakagami fading channels are assumed, the power gain  $f_\eta(x)$  (squared-envelope) is given by a Gamma distribution [24, pp. 54]

$$f_\eta(x) = \frac{m^m x^{m-1}}{\Gamma(m)} \exp(-mx), \quad m \geq \frac{1}{2} \quad (5)$$

where  $m$  is the Nakagami shape factor and  $\Gamma(\cdot)$  denotes the gamma function.

Let  $\kappa_{i,j}^I = \xi_{i,j}^I \eta_{i,j}^I$  and  $\kappa_j^A = \xi_j^A \eta_j^A$  denote the composite shadowing and fading factor of the interference channel and access channel, respectively. It follows that all  $\kappa_{i,j}^I$  and  $\kappa_j^A$  follow the same Gamma-log-normal distribution [24, pp. 102] whose probability density function (PDF) is denoted as  $f_\kappa(x)$ . Such a Gamma-log-normal distribution can be approximated by a log-normal distribution as [24, pp. 102]

$$f_\kappa(x) \approx \frac{1}{K_\epsilon \sqrt{2\pi\sigma x}} \exp\left\{-\frac{(10\log_{10} x - \mu)^2}{2\sigma^2}\right\}. \quad (6)$$

In (6), the mean  $\mu$  and variance  $\sigma^2$  are given by [24]

$$\mu = K_\epsilon^{-1} [\psi(m) - \ln(m)] \quad (7)$$

$$\sigma^2 = K_\epsilon^{-2} \zeta(2, m) + \sigma_\xi^2 \quad (8)$$

respectively, where  $K_\epsilon = \ln(10)/10$  is a constant. In (7),  $\psi(\cdot)$  is the Euler psi function given by [24]  $\psi(m) = -K_{Eu} + \sum_{k=1}^{m-1} (1/k)$  ( $m=1, 2, \dots$ ), where  $K_{Eu} \approx 0.5772$  is Euler's constant. In (8),  $\zeta(\cdot, \cdot)$  is Riemann's zeta function given by [24]  $\zeta(2, m) = \sum_{k=0}^{\infty} \frac{1}{(m+k)^2}$  ( $m=1, 2, \dots$ ).

When  $m=1$  (i.e., Rayleigh fading) the approximation in (6) is valid for  $\sigma_\xi > 6$  dB, and for  $m > 2$  the approximation is valid for all ranges of  $\sigma_\xi$  of interest [24].

### 2.3 Access Procedure and Interference Averaging

We assume that the CR network coexist with a primary system in a licensed frequency band. The licensed band is further divided into a number of orthogonal channels (e.g., subcarriers), each of which is subject to an interference outage constraint to protect the primary system. Without loss of generality, identical interference constraints are assumed for all channels. Multiple CR users are scheduled by the CR BS to transmit in orthogonal channels to avoid mutual interferences. Furthermore, frequency hopping is adopted as an interference averaging technique to protect the primary system. Fig. 2 illustrates a possible frequency hopping scheme of a six-user cell, where users are indexed by letters ( $a$  to  $f$ ) and time is divided into slots. Frequency hopping works in a way that the same frequency band is randomly allocated to different CR users within the cell in different time slots. In other words, a CR user hops to a different frequency channel is each time slot. Given that CR users are randomly distributed

and frequency hopping is fast enough, the usage of a particular frequency band will be geographically randomized, thereby reducing the probability of interference outage to a particular primary receiver in a particular band.

When only one particular channel is under concern, we can consider an equivalent network that has only one (target) CR user, whose location is uniformly distributed within the cell and updates in every time slot. Since our subsequent analysis will deal with a single target CR user (with random locations) and a single primary receiver (with an arbitrary location), we can drop the CR user index  $i$  and primary receiver index  $j$  to simplify the notations.

## 2.4 Assumptions

The following assumptions are made for our subsequent analysis. First, we assume that a CR user have perfect knowledge of the channel between itself and the CR BS. Power control is used by a CR user to adjust its transmit power according to the instantaneous channel gain, so that a predefined SNR is maintained at the BS. Such a power control mechanism is similar to what has been widely applied in CDMA networks. Unlike many works in the literature (e.g., [10][20][25][26]), we assume that a CR user does not have information about the interference channel (i.e., channel from CR user to primary receiver). We note that this is more realistic as most primary receivers are hidden terminals in practice.

Second, we assume that the maximum transmit powers for CR devices are very high. Therefore the transmit powers of CR users are always limited by the interference constraints rather than CR device capabilities. This assumption allows us to study the fundamental limit of CR system capacity, which is entirely constrained by primary networks. We note that such a dependence on the primary network is what distinguishes CR networks from conventional networks, and is therefore the focus of our study.

## 3. Capacity of QoS-Guaranteed CR Network

In this section, we study the system capacities of the CR networks designed to support QoS guaranteed traffics in non-fading channels and fading channels, respectively. The closed-form capacities in both cases are derived. Assume that a received power  $Y_0$  is required at the CR BS to support certain QoS. We can then write

$$P = Y_0 / h^A \quad (9)$$

where  $P$  is the transmit power of the target CR user and  $h^A$  is the access channel gain. The interference perceived by the primary receiver is given by

$$I = Ph^I = Y_0 h^I / h^A \quad (10)$$

where  $h^I$  is the interference channel gain. Clearly, the interference  $I$  would appear to be a time-varying stochastic signal when we consider the location of the target CR user changes in every time slot. We further assume that in order to protect the primary services, the probability that  $I$  exceeds a threshold  $I_{out}$  should be smaller than an outage probability limit  $\chi$ . Such an interference outage constraint in turn imposes a limit on  $Y_0$  and allows us to express the capacity

$$C = \log_2(1 + Y_0 / \Omega_0) \quad (11)$$

as a deterministic function of  $I_{out}$  and  $\chi$ . In (11),  $\Omega_0$  denotes the total interference and noise power perceived at the BS.

### 3.1 Capacity for Non-fading Channels

Here we consider only the path loss and aim to express  $C$  in (11) as a closed-form function of  $I_{out}$  and  $\chi$ . First, we need to obtain the PDF/CDF of the interference  $I$ . Substituting (1) and (2) into (10) we get

$$I = Y_0 \frac{K_I}{K_A} \left( \frac{d^A}{d^I} \right)^\alpha \quad (12)$$

where  $K_I$  and  $K_A$  are constants related to path loss and antenna gains in the interference and access channels, respectively,  $d^I$  is the distance between the target CR transmitter and the primary receiver,  $d^A$  is the distance between the target CR transmitter and the BS. In (12),  $d^A$  and  $d^I$  are correlated random variables, making it difficult to directly calculate the interference distribution. To solve the problem, let us define a new random variable  $U = (d^I / d^A)^2$ , the CDF of  $U$  can be derived as (see Appendix)

$$F_U(x) = \begin{cases} \frac{k^2 x}{(1-x)^2}, & x \in [0, (1-k)^2], \\ \frac{1}{\pi} \left[ \theta_1 + \theta_2 \frac{k^2 x}{(1-x)^2} - \frac{k \sin \theta_1}{|1-x|} \right], & x \in [(1-k)^2, 1], \\ \frac{1}{\pi} \arccos(k/2), & x = 1, \\ 1 - \frac{1}{\pi} \left[ \theta_1 + \theta_2 \frac{k^2 x}{(1-x)^2} - \frac{k \sin \theta_1}{|1-x|} \right], & x \in (1, (1+k)^2), \\ 1 - \frac{k^2 x}{(1-x)^2}, & x \in ((1+k)^2, \infty). \end{cases} \quad (13)$$

where  $k = r/R$  ( $0 \leq k \leq 1$ ) and  $\arccos(\cdot)$  is the inverse cosine function. In (13),  $\theta_1$  and  $\theta_2$  are given by

$$\theta_1 = \arccos \left( \frac{(1-x)^2 + k^2 - xk^2}{2k|1-x|} \right) \quad (14)$$

$$\theta_2 = \arccos \left( \frac{xk^2 + k^2 - (1-x)^2}{2k^2 \sqrt{x}} \right) \quad (15)$$

respectively. For convenience in subsequent derivation, let us rewrite (12) in the dB scale

$$(I)_{dB} = (Y_0)_{dB} - (K_A)_{dB} + (K_I)_{dB} - (V)_{dB} \quad (16)$$

where  $(I)_{dB} = 10 \log_{10} I$ ,  $(K_A)_{dB} = 10 \log_{10} K_A$ ,  $(K_I)_{dB} = 10 \log_{10} K_I$ ,  $(Y_0)_{dB} = 10 \log_{10} Y_0$ , and  $(V)_{dB} = 5\alpha \log_{10} U$ . Using the transformation of random variables [27], the CDF of  $(V)_{dB}$  can be written as

$$F_{(V)_{dB}}(x) = F_U \left( 10^{\frac{x}{5\alpha}} \right). \quad (17)$$

From (16) and (17), we can easily obtain the CDF of  $(I)_{dB}$  as

$$F_{(I)_{dB}}(x) = 1 - F_{(V)_{dB}}(Y_0 K_I / K_A - x). \quad (18)$$

We now consider an interference outage constraint which specifies that  $(I)_{dB}$  should not exceed a threshold  $(I_{out})_{dB} = 10 \log_{10} I_{out}$  with an outage probability of  $\chi$ , namely

$$1 - F_{(I)_{dB}} \left[ (I_{out})_{dB} \right] \leq \chi. \quad (19)$$

Substituting (18) into (19) we get

$$(Y_0)_{dB} \leq (I_{out})_{dB} + (K_A)_{dB} - (K_I)_{dB} + F_{(V)_{dB}}^{-1}(\chi) \quad (20)$$

where  $F_{(V)_{dB}}^{-1}(\chi)$  is the inverse function of  $F_{(V)_{dB}}(x)$  given by (17). From (20) we can see that the maximum received signal power at the secondary BS is bounded by a function of the interference outage constraint parameters  $I_{out}$  and  $\chi$ . The uplink capacity is then given by

$$C = \log_2 \left( 1 + 10^{\left[ (I_{out})_{dB} + (K_A)_{dB} - (K_I)_{dB} - (\Omega_0)_{dB} + F_{(V)_{dB}}^{-1}(\chi) \right] / 10} \right) \quad (21)$$

where  $(\Omega_0)_{dB} = 10 \log_{10} \Omega_0$  is the total interference and noise power perceived at the BS in the dB scale.

### 3.2 Capacity for Fading Channels

Substituting (3) and (4) into (10) and dropping the user indices we have

$$I = Y_0 \frac{K_I}{K_A} \left( \frac{d^A}{d^I} \right)^\alpha \frac{\kappa^I}{\kappa^A}. \quad (22)$$

In the dB scale (22) can be written as

$$(I)_{dB} = (Y_0)_{dB} - (V)_{dB} + (K_I)_{dB} - (K_A)_{dB} + (\kappa^I)_{dB} - (\kappa^A)_{dB} \quad (23)$$

where  $(I)_{dB}$ ,  $(Y_0)_{dB}$ , and  $(V)_{dB}$  are given in (16),  $(\kappa^I)_{dB}$  and  $(\kappa^A)_{dB}$  are corresponding dB values of  $\kappa^I$  and  $\kappa^A$ , respectively. In (23),  $(V)_{dB}$  is a random variable whose PDF  $f_{(V)_{dB}}(x)$  can be obtained by taking the derivative of its CDF  $F_{(V)_{dB}}(x)$  given by (17). In addition,  $(\kappa^I)_{dB}$  and  $(\kappa^A)_{dB}$  are independent Gaussian random variables. Let us define  $(T)_{dB} = -(V)_{dB} + (\kappa^I)_{dB} - (\kappa^A)_{dB}$ . Its PDF  $f_{(T)_{dB}}(x)$  can be obtained numerically as the convolution of the individual PDFs of  $(V)_{dB}$ ,  $(\kappa^I)_{dB}$ , and  $(\kappa^A)_{dB}$ . From  $f_{(T)_{dB}}(x)$ , we can get the corresponding CDF  $F_{(T)_{dB}}(x)$  through numerical integration. Since  $(I_{out})_{dB} = (P)_{dB} + (T)_{dB}$ , it follows that the CDF of  $(I_{out})_{dB}$  is related to  $F_{(T)_{dB}}(x)$  by

$$F_{(I)_{dB}}(x) = F_{(T)_{dB}} \left[ x - (Y_0)_{dB} + (K_A)_{dB} - (K_I)_{dB} \right]. \quad (24)$$



Considering the interference outage constraint specified in (19), from (24) it is easy to show that  $(Y_0)_{dB}$  is bounded by

$$(Y_0)_{dB} \leq (I_{out})_{dB} + (K_A)_{dB} - (K_I)_{dB} - F_{(T)_{dB}}^{-1}(1-\chi) \quad (25)$$

where  $F_{(T)_{dB}}^{-1}(\cdot)$  is the inverse function of  $F_{(T)_{dB}}(x)$ . We can then obtain the uplink cell capacity as

$$C = \log_2 \left( 1 + 10^{\left[ (I_{out})_{dB} + (K_A)_{dB} - (K_I)_{dB} - (\Omega_0)_{dB} - F_{(T)_{dB}}^{-1}(1-\chi) \right] / 10} \right). \quad (26)$$

#### 4. Numerical Results and Discussions

In this section, we present the capacities of the proposed CR networks designed to support QoS guaranteed traffics in realistic scenarios. As a practical application, we consider a fixed microwave communication (FMC) system as the primary network. FMC systems (e.g., fixed WiMax) are characterized by highly directive transmissions that, intuitively, present opportunities for other systems to share their spectrum. Non-hierarchical spectrum sharing with FMC systems has been considered in e.g., [28][29], for non-cognitive systems. Here we will consider (hierarchical) spectrum sharing in the new context of CR networks. It is interesting to note that interweave CR networks may not be able to operate in these bands because highly directive primary transmissions are difficult to be detected reliably. The overlay CR then becomes the only feasible CR system to exploit spectrum opportunities in these bands.

We assume an omnidirectional antenna at the CR BS and directional antenna at the primary receivers. Based on the practical parameters discussed in [28] we assume that  $K_A / K_I = 10$ ,  $I_{peak} / \Omega_0 = 1$ ,  $I_{out} / \Omega_0 = 1$ , the path loss exponent  $\alpha = 4$ , and the shadowing standard deviation  $\sigma_\xi = 8$  dB. Three different channel configurations, i.e., the path loss-only model, the path loss-shadowing model, and the path loss-shadowing-fading models will be used for comparison purpose to reveal the impacts of shadowing and fading on the capacity.

First, as revealed by (13), it is important to note that the capacity is not dependent upon the absolute value of the cell radius  $R$ , but rather on the relative location of primary users in the cell  $k = r / R$ . Based on (21), Fig. 3 shows the capacity  $C$  as a function of  $\chi$  for non-fading channels. As expected, the capacity increases with increasing outage limit  $\chi$ . A bigger value of  $\chi$  means that the primary network is more tolerant to the interference, thereby allowing the CR network to gain more capacity. Simulation results are shown to match well with the numerical results.

Changing to fading channel models, Fig. 4 shows the capacity  $C$  as a function of  $\chi$  based on (26). Again, good matches between numerical and Monte Carlo simulation results are observed. Comparing Fig. 4 with Fig. 3, we found that log-normal shadowing has a significant impact on the capacity. Furthermore, the impact of small scale fading on the capacity is investigated in Fig. 5. We choose an example  $r = 0.3R$  and vary the value of the Nakagami shape factor  $m$  to represent different small-scale fading scenarios. The case of  $m = 1$  corresponds to Rayleigh fading. Small-scale fading is found to have a smaller impact on the capacity compared with shadowing. The reason is that, mathematically speaking, the

Nakagami fading distributions have shorter tails than the log-normal shadowing distribution, and therefore have less impact on a outage-based metric.

In **Figs. 3** and **4**, the capacities are shown to be sensitive to the exact location of the primary receiver. In **Fig. 3**, primary receivers closer to the edge of the cell have greater impact on the capacity when the interference outage limit is small. When the outage limit increases, the receivers near the BS tend to limit the performance. This is because under the assumptions of power control and deterministic (non-fading) channels, primary receivers closer to the CR BS have smaller dynamic ranges of the received interference power, which translates to a flatter capacity curve under interference outage constraints. On the contrary, in **Fig. 4**, the interference has a large dynamic range due to channel fading, therefore the capacity curves change accordingly. **Fig. 4** shows that the capacity is limited by the primary user with the shortest distance to the CR BS. In an extreme case, if the primary receiver is colocated with the CR BS, the system capacity will decrease to zero for non-zero  $\chi$ . We note that when there are multiple primary users in the cell, the capacity is limited by the minimum value among all the capacity curves.

In summary, the following guidelines are provided for the CR network supporting QoS-guaranteed traffic. First, deployment of such CR networks in the FMC band seems feasible with the use of interference averaging techniques. Second, in non-fading channel scenarios (e.g., the primary receivers and CR BS are high raised to result in a high probability of light-of-sight (LoS) channels), **Fig. 3** suggests that the location of the CR BS should be carefully chosen with respect to primary user locations and  $\chi$  to maximize the system capacity. Third, in fading channel scenarios, **Fig. 4** suggests that the CR BS location should be chosen to maximize its shortest distance to any primary receivers. Fourth, since the CR network performance is shown to depend heavily on the propagation environment (e.g., shadowing), scenario-specific network optimization would be important.

## 5. Conclusions

In this paper, we have proposed a centralized CR network with interference averaging techniques to support QoS-guaranteed traffic under interference outage constraints. The system capacities of the network have been analyzed at the single-cell level, taking into account all major system parameters (e.g., outage probability limit, channel fading and relative location of primary users). It has been found that the capacity is susceptible to the channel shadowing as well as the locations of primary users in the cell, but insensitive to the cell radius and small-scale fading. Numerical and simulation results have shown promising capacity potential for deploying QoS-guaranteed CR networks in the FMC bands. Important deployment guidelines have been discussed. We expect that the flexible analytical framework provided in this paper can be adopted for the strategic planning of a wide range of centralized CR networks.

## Appendix: Derivation of (13)

In this Appendix, we wish to find the CDF  $F_U(x)$  of the random variable  $U$  given by  $U = (d^I / d^A)^2$ , where  $d^A$  and  $d^I$  are distances from the target CR user to the CR BS and the primary user, respectively. As illustrated in **Fig. 3**, CR users are uniformly distributed in a circular CR cell of radius  $R$ . The BS is at the center of the cell and the distance between the

primary user and the CR BS is  $r$ .

One approach to find the PDF of  $U$  is by using transformations of random variables. This approach, however, leads to a definite integral which cannot be solved in a closed-form. Instead, a geometry-based method will be introduced in this Appendix to give a closed-form expression of  $F_U(x)$ . First of all, as shown in Fig. 6, we put the CR network into a  $(x, y)$  coordinate system. The coordinates of the BS (point  $O_1$ ), primary user (point  $P$ ) and CR user are given by  $(r/2, 0)$ ,  $(-r/2, 0)$ , and  $(x, y)$ , respectively. We consider this coordinate system as a probability plane corresponding to the location distribution of the CR users. Since the CR users are uniformly distributed within the circle  $O_1$ , the disk area  $O_1$  has a probability density of  $1/R^2$  and elsewhere the probability density is zero.

Under this coordinate system, we have

$$U = \frac{(x+r/2)^2 + y^2}{(x-r/2)^2 + y^2}. \quad (27)$$

The set of the points  $(x, y)$  that fulfill (27) forms a circle  $O_2$  with an origin located at  $\left(-\frac{r}{2} \left(\frac{1+U}{1-U}\right), 0\right)$  and a radius  $\tilde{r} = |1-U|$ . The CDF of  $U$  can be obtained by investigating the relationships between circle  $O_1$  and circle  $O_2$ . For convenience, let us denote  $k = r/R$ . With  $U$  increasing from 0 to  $\infty$ , we have the following five stages:

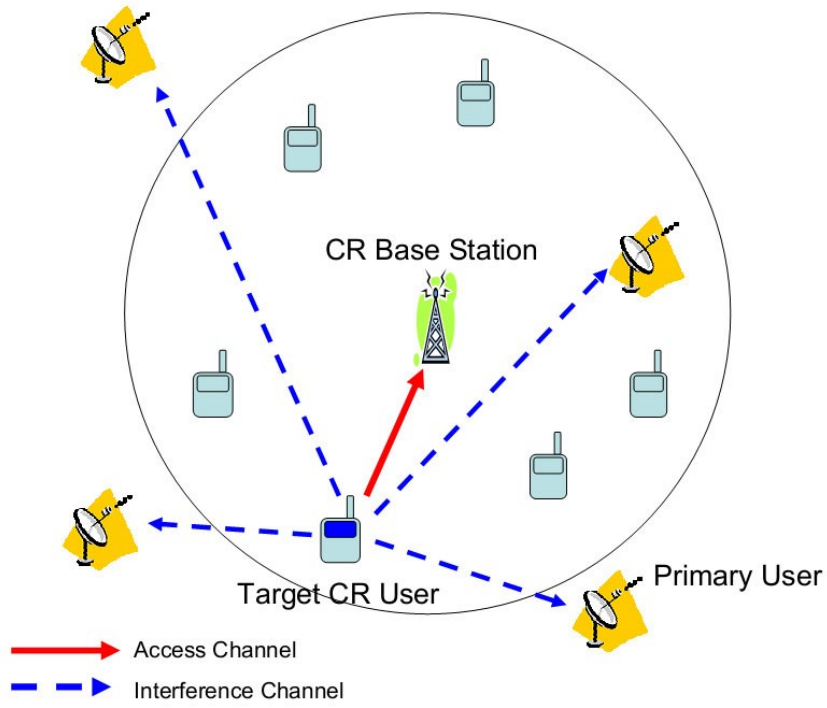
- 1) When  $U$  increases from 0 to  $(1-k^2)$ , circle  $O_2$  starts as a small circle around point  $P$  and become a circle that internally tangents circle  $O_1$ . The CDF  $F_U(x)$  is area of disk  $O_2$  times the probability density, i.e.,  $F_U(x) = \tilde{r}^2 / R^2$ ,  $x \in [0, (1-k)^2]$ .
- 2) When  $U$  increases from  $(1-k)^2$  to 1, circle  $O_2$  intersects circle  $O_1$ . The CDF  $F_U(x)$  is the common area of circles  $O_1$  and  $O_2$ , denoted as  $S$ , times the probability density, i.e.,  $F_U(x) = S / \pi R^2$ ,  $x \in [(1-k)^2, 1]$ , where  $S$  can be easily obtained using basic geometry  $S = \theta_1 R^2 + \theta_2 \tilde{r}^2 - \frac{1}{2} D^2 \sin(\theta_1)$ . Here, we have  $\theta_1 = \arccos\left[\frac{R^2 + D^2 - \tilde{r}^2}{2RD}\right]$ ,  $\theta_2 = \arccos\left[\frac{\tilde{r}^2 + D^2 - R^2}{2\tilde{r}D}\right]$ , and  $D = r/|1-x|$  is the distance between point  $O_1$  and point  $O_2$ .
- 3) When  $U = 1$ , the CDF  $F_U(x)$  is the probability density times the area which is within circle  $O_1$  and left to the  $y$ -axis. Using basic geometry we can get  $F_U(x) = \arccos[r/(2R)]$ ,  $x = 1$ .
- 4) When  $U$  increases from 1 to  $(1+k)^2$ , circle  $O_2$  intersects circle  $O_1$ . The complementary CDF  $1 - F_U(x)$  is the common area  $S$  times the probability density. It follows that the CDF is given by  $F_U(x) = 1 - S / (\pi R^2)$ ,  $x \in (1, (1+k)^2)$ , where  $S$  is given in stage 2.
- 5) When  $U$  increases from  $(1+k)^2$  to  $\infty$ , circle  $O_2$  is inside circle  $O_1$  and gradually

converges toward point  $O_2$ . The complementary CDF  $1 - F_U(x)$  is given by the area of disk  $O_2$  times the probability density. It follows that the CDF is given by  $F_U(x) = 1 - \tilde{I}$ ,  $x \in ((1+k)^2, \infty)$ . Finally, (13) can be obtained by summarizing the above five steps.

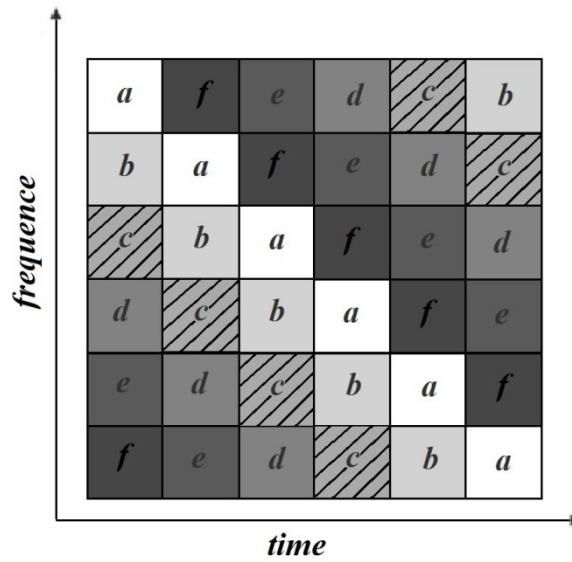
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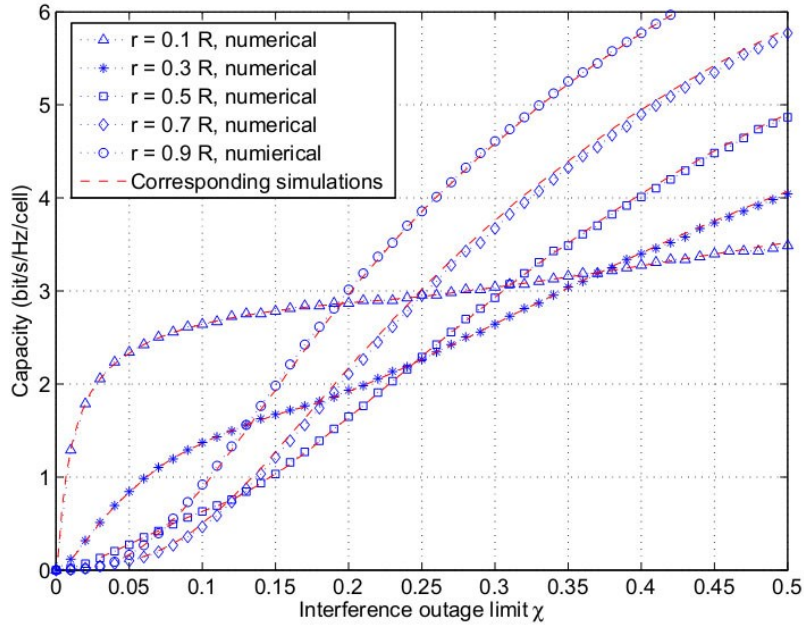
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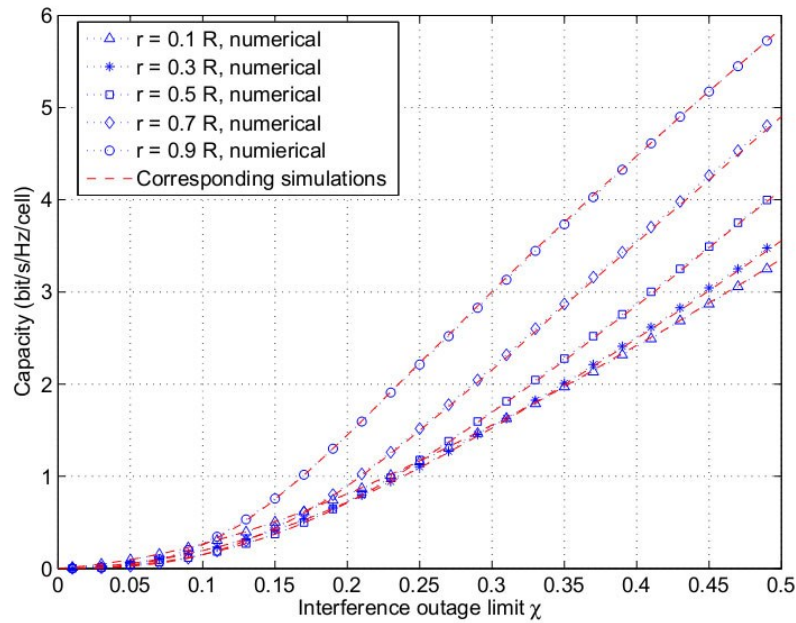
**Fig. 1.** System model of centralized CR networks.



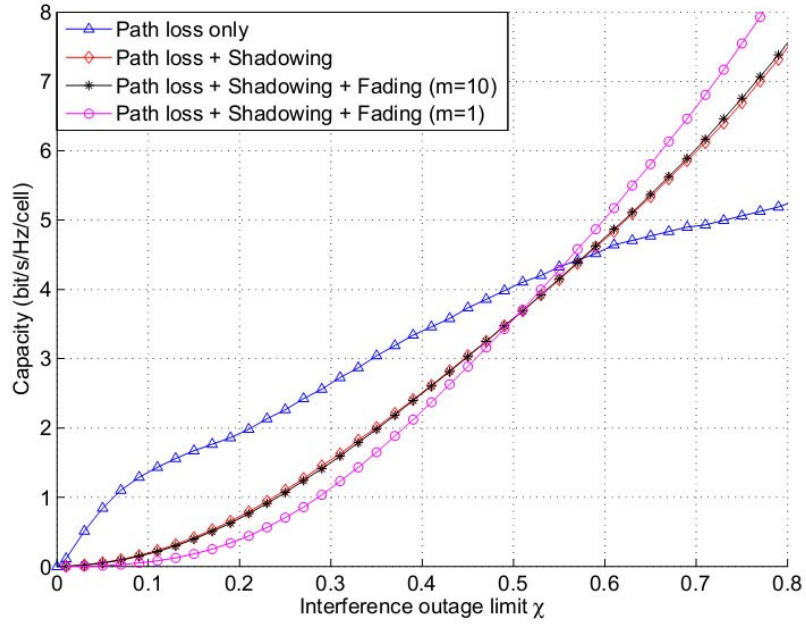
**Fig. 2.** Illustration of interference averaging techniques.



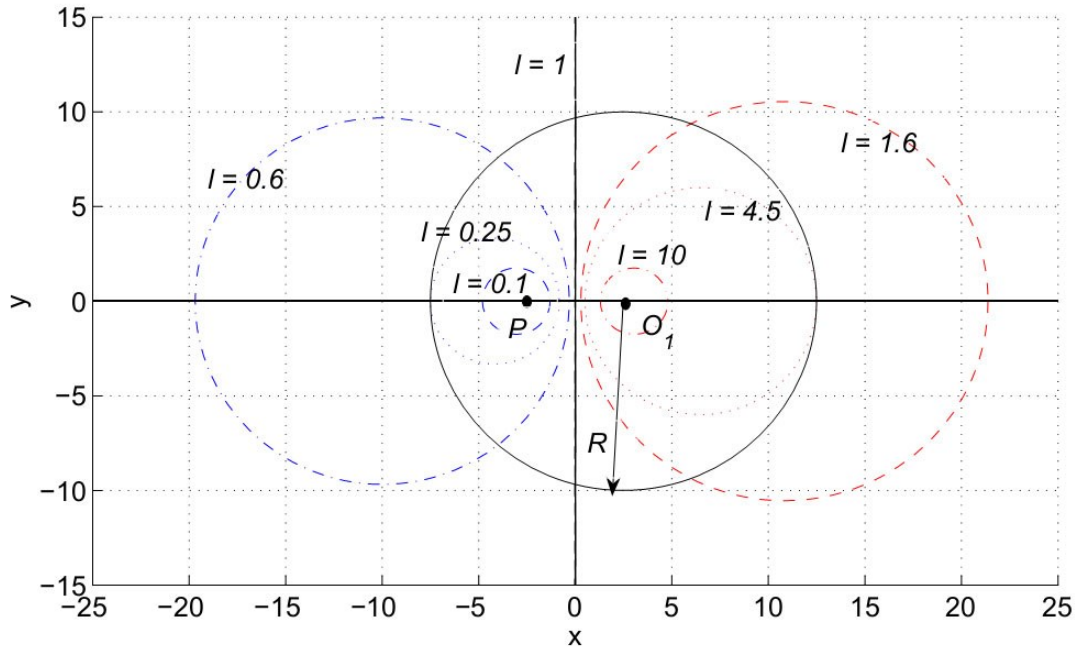
**Fig. 3.** Capacity of CR networks supporting QoS-guaranteed traffic as a function of  $\chi$  with different values of  $r/R$  ( $I_{out}/\Omega_0=1$ ,  $K_A/K_I=10$ , path loss-only channel model).



**Fig. 4.** Capacity of CR networks supporting QoS-guaranteed traffic as a function of  $\chi$  with different values of  $r/R$  ( $I_{out}/\Omega_0=1$ ,  $K_A/K_I=10$ ,  $\sigma_\xi=8$  dB, path loss-shadowing channel model).



**Fig. 5.** Capacity of CR networks supporting QoS-guaranteed traffic as a function of  $\chi$  under different channels ( $I_{out}/\Omega_0=1$ ,  $K_A/K_I=10$ ,  $r/R=0.3$ ,  $\sigma_\xi=8$  dB).



**Fig. 6.** Illustration of the geometry-based approach for the derivation of (13).





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