

Examination of Prospective Teachers' Perceptions on Mathematical Concepts and Their Potential Teaching Strategies¹

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(Received November 24, 2013; Revised March 20, 2014; Accepted March 21, 2014)

This study examined the potential teaching strategies of prospective elementary teachers and their perceptions of the procedural/conceptual nature of examples. Fifty-four prospective teachers participated in this study, engaging in two-phase tasks. Analysis of data indicated that:

- (a) Overall, the participants' perceptions were geared toward putting emphasis on conceptual understanding rather than procedural understanding; but
- (b) Generally, procedure-oriented strategies were more frequently incorporated in participants' potential teaching plans. This implied that participants' preconceived ideas regarding math examples were not always reliable indicators of their potential teaching strategies. Implications and suggestions for mathematics teacher preparation are discussed.

Keywords: pre-service teacher education, conceptual/procedural knowledge, teaching strategies, beliefs

MESC Classification: D40, D30, C29, C49, C79

MSC2010 Classification: 97C70, 03B42

INTRODUCTION

“We have to teach for conceptual understanding.”

“It is more important to explain mathematical processes rather than simply execute the memorized rules.”

“It is important to represent a concept in more than one mode to support various learners and to enhance their understanding.”

¹ This paper will be presented at KSME 2014 Spring Conference on Mathematics Education at Hankuk Univ. of Foreign Studies, Dongdaemun-gu, Seoul 130-791, Korea; April 4–5, 2014.

How wonderful it is to hear prospective teachers say these statements! In fact, these prospective teachers' statements are inline with the general consensus of the mathematics education community in that effective teachers focus on promoting students' understanding by employing a variety of teaching strategies, rather than delivering isolated facts or rules (e.g., National Council of Teachers of Mathematics [NCTM], 2000; NCTM Position Statement, 2005). For the prospective teachers in a K-8 mathematics methods class who enter the final stage of pre-service teacher education, these statements seem natural and actionable. However, one task in the mathematics methods class shows a large variation in the perceptions and interpretations of the aforementioned statements among these prospective teachers. This paper reports on an examination of elementary prospective teachers' perceptions of sample mathematical tasks and proposed teaching strategies. The aim is to unpack ideas related to what prospective teachers believe as compared to their potential teaching strategies, with a focus on conceptual versus procedural perspectives.

RELATED ISSUES IN THE LITERATURE

Knowledge, Skills, and Understanding for Effective Teaching

The aspects of knowledge, skills, and understanding are essential in the discussion of effectively teaching and learning mathematics. Many educators have strived to explain various aspects of the learning and teaching process, though the terms they have used vary. Several frames suggested by research include procedural/conceptual knowledge (Hiebert & Lefevre, 1986), instrumental/relational understanding (Skemp, 1987), ritual/principled knowledge (Edwards & Mercer, 1987), and operational/structural conceptions (Sfard, 1991). The mathematics education community holds the general opinion that none of these distinctions are absolutely dichotomous. For example, the National Council of Teachers of Mathematics' (2000) *Principle and Standards for School Mathematics* clearly depicts the importance of "the alliance of factual knowledge, procedural proficiency, and conceptual understanding" (p. 20) as a powerful way of teaching and learning mathematics. However, it seems the public perception of these messages tells a slightly different story. For instance, in the survey conducted by Bossé & Bahr (2008), a group of mathematics teacher educators claimed that conceptual understanding and procedural knowledge are both necessary and important as general statements. However, what appeared in the features of each end identified by the participants was a strongly polarized view on the conceptual-procedural frame. The characteristics of conceptual understanding described by the respondents could be understood as unanimously positive, whereas procedural knowledge was described in an overwhelmingly negative tone and context. There is a line of research that reemphasizes the fact that various terminological distinctions ac-

tually combine many dimensions into one. This demonstrates that multiple layers of characteristics and relationships can be learned and acquired in tandem, rather than independently, and there is a need to develop useful theory to explain how conceptual knowledge and procedural knowledge are related (e.g., Baroody, Feil, Johnson, 2007; Rittle-Johnson, Siegler, Alibali, 2001; Star, 2000, 2005). The findings from prior research prompted this study to further investigate how future teachers generally understand the conceptual/procedural frame and what specific thoughts are ingrained in their perceptions.

Teachers' Beliefs, Practice, and the Noted Gap

Numerous research studies have examined teachers' beliefs in multiple areas, including the nature of subject matter, teaching and student learning, and the influence of beliefs on instructional practices (e.g., Barlow & Reddish, 2006; Cooney, Shealy, & Arvold, 1998; Cross, 2009; Ernest, 1988; Levitt, 2001; Ogan-Bekiroglu & Akkoc, 2009; Pajares, 1992; Stipek, Givvin, Salmon, & MacGyvers, 2001; Thompson, 1992). These studies garnered mixed results regarding changes in teachers' or prospective teachers' beliefs toward teaching and learning various subjects. In Barlow and Reddish's (2006) study, 76 teacher candidates completed a survey that was identical to that which was used in Frank's (1990) study that examined beliefs and myths about mathematics. It concluded that various mathematical myths have remained constant over 15 years despite changing educational standards. One of the main themes identified was that prospective teachers view mathematics as a set of rules, procedures, or facts that must be memorized. In contrast, other research studies in science education indicated that most prospective teachers held instructional beliefs aligned with the philosophy of current science education reform (Levitt, 2001; Ogan-Bekiroglu & Akkoc, 2009).

Another line of research examined the relationship between teachers' beliefs or generally manifested curriculum ideas and teachers' actual classroom practice. Most of the research affirms that what teachers claim to believe and what they actually perform in the classroom are often inconsistent for various reasons (e.g., Raymond, 1997; Schorr, Firestone, & Monfils, 2001; Spillane & Zeuli, 1999). For example, teachers in Spillane and Zeuli's (1999) study addressed their familiarity of reform-oriented practice by frequently mentioning phrases like "hands-on activities," "conceptual understanding," "multiple representations," and "mathematical connections" to describe their teaching. However, these descriptions were distinctly different from their actual classroom teaching practices, which appeared to remain quite traditional.

Studies with real-life connections also show similar discrepancies between manifested ideas in written curriculum and enacted teaching practice. The TIMSS 1999 Video Study (Hiebert et al., 2003), for example, showed that most of the countries studied contained-

low amounts (9–27%) of problems connected to real life in their curriculum materials. The distinct exception was the case of the Netherlands (42%). However, Mosvold's (2008) study provides a different perspective on this result. Using the TIMSS 1999 Video Study materials, Mosvold (2008) compared the cases of Japan and the Netherlands with respect to real-life connections in actual teaching implementation. His study revealed that Japanese teacher participants adopt the principles of real-life connections more closely than their Dutch counterparts in the actual teaching implementation. This signifies that the amount of real-life connections in the written curriculum is not a sole indicator of the quality of real-life connections happening in the actual classroom.

The previous research in this area mainly focuses on in-service teachers' beliefs and teaching practices, reporting that many external factors, such as school culture, curriculum mandates, and class size, impact the discrepancy between teachers' thinking and practice (Hart, 2002; Valderrama-Aguelo, Clarke, & Bishop, 2007). Informed by previous research, this study focuses on prospective teachers' thinking and doing in a context in which the presence of the aforementioned external factors is not evident. It is expected that the similarities and differences between the previous research with in-service teachers and this study with teacher candidates will provide some insight for bridging the gap, if one exists.

Supporting Prospective Teachers: Problems and Possibilities

Noting various concerns, such as the lack of sound subject matter knowledge and the strong influence of prospective teachers' past experiences as students, the consensus in the mathematics education community is that it is essential to provide prospective teachers with ample opportunities to improve and transform their knowledge, beliefs, and actual teaching practice. However, what prospective teachers bring to teacher education programs is, admittedly, often difficult to greatly change in the limited time period. For example, candidates' beliefs about what to teach and how to teach it have been consciously or unconsciously developed through many years of observations of their own K–12 teaching practices, and it is almost improbable to completely transform their beliefs (e.g., Ambrose, 2004; Doerr & Lesh, 2003; Eisenhart, Borko, Underhill, Brown, Jones, & Agard, 1993; Thompson, 1992).

With these factors considered, several research studies suggest searching for alternative ways to support prospective teachers' ability to teach. Many researchers urge teachers to reflect upon their own teaching as a way of transforming their beliefs and making sound instructional decisions for their students (e.g., Cooney, 1999; Schön, 1983; Simon, 1995). One problem is that the prospective teachers' actual teaching practice is usually limited to only a few hours in a field experience setting while taking the mathematics me-

thods course. With this in mind, more recent studies are geared toward creating explicit, task-oriented environments that provide more realistic classroom conditions. For example, Timmerman (2004) reported a positive shift in the prospective teachers' beliefs using three interventions (e.g., problem-solving journals, structured interviews, and peer teaching). Knowledge of mathematics, knowledge of children's thinking, and knowledge of teaching practice were identified by these interventions. Hiebert et al. (2007) proposed a framework for teacher education programs that intends to better support teacher candidates with learning how to teach by studying teaching. The framework contains four skills:

- (a) Specifying the learning goal(s) for the instructional episode (what are students supposed to learn);
- (b) Conducting empirical observations of teaching and learning (what did students learn);
- (c) Constructing hypotheses about the effects of teaching on students' learning (how did teaching help or not help students learn?); and
- (d) Using analysis to propose improvements in teaching (how could teaching more effectively help students learn?).

In the follow-up study, Morris & Hiebert (2009) report the successes and challenges that prospective teachers are likely to experience as they identify subconcepts and subskills for target learning goals. This study reports that prospective teachers can identify mathematical subconcepts of learning goals in supportive contexts (e.g., when a student gives an incorrect response) but do not spontaneously apply a strategy of unpacking learning goals to plan for, or evaluate, teaching and learning.

Overall, there is little research that has been done on prospective teachers' thinking process at the undergraduate level. Prior research in this area also informed this study that it is imperative to design explicit course activities that provide opportunities for teacher candidates to think, reflect, and practice in the situations similar to what most teachers are doing in their classrooms. It is expected that the present study will contribute to this line of research efforts by providing prospective teachers the opportunity to think like teachers and to continuously craft their teaching practice.

Situating the Study

Informed by the results and suggestions from prior research, the present study identifies the following needs for further investigation:

- (a) To probe prospective teachers' views and understanding of the conceptual/procedural framework;
- (b) To further investigate how prospective teachers' beliefs impact their potential teach-

- ing practice; and
- (c) To provide explicit opportunities for prospective teachers to think and act in contexts that are similar to teachers' daily lives.

Building on earlier work, this study explores how a group of prospective teachers acknowledge and interpret the conceptual/procedural framework via two-phase tasks that encourage them to view the context from the teacher's perspective. Therefore, this study's approach was designed to stimulate more active views from future teachers, beyond their previous role as learners. Specifically, this study examines the following questions:

1. In relation to the conceptual/procedural framework, how do prospective teachers interpret the nature of the mathematics examples provided?
2. What types of teaching strategies do prospective teachers propose as intervention when student errors are made on the given mathematics examples?

This study notes that teaching is not black and white, but rather a complex process, and that each individual teacher has his or her unique circumstances. The purpose of this study is not to suggest a definite, exemplary model of what teachers should think and do. Rather, it is to gain insight into why prospective teachers think or act in certain ways and what factors influence their instructional decisions. Several participants had the chance to actually implement their proposed instructional strategies in their field setting, to interpret their classroom activities, and to reset their hypothetical learning trajectories. However, this process was not required for all participants. Thus, the focus of this study will be on a snapshot of the participants' ways of thinking as a whole.

METHODS

Participants

This study involved 54 undergraduate prospective teachers enrolled in two sections of a K–8 mathematics methods course at a Midwestern university in the United States. The author of this study was the instructor for the two sections. All participants had successfully completed mathematics content courses prior to this methods course. For most of the participants, it was their final semester before student teaching.

Tasks and Data Collection

There are two impetuses for the design of tasks. First, it grew from the author's interest in unwrapping elementary prospective teachers' stance on teaching and learning mathematics in general, and its embodiment in more specific contexts. Through personal teach-

ing experience, the author noted that many prospective teachers frequently address the importance of “teaching for understanding” in various course assignments and class discussions to express desirable teaching practices. However, it was uncertain whether these prospective teachers’ opinions were internalized thoughts or merely iterations of what they kept hearing from external authority figures. Second, considering the limited amount of time given to the course, it is unrealistic to cover all detailed teaching strategies for the nine years of mathematics curriculum (grades K-8). Thus, it is evident that prospective teachers need to develop their ability to expand their prior knowledge when they encounter new, unfamiliar situations, especially when they unexpectedly face students’ incorrect responses. With these thoughts in mind, a series of tasks were designed to uncover participants’ perceptions of the nature of sample mathematics concepts, their subsequent teaching strategies for the same concepts, and the level of accordance between their initial perceptions and teaching strategies.

Prior to the present study, the investigator prepared a questionnaire containing 5 sample students’ errors that were commonly observed in elementary and middle school levels. The participants engaged in two tasks:

- (1) Individual ratings of the procedural versus conceptual nature of the given examples, and
- (2) Developing and sharing their potential intervention strategies.

Task 1. Participants were asked to indicate their judgment on the nature of the following examples on the continuum of a 5-point scale:

1) $\frac{2}{3} \div \frac{3}{5}$

2) $10 - 2 \times 3$

3) $\frac{2}{3} = \frac{x}{9}$

4) $2x - 3 = 7$

5) $0.2 \overline{)3}$

The terms “rules/conventions” and “understanding” were used to highlight participants’ judgments. “Rules/conventions” refer to things about which students need to be informed. Whereas, “understanding” refers to when students need to reference their own prior knowledge in order to discover why they are true. However, it was clarified for the participants that the “rule/convention” versus “understanding” distinction was not absolutely dichotomous. Participants were encouraged to use the scale as a simple tool to represent their level of understanding of the given math examples, which certainly could not be evaluated as right or wrong.

A “1” indicates that participants feel the given example fits very well with their idea of mathematical conventions or rules while a “5” indicates that participants feel the learners should possess mathematical understanding to solve or communicate the concept/skill involved in the given example. They were instructed to use the other numbers to indicate intermediate judgments. Participants were told that this questionnaire had to do with what they have in mind when they reference mathematical concepts and procedures and how they feel regarding the application of these procedures in the classroom. They were also reminded that this kind of judgment has nothing to do with how much they like or how well they understood mathematics. Participants were asked to complete their ratings individually during the first week of semester.

Task 2. After the completion of Task 1, participants engaged in a series of online discussions for two weeks at the beginning of the semester. To begin the online group discussion, the following five samples were given along with students’ incorrect solutions. All of the examples contained the corresponding mathematical concepts that were used in the first task:

$$1) \frac{2}{3} \div \frac{3}{5} = \frac{2}{5}$$

(The student canceled the numerator 3 in the second fraction and denominator 3 in the first fraction.)

$$2) 10 - 2 \times 3 = 8 \times 3 = 24$$

(The student calculated from left to right, in order.)

$$3) \frac{2}{3} = \frac{x}{9}$$

$$2x = 27$$

$$x = 13.5$$

(The student multiplied denominators and numerators.)

$$4) 2x - 3 = 7$$

$$2x = 7 - 3$$

$$2x = 4$$

$$x = 4 \div 2$$

$$x = 2$$

(The student moved -3 to the other side to get 2x alone and then solved the problem.)

$$5) 0.2 \overline{)3} \rightarrow 2 \overline{)3} \begin{array}{r} 1.5 \\ 3 \end{array}$$

(The student moved the decimal point to the right to make the divisor a whole number

and then solved the problem.)

The sample students' incorrect solutions are common error patterns frequently encountered in classrooms (e.g., Ashlock, 1994). A total of 10 groups consisting of 4-5 participants discussed the given examples using an online discussion tool (Moodle Discussion Forum) for approximately two weeks. The prospective teachers were required to post at least one personal intervention plan and responded to at least one group member's posting for each sample. The instructor occasionally contributed in an effort to facilitate the group discussion or encourage participants to articulate their statements. The instructor merely observed the conversation and did not interject beyond the nature of these posts. All students' online postings were collected as data.

Data Analysis

Both quantitative data from Task 1 and the qualitative data from Task 2 were analyzed on multiple levels.

Analysis of quantitative data from Task 1. The mean score for each participant's numeric ratings on the five examples was calculated. Each participant's individual mean score was used as a global representation of his or her overall stance on how the given examples should be taught in mathematics classrooms.

Analysis of qualitative data from Task 2. Qualitative data obtained from the participants' online postings were analyzed following aspects of a double-coding procedure suggested by Miles & Huberman (1994). Initially, the researcher and a research assistant reviewed all entries and identified emerging themes, independently focusing on significant features revealed by the participants. Then, the themes each person identified were compared and combined to determine the final coding scheme, as shown in Table 1. The coding process focused on whether the specific theme was present or absent in each participant's posting. Since some participants addressed multiple thoughts/strategies, it was possible that one posting could be coded into multiple categories. The researcher and another research assistant coded the data from Task 2 independently. The interrater reliability was calculated as the number of agreements divided by the number of items coded. After the first round of coding, the agreement between two coders reached 82%. The discrepancies between two coders were mostly found in categories C and D. These disagreements were discussed and resolved. For example, a participant suggested an online-based game. One coder interpreted it as a representation tool while the other coder interpreted it as the theme of informing process. Two coders reviewed the online game and found that although the game had many pictorial, manipulative features, it was merely a device to inform the steps to follow.

Table 1. Coding Scheme

Code	Emerging Category	Description	Examples
A	Informing Fact	<ul style="list-style-type: none"> • Suggest to teach the example as a set of rules to memorize • No attempt to explain underlying concepts/background concepts and their relationships 	<ul style="list-style-type: none"> • Memorize • Practice • Repeat • Rule • Convention • Charts/posters as a reminder
B	Informing Process	<ul style="list-style-type: none"> • Suggest various tricks to help retrieve the process (non-mathematical) 	<ul style="list-style-type: none"> • Various mnemonics (e.g., PEMDAS) • Various phrases (e.g., “cross-multiply”, “do the same thing on the other side”, “add one more zero”)
C	Suggesting Representational Tools	<ul style="list-style-type: none"> • Suggest various modes of representational tools to unpack the concepts 	<ul style="list-style-type: none"> • Pictorial representations (e.g., number line, area model, diagrams) • Math stories (e.g., various contexts) • Manipulatives
D	Providing Reasons/Justifications	<ul style="list-style-type: none"> • Fully explaining underlying concepts/background concepts and their relationships 	<ul style="list-style-type: none"> • Equivalency • Place value • Regrouping • Part/whole relationship
E	Generic Statement	<ul style="list-style-type: none"> • Merely state the importance of some aspect of teaching (e.g., understanding) without any specific explanations 	<ul style="list-style-type: none"> • “It is important to help students understand the concept” (no additional explanation)
F	No Confidence	<ul style="list-style-type: none"> • Could not suggest any strategies • Admit that they are not strong in the subject 	<ul style="list-style-type: none"> • “Not sure what to do” • “Do not know” • “Not good at [math]”

RESULTS

Analysis of quantitative data from Task 1

The overall mean scores for each participant’s numeric ratings on five examples as a global representation of the individual participant’s view ranged from 2.5 to 5.0. Table 2 depicts the distribution of individual overall mean scores among participants. This result indicates that the participants’ perceptions are generally geared toward putting emphasis on the conceptual understanding aspect rather than the procedural aspect.

Table 2. Distribution of individual overall mean scores

Range of individual mean score (m)*	Number of participants (frequency)
$1 \leq m < 2$	0
$2 \leq m < 3$	10
$3 \leq m < 4$	31
$4 \leq m < 5$	13

Note: *Based on a 5-point scale, as explained in the description of Task 1 above.

Analysis of qualitative data from Task 2

Table 3 shows the percentage of categories appearing in participants' teaching strategy discussion on each example. For example, 81.5% of postings regarding Example 1 used informing facts or rules as a way of assisting students' that had errors in relation to the division of fractions. Although there are some differences by example, themes A and B (informing facts and procedures) were more frequently incorporated in participants' teaching strategies than themes C and D (suggesting representational tools and providing justifications).

Table 3. Analysis of qualitative data from Task 1 (proposed teaching strategies)

Categories	Sample number				
	1	2	3	4	5
A: Informing Fact	81.5 %	65.9 %	85.7 %	91.7 %	57.1 %
B: Informing Process	66.7 %	63.4 %	53.6 %	75.0 %	57.1 %
C: Suggesting Representational Tools	51.9 %	31.7 %	32.1 %	33.3 %	21.4 %
D: Providing Reasons/Justifications	33.3 %	31.7 %	14.3 %	61.1 %	42.9 %
E: Generic Statement	29.6 %	61.0 %	53.6 %	30.6 %	42.9 %
F: No Confidence	7.4 %	4.9 %	7.1 %	2.9 %	35.7 %

Examples of Participants' Responses

The data analyzed above depict an overall snapshot of this group's perceptions as a whole. The following examples of several individual responses from Tasks 1 and 2 provide some insight into the characteristics revealed by individual participants (see Table 4). All of these examples are related to the discussion on the third question in Tasks 1 and 2:

Task 1: Rate the mathematical nature of this example on a 5-point scale

$$\frac{2}{3} = \frac{x}{9}$$

Task 2: How can you help the student correct the error?

$$\frac{2}{3} = \frac{x}{9}$$

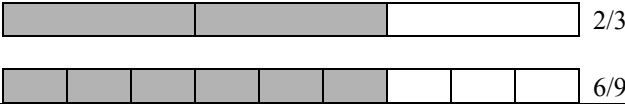
$$2x = 27$$

$$x = 13.5$$

(The student multiplies denominators and numerators.)

Table 4. Examples of Participants' Responses*

Participant	Rating in Task 1	Excerpts from responses in Task 2
A	4	"I will explain that the student needs to use the cross-multiply method. To cross multiply is to go from this (a/b=c/d) to this (ad=bc)."
B	3	"A student's first instinct is going to be to read something from left to right. So I think teachers need to come up with some kind of catchy/engaging strategy to get students to remember the cross-multiply method.... Maybe, if students used a highlighter to show which numbers they will be switching, this might get them to remember Also, letting them draw arrows between the numbers will help them see which numbers they are switching. Both of these strategies are visual helpers."
C	3	"Using a visual can help students remember which numbers need to be multiplied together.... You can have students make a big multiplication sign (X) through the equal sign. The lines of the X connect the numbers that need to be multiplied together."
D	4	In response to Participants B & C: "I agree that some visuals (e.g., X, different colors, arrows) would be helpful. However, I think it is important for students to see why cross-multiplying works. I would explain like this. $a/b=c/d$, $(a/b)b=(c/d)b$, $a=(bc)/d$, $ad=(bc/d)d$, $ad=bc$."
E	3	"I could display this [cross-multiply rule] in my classroom to remind students how to cross-multiply.... It is important to tell your students to write down every step they do.... The student in this example needs to understand when to use this concept Giving some extra practice after the teacher initially explains this rule would be helpful."
F	3	"I don't know why we cross-multiply... I remember learning this in school, but I don't remember how it was taught.... Why wouldn't we just multiply each side by 9 to leave the x alone – just like the same way we solve equations : $2/3 = x/9$, $(2 \times 9)/3 = x$, $6 = x$. Does it matter if I show it in this way without using the word 'cross-multiply'?"

G	2	"I would begin <i>explaining</i> to the students that they first need to cross multiply. ($2/3 = X/9$, Do 2×9 and $3 \times X$, this would give you $3X=18$, They to get X by itself, divide both sides by 3. $X=6$, Check: $2/3 = 6/9$)
H	4	"I would show students various examples of <i>equivalent fractions</i> , so they can see that the cross-multiply method always works with equivalent fractions."
I	5	"Cross-multiplying is difficult for students because, if they miss any step, the problem is wrong....I would show all <i>step-by-step procedures</i> [cross-multiply].It will help the student <i>visualize the process</i> ."
J	4	"I really don't think that it is sufficient to just tell a student that they were supposed to cross-multiply.Many students do not even know why we cross-multiply...I also looked for videos on YouTube, but none of them showed the reasoning behind it.... I might just draw those fractions and show students that they must be <i>equivalent</i> ." 
K	5	In response to Participant J: "I like your <i>visual</i> representation.I think the <i>fraction bars</i> or <i>fraction circles</i> can be used to show the <i>equivalency</i> ."

Note: * Italic/bold added.

As shown in the above sample responses, participants' ratings in Task 1 do not accurately predict the nature of teaching strategies they would potentially use in teaching. Participants exhibited inconsistency between what they believe and what they plan to do. The general tendency was for the majority of participants to rate high in Task 1, but to propose very rule-based, procedure-oriented strategies.

It was also noted that there exists a wide range of variety in using several terms among the participants. For example, in Table 4, Participants I and K both addressed the use of "visual" representation as part of their potential teaching strategies. However, Participant I's "visual" is more likely a tool for memorization, whereas Participant K's "visual" is a tool to explain the equivalency of two fractions. The same is true for the term, "explanation". Some participants (e.g., Participants A and E) use this term synonymously with informing while Participant D uses it as a way to prove the given mathematical statements in relation to students' prior knowledge.

DISCUSSION

The results of participants' ratings on given mathematical examples and their potential teaching strategies provide some insight into in-depth understanding of current mathematics teacher preparation and implications for future research. In this section, several aspects

are further discussed.

Conceptual/ Procedural Framework: Manifested Level

Barlow & Reddish (2006) claimed that various mathematical myths remained the same despite changing educational standards over 15 years, and in particular, the view of mathematics as a set of rules, procedures, or facts was constantly appearing regardless of the time period. Unlike Barlow and Reddish's study, these participants demonstrated their understanding of the given math examples as conceptual rather than procedural (as shown in Task 1). In this regard, the participants in this study affirmed that they held similar instructional beliefs that are aligned with the philosophy of current mathematics education reform, as shown in the case of science education research (e.g., Levitt, 2001; Ogan-Bekiroglu & Akkoc, 2009). While this study abstains from drawing an evaluative conclusion from these results, it is noted that, at least at its manifested level, the general perceptions of the participants of this study have evolved as these new perceptions emphasize the conceptual understanding of mathematics questions.

Conceptual-Procedural Framework: Behind the Scene

In contrast to the participants' manifested beliefs shown in Task 1, the teaching strategies proposed by participants in Task 2 revealed different layers of participants' perceptions. Despite the tendency to emphasize the conceptual nature of the given example, participants' potential instructional emphases, as reflected in their proposed teaching strategies, were geared toward informing facts and procedures. Moreover, the participants' sample responses, shown in Table 4, demonstrated that the associations between participants' responses to Task 1 (rating examples) and Task 2 (types of potential instructional strategies proposed) did not go hand-in-hand. This discrepant result is similar to what previous research revealed from in-service teachers' thinking and practice. This deviation originated from many factors such as school culture, curriculum mandates, class sizes, time restraints, etc. (Hart, 2002; Valderrama-Aguelo, Clarke & Bishop, 2007). Considering the absence of these types of external factors in the context of this study, the results tell us that there should be other significant factors that affect the inconsistency teachers or prospective teachers exhibit. It also indicates that the formation of this inconsistency may occur before prospective teachers faced the external factors.

Implications for Mathematics Teacher Preparation: Noted challenges and Possibilities

Noting the incongruous relationship between participants' beliefs and their potential teaching strategies, this study identifies several challenges that will help to refine the

goals of teacher education programs and future teacher education research.

As the first, most fundamental implication, there is a need to clarify how the conceptual/procedural paradigm is presented and referred to in the mathematics teacher education program. The inconsistency between participants' ratings in Task 1 and the instructional strategies shown in Task 2 informed the absence of an internalization process about the idea of conceptual understanding. We often say that teachers teach the way they were taught when we criticize their unsatisfactory traditional teaching methods. The same situation could happen in the teacher education program. It is possible that participants emphasized the aspect of conceptual understanding simply because it was drawn from a phrase they continuously heard in the teacher education program. In other words, it could be nothing more than a memorized idea from various teacher education courses. Although many teacher educators agree conceptual and procedural knowledge should be taught and acquired in tandem rather than independently (e.g., Rittle-Johnson, Siegler & Alibali, 2001; Star, 2000), it is uncertain how this message is delivered to teacher candidates. Given this situation, one immediate challenge for the current mathematics teacher education program is to unpack the conceptual/procedural frame by providing teacher candidates with specific activities, examples, and the explicit opportunities to engage in reflections in an effort to elicit their thinking process and have personally meaningful experiences.

Second, there should be deliberate opportunities to examine the justifications behind teachers' or teacher candidates' instructional decisions. Participants in this study demonstrated a visible tendency to look for tricks or gimmicks from easily accessible resources. This implied that most participants positioned themselves as individuals who are responsible for conveying the correct way to solve the given examples. It is not to say that certain resources or strategies are better than the others. The challenge for teacher educators is to help teacher candidates to extract justifications behind their instructional decisions rather than the materials themselves. This study attempted to provoke this justification process by utilizing the collaborative online discussion forums, anticipating that this format would encourage participants to share their thoughts on the instructional decisions they proposed. However, when the format of discussion was open to the groups, the justification process was not clearly visible. In this study, the instructor was involved only in the initial stage of group discussion to minimize the influence of the authority figure and to let participants' thoughts flow freely. A more-structured format of discussion will be beneficial to refine teacher candidates' justifications on their instructional decision based on feedback the instructor/research had from the results of this study. It would have been more productive for participants to act as critics early on although toward the end participants stepped into that role. The challenge in creating this kind of sharing environment is maintaining a careful balance as Chazan & Ball (1999) stated using an analogy of "intellectual ferment":

Mere sharing of ideas does not necessarily generate learning. For a discussion to be productive of learning, different ideas need to be in play, the air filled with a kind of 'intellectual ferment' in which ideas bubble and effervesce.... It can be accelerated by the presence of catalysts. Disagreement – the awareness of the presence of alternative ideas – can be an important catalyst.... However, fermentation requires a delicate balance: for example, too much heat will kill yeast. Similarly, though disagreement can be a catalyst, it can also shut discussions down. Students' disagreements can lead to confrontation rather than learning (p. 7–8).

This study implied that the intellectual ferment could have many different layers. For example, some participants revealed rule-oriented perceptions because that was the way they were taught or because they did not have a thorough understanding. In contrast, a participant perceived that some examples needed to be taught as a set of rules even though she exhibited sound background knowledge during the discussion. The challenge teacher educators face is creating the optimal intellectual ferment for each teacher candidate by placing more emphasis on their thinking process rather than the end results.

Third, it will be beneficial to introduce teacher candidates to the plausible contexts that teachers see daily in their classroom much earlier, prior to working with real students. As mentioned before, this study showed that the gap between what teachers believe and what they actually do exists, not only in the in-service teacher community but also in the teacher candidate population. It signifies that there must be fundamental obstacles that in-service teachers encounter. Thus, this issue must be addressed much earlier in the teacher preparation process so that teacher candidates can better understand these situations and get involved in the active reflection process. This study supports the recent research efforts presented in the aforementioned research studies (e.g., Hiebert et al., 2007; Morris & Hiebert, 2009; Timmerman, 2004) in the sense that more effective teacher preparation can be accomplished in realistically supportive contexts. This study encouraged participants to develop their hypothetical learning trajectories to remediate the given examples. Although this study was not able to document the subsequent transformation process participants would have over time, it did provide some guidance for further investigations.

CONCLUDING REMARKS

The study presented here indicates that the gap between knowing and doing is not an issue limited to in-service teachers' professional development. This study showed that the current teacher candidates already appear to exhibit the same kind of gap. The participants' dominant mathematics teaching beliefs, which placed emphasis on conceptual learning, were promising. It is unclear whether their beliefs are the results of the imposed instruction by teacher educators or their internalization process of examining the gap between

their manifested beliefs and instructional strategies. While many research studies focus on changing teachers' beliefs, this study suggests that the goal of the teacher education program is for teacher candidates to experience the decision-making process when they encounter unfamiliar teaching situations. This goal allows the candidates to elaborate on what they really think, why they believe this, and what they plan to do without worrying about what others say. It is believed that getting a snapshot of teacher candidates' current status is a first step toward more in-depth research efforts for tracking their thinking process. Currently, a follow-up study is being undertaken in this direction and a focus group was formed from the participants of this study. The members of the focus group are presently in the student-teaching stage and spending their whole semester in K–8 classroom settings. This focus group is continuously discussing the given examples and refining their initial instructional strategies. In this follow-up study, participants are now thinking about their instructional strategies in an actual setting, rather than working in a plausible context. They are positioning themselves more closely as classroom teachers and working through full rounds of the mathematics teaching cycle by refining their hypothetical learning trajectories, teaching, and assessing. This study will show a more detailed transformation of teacher candidates' thinking that impacted their instructional decisions. Hopefully, teacher educators will use the results of this study to design and implement course activities that accommodate teacher candidates' thinking process.

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